# Vertical tunneling between two quantum dots in a transverse magnetic field

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Tunneling between two quantum dots is studied at low temperatures. The quantum dots are formed by the combined sidewall confinement and vertical confinement in an $Al_xGa_{1-x}As$-GaAs triple-barrier diode with a conducting diameter of 180 nm. The fine structure that is observed in the main resonance peaks of the current-voltage characteristics is related to lateral quantization effects. Electrons tunnel between zero-dimensional (0D) states in the two coupled quantum dots. A magnetic field applied perpendicular (transverse) to the tunneling direction shifts the main (2D) resonance peaks to higher bias and causes a substantial broadening. Within the fine structure we find that the resonance positions are virtually magnetic-field independent, whereas the resonance amplitudes show significant variations with increasing magnetic field; a simple model is developed to describe this behavior in terms of the magnetic-field dependence of the interdot transition probabilities.

Tunneling is a powerful tool with which to investigate the electronic properties of artificially fabricated, three-dimensional quantum boxes ("quantum dots") in semiconductor nanostructures. Devices suited to studying vertical tunneling through quantum dots have been realized in $Al_xGa_{1-x}As$-GaAs double- or triple-barrier diodes (DBD's or TBD's) with diameters between 0.1–1 μm.[1–6,8–10] The main resonance structure in the current-voltage ($I$-$V$) characteristics is a manifestation of the barrier-induced (i.e., vertical) quantum confinement. Additional fine structure which appears in the $I$-$V$ resonances of small diameter DBD's (TBD's) at low temperatures has been attributed to the "in plane," or lateral, quantization of the electron motion in the quantum dot. However, due to the limitations of nanostructure processing, the lateral confinement is much weaker than the vertical confinement. Although there is currently a lively debate as to the origin of the lateral confinement potential—surface depletion from the diode sidewalls,[1–4] single impurity states,[5] and random potential fluctuations[6] have been discussed—there seems to be a consensus of opinion that the observed fine structure is at least partly due to quantization. Additional effects such as single electron charging[2,4,7] and mode-mixing transitions[8] complicate the tunneling current characteristics.

In this paper, we report tunneling between *two* quantum dots separated by a potential barrier, with a high magnetic field applied perpendicular (i.e., transverse) to the tunneling current. The transverse magnetic field strongly affects the transition probabilities between the discrete (0D) states in both dots, and the study and modeling of this effect leads to a better understanding of the electronic properties of quantum dots.

The system of two coupled quantum dots investigated here was realized using an $Al_xGa_{1-x}As$-GaAs triple-barrier heterostructure with a diameter of 400 nm.[1,9,10] The heterostructure was grown by molecular-beam epitaxy on top of an $n^+$ (001) GaAs substrate. The layer sequence is as follows [see Fig. 1(a)]: (i) 0.5-μm $n^+$-type GaAs bottom contact layer (doped to $1\times10^{18}$ $cm^{-3}$), (ii) 5-nm GaAs spacer layer, (iii) 5-nm $Al_{0.32}Ga_{0.68}As$ (bottom) barrier, (iv) 7-nm GaAs well, (v) 7-nm $Al_{0.32}Ga_{0.68}As$ (middle) barrier, (vi) 5-nm GaAs well, (vii) 5-nm $Al_{0.32}Ga_{0.68}As$ (top) barrier, (viii) 5-nm GaAs spacer, and (ix) 0.3-μm $n^+$-GaAs top contact layer. Sil-

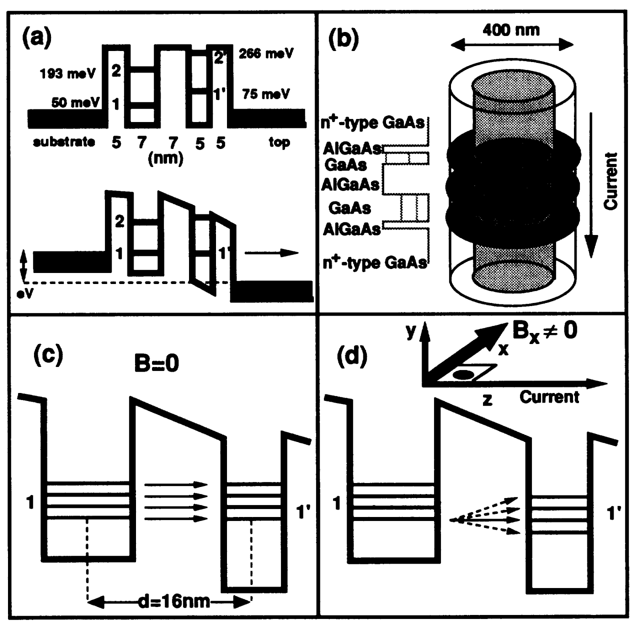


FIG. 1. (a) Potential profile of the asymmetric triple-barrier heterostructure. In large area diodes, 2D subbands $s$ ($t'$) are formed in the bottom (top) quantum well at the energies indicated. Upon matching the energies with an applied bias $V$, the subbands $s$ and $t'$ are in resonance, and enhanced current is observed in the $I$-$V$ characteristics. (b) Schematic sketch of the double quantum dot structure. Lateral sidewall confinement defines 0D electronic states between the barriers. (c) Tunneling between the 0D states $j,m,n$ and $j',m',n'$ will conserve quantum number. (d) A transverse magnetic field causes some of the electrons to change quantum number during tunneling ($j\neq j'$ and/or $m\neq m'$).

 

icon was used as $n$-type dopant. The layers (ii)–(viii) are all undoped. Diodes with diameters between 2 $\mu$m and 400 nm were processed from the heterostructure using electron-beam lithography and metalorganic reactive ion etching techniques.[2] The samples were measured in a dilution refrigerator operating at a base temperature of 50 mK.

We will first discuss data obtained for large area diodes [Fig. 2(a)]. In asymmetric triple-barrier diodes, two-dimensional (2D) subbands with different vertical confinement energies are formed in the quantum wells. When the bias voltage in the contacts is such that subband $s$ in the bottom quantum well and subband $t'$ in the top well match in energy, the tunneling current is enhanced and exhibits a (2D) resonance peak.[11,12] These 2D resonances are evident in the $I$-$V$ characteristics of the 2-$\mu$m diameter diode at $B=0$, as shown in Fig. 2(a). The resonances have been identified earlier[12] as the 1-1′, 1-2′, and 1-3′ transitions in forward bias, and as the 1′-2 and 1′-3 transitions in reverse bias. In a transverse magnetic field the 2D-resonance peaks broaden and shift to higher bias, an effect that has also been observed in tunneling through double-barrier heterostructures.[13] The shifts can be explained in terms of the change of electron momentum $k_y$ (which is oriented parallel to the barriers) by the amount $\Delta k_y = eBd/\hbar$, due to the circular motion of the electron in a magnetic field $B$ while tunneling through the center barrier with thickness $d$. The change $\Delta k_y$ shifts the 2D-resonance bias by $\Delta V = (\beta/e)[\hbar^2 k_y^2 - \hbar^2 (k_y + \Delta k_y)^2]/2m^*$ (where the conversion factor $\beta$ between the shift of resonance bias $\Delta V$ and energy $\Delta E$ in the quantum well was determined to be $\beta = 2.3$ in Ref. 12, using parallel magnetotunneling data on the same heterostructure). Between $B=0$ and 15 T, the 1-1′ resonance shifts by $\Delta V = 0.18$ V. From the formulas for $\Delta k_y$ and $\Delta V$ we can estimate the effective separation of the charge distributions between the wells as $d' \approx 16$ nm. This value exceeds the barrier width, because charges are located in the centers of the wells.

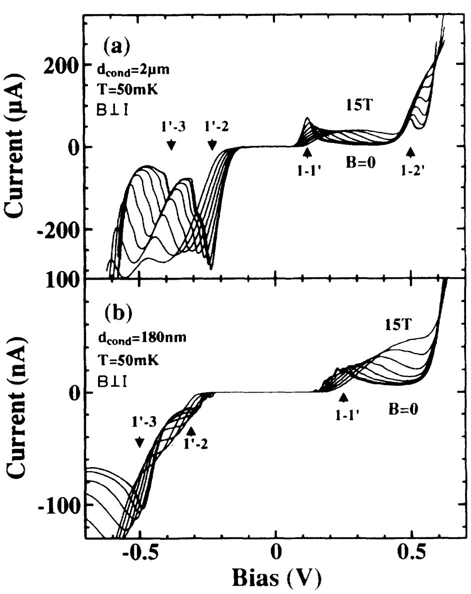


FIG. 2. (a) $I$-$V$ characteristics taken at $T \approx 50$ mK of a triple-barrier diode with diameter 2 $\mu$m, as a function of transverse magnetic field. (b) A similar $I$-$V$ plot for a 400-nm-diameter triple-barrier diode ($d_{\rm cond} = 180$ nm).

In small diameter diodes, see Fig. 1(b), electrons are also laterally confined and two quantum dots with 0D states are formed between the barriers. Tunneling between 0D states in each dot can be observed when the thermal broadening $k_B T$ is smaller than the spacing of the 0D states. Transitions between 0D states are only possible if the 0D energies coincide, leading to sharp resonance peaks in the $I$-$V$.[9] Figure 2(b) shows the $I$-$V$ characteristics at $T \approx 50$ mK of a 400-nm-diameter triple-barrier diode fabricated from the same heterostructure shown in Fig. 1(a). Resonance peaks corresponding to the 1-1′, 1′-2, 1′-3, and 1-2′ (not shown) transitions in the larger diameter diode can still be observed, with a small shift to higher biases. The low bias peak amplitudes (1′-2 and 1-1′) are now much smaller than the corresponding high bias peaks (1′-3 and 1-2′). This decrease of low bias peak amplitudes has been reported previously,[9] and can be assigned to charging effects. We have therefore used the high bias peak currents (1-2′ and 1′-3 resonances) to estimate the effective diode diameter as $d_{\rm cond} = 180$ nm, by extrapolating from the peak currents of larger area diodes.[14] The interesting feature in Fig. 2(b) is the fine structure that has appeared in the low bias resonances (1-1′ and 1′-2). The structure can be seen more clearly in Fig. 3, where only the 1-1′ peak (forward bias) is plotted. Previously, the resonancelike fine structure has been assigned to tunneling between discrete (0D) quantum dot states.[9] However in Fig. 3 the individual peaks are more smeared than the structure observed in smaller diameter diodes in Ref. 9. This may indicate that 0D-0D transitions are not resolved because they are too closely spaced, and that only the envelope shape of the resonance sequence can be observed.[15] Smearing of the fine structure makes it difficult to determine the spacing of 0D levels and therefore the lateral confinement mechanism (i.e., surface depletion or potential fluctuations) could not be identified in this diode.

In the small area diodes, increasing the transverse magnetic field leads to a shift to higher bias of the 1-1′, 1′-2, and 1′-3 resonances as well as substantial broadening of the peaks, similar to the behavior observed in the large

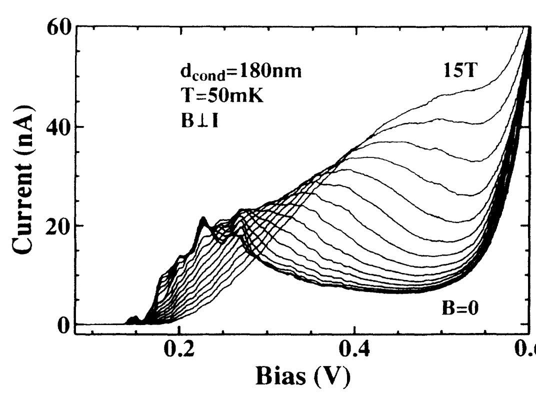


FIG. 3. Expanded view of the 1-1′ $I$-$V$ resonance of the 180-nm-diameter triple-barrier diode, as a function of transverse magnetic field between 0 and 15 T.

area diodes. However, Fig. 3 shows that in a transverse magnetic field the *fine structure* does not shift significantly in bias or broaden, but the amplitude is sensitive to magnetic field. This indicates that the resonant energies for tunneling between 0D states do not depend on magnetic field, whereas the transition probabilities between 0D states do.

At high magnetic fields, i.e., when the 1-1′ resonance peaks at $\approx 0.4$ V, the area under the peak has increased, an effect that is not so evident in the large area diodes. Increased resonance peaks can be assigned to increased charging of the first quantum dot, because the current is proportional to the charge in the first dot (only the first dot will charge up strongly with bias, because electrons accumulate before the thick center barrier). Note also that new fine structure, which was not visible at $B=0$, can be observed at $V \approx 0.5$ V at higher magnetic fields.

We shall investigate theoretically whether the observed fine structure, which is sensitive to the magnetic field only in its amplitude variation, can be explained by a model that allows for lateral quantum confinement (neglecting charging effects). Let us assume that electrons tunnel between two disk-shaped quantum dots, separated by a barrier in the $z$ direction, as depicted in Figs. 1(c) and 1(d). Electrons tunnel from 0D levels in the left-hand quantum dot [quantum number $s=(j,m,n)$] into 0D levels in the right-hand dot [quantum number $t'=(j',m',n')$]. The center barrier of the heterostructure is much thicker than the two outer barriers and limits the tunnel current through the structure. The Hamiltonian for each quantum dot in the presence of a transverse magnetic field $B=(B,0,0)$ is

$$H=(\mathbf{p}+e\,\mathbf{A})^2/2m^*+V_{\text{conf}}(x,y,z)\ ,$$

with the vector potential $\mathbf{A}=(0,-zB,0)$ and the three-dimensional confining potential of the quantum dot $V_{\text{conf}}(x,y,z)$. First-order perturbation theory in $B$ relates the wave functions for each quantum dot state $(j,m,n)$ in the presence of a transverse magnetic field $B$ to those at $B=0$. The quantum dot eigenenergies are then increased by the diamagnetic shift term $\Delta E_{j,m,n}=(e^2B^2/2m^*)(\langle z^2\rangle-\langle z\rangle^2)$. We estimate this shift for a well width of 5–7 nm at $B=10$ T to be smaller than 0.5 meV.[16] The shift is too small to be resolved in experiment, in agreement with our observations. We note that first-order perturbation theory may not be valid if the magnetic length $l=\sqrt{\hbar/eB}$ is smaller than $d$ which is already the case for magnetic fields exceeding $B=2.5$ T. However, the model using first-order perturbation theory will still provide a good qualitative understanding of the physics involved.

Having examined the shift of the fine structure in transverse magnetic field, we now proceed to consider the resonance amplitudes. The tunnel current between 0D states can be evaluated using Bardeen's transfer Hamiltonian method. The current $I \propto |T_{12}|^2$ is proportional to the square of the matrix element

$$T_{12}=\hbar^2/2m^*\int\int\{\psi_1^*(x,y,z)[\partial\psi_2(x,y,z)/\partial z] -[\partial\psi_1^*(x,y,z)/\partial z] \times\psi_2(x,y,z)\}dx\,dy \qquad (1)$$

at all energies for tunneling (i.e., $E_{j,m,n}=E_{j',m',n'}+eV_{sd}$), where $\psi_1(x,y,z)$ and $\psi_2(x,y,z)$ are the perturbed wave functions of quantum dots 1 and 2, respectively. The surface integral is evaluated at constant $z$ anywhere inside the barrier.

The matrix elements can be calculated explicitly if the confinement potential, and thus the wave functions, are known. We can separate $T_{12}=X_{jj'}Y_{mm'}Z_{nn'}$, where $X_{jj'}$ and $Z_{nn'}$ are independent of transverse magnetic field $B$. All dependence of the tunnel current on transverse magnetic field is contained in $Y_{mm'}$ and, in order to calculate it only the $y$ confinement has to be considered. Assuming a parabolic lateral confinement potential $V_{\text{conf}}(x,y)=(\frac{1}{2})m^*\omega_0^*(x^2+y^2)$ in each dot, giving equidistant eigenenergies $E_{jm}=\hbar\omega_0(j+m+1)$, the matrix elements become

$$|T_{12}|^2(m=0,m'=s)=A\alpha^{2s}e^{-\alpha^2/2}/2^s s!\ , \qquad (2)$$

$$|T_{12}|^2(m=1,m'=s) =A(s-\alpha^2/2)^2\alpha^{2(s-1)}e^{-\alpha^2/2}/2^{s-1}s!\ , \qquad (3)$$

with $\alpha=(eBd/\hbar)(m^*\omega_0/\hbar)^{-1/2}$ and $A$ being a numerical constant that depends on the transmission probability of the center barrier, i.e., the effective barrier thickness $d$ and the quantum numbers $j,m,n$ $(j',m',n')$ in dot 1 (dot 2).

Using the matrix elements $T_{12}$ and $I\propto|T_{12}|^2$, Fig. 4 shows the tunnel current through a double quantum dot system calculated for various transverse magnetic fields as a function of bias $V_{sd}$ over the center barrier. The center barrier is 7 nm thick, but the centers of charge are assumed to be separated by $d=16$ nm. The voltage $V_{sd}$ applied over the barrier shifts the 0D states in the two dots with respect to each other by $\Delta E=eV_{sd}$. The voltage is given in units of the lateral quantization energy $\hbar\omega_0$. Transitions are only possible when 0D states match in energy, i.e., when $E_{j,m,n}=E_{j',m',n'}+eV_{sd}$. At $B=0$ there is only one peak evident which results from all mode conserving 0D transitions $j=j'$, $m=m'$ $(n=n'=1)$ occurring at the same bias. The linewidth has been taken to be $\Gamma=0.2$ meV under the assumption that it includes several broadening mechanisms like tunneling ("escaping") through the thinner barriers and scattering processes. Upon increasing $B$ [equivalent to an increase of $\alpha=eBd/\hbar\,(m^*\omega_0/\hbar)^{-1/2}$ to $\alpha=0$, 0.2, 0.4, 0.6, 0.8, 1.0, and 1.2], we observe a decrease of the $m=m'$, $j=j'$ resonance amplitude. Satellite peaks, with shift in multiples of $\hbar\omega_0$ according to mode-mixing transitions ($m\neq m'$ and/or $j\neq j'$) appear. It can clearly be seen that the maximum of the envelope function shifts to higher bias. Assuming $m^*=0.067m_0$ and $d=16$ nm, the magnetic field is related to $\alpha$ by $B(T)=1.85\alpha\sqrt{\hbar\omega_0\ (\text{meV})}$. If we assume $\hbar\omega_0=6.25$ meV, the value $\alpha=1.2$ corresponds to $B\approx 5.5$ T in Fig. 4.

The dependence of the 0D transitions on transverse magnetic field can be explained as follows. If the lateral confinement potential is translationally invariant in the $z$ direction, for $B=0$ the only allowed transitions are those that conserve the lateral quantum number during tunneling. Upon applying a transverse magnetic field $B$, the

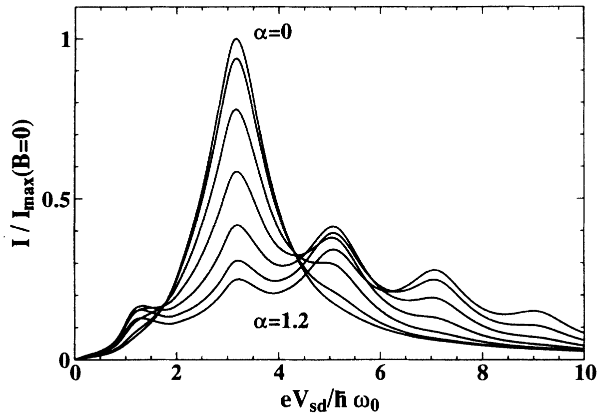


FIG. 4. Calculated *I-V* characteristics of a triple-barrier diode under transverse magnetic field. The model and parameters used are described in the text. The parameter $\alpha$ is related to the transverse magnetic field $B$ and the lateral quantization energy $\hbar\omega_0$ via the relation $\alpha = eBd/h(m^*\omega_0/\hbar)^{-1/2}$. Assuming $d=16$ nm for the structure shown in Fig. 1(a), $B(T)=1.85\alpha\sqrt{\hbar\omega_0}$ (meV).

symmetry is broken and transitions between states with different quantum numbers become possible. The transition probability goes through a maximum for ever higher-order transitions with increasing magnetic field due to the increased $\Delta k_y$ during tunneling. This is in analogy to the quasiclassical model for tunneling between 2D subbands described above in the context of large area diodes. The change of $\Delta k_y$ corresponds to the change in transition probability between different 0D states.

Experiment and theory bear some analogy to resonant tunneling in *tilted* magnetic fields in large area double-barrier diodes.[17] The component $B_{\parallel}$ of the magnetic field parallel to the current creates quantized Landau levels in the emitter and well, comparable to highly degenerate 0D states. The transverse field component $B$ causes tunneling electrons to change Landau-level index during tunneling due to a change of transverse momentum under the Lorentz force. Thus, satellite peaks can be observed, with the amplitude going through a maximum with increasing magnetic field.

Our experimental results in a transverse magnetic field are qualitatively similar to the simulation in Fig. 4. However, at zero magnetic field there is not just one peak ($j=j', m=m'$) evident in the experimental data in Fig. 2(b), but a number of closely spaced resonances. This is not surprising, since (i) the lateral quantization energies $\hbar\omega_0$ are unlikely to be identical in both quantum dots. It is also unlikely that the energy states are equidistant; since the lateral confinement potential will in reality not be parabolic. Therefore, not all transitions will occur at the same bias and a multiple peak structure can be observed. The resonances are so closely spaced that they are not resolved and just the envelope is observed.[15] (ii) A perturbation of the translational invariance of the lateral confinement potential makes mode mixing transitions possible.[8] For reasons (i) and (ii) the model calculation shown in Fig. 4 appears to be an oversimplification. However, it demonstrates nicely the physics governing tunneling between quantum dots in a transverse magnetic field.

It was noted above that electron charging alters the electronic properties of the double-dot system considerably.[18] These effects have been neglected in the model, since they exhibit no first-order magnetic-field dependence.

The experiment does not clarify the question as to which mechanism causes the lateral confinement. This is only possible in diodes small enough so that single resonance peaks can be resolved. Qualitatively, the data could be modeled with the above simplified theory by assuming lateral confinement from the surface depletion potential at the free diode surfaces. However, other models that have been proposed as the origin of the lateral confinement, such as potential fluctuation induced lateral confinement[6] or single impurity states in the well,[5,6] could explain the data as well.

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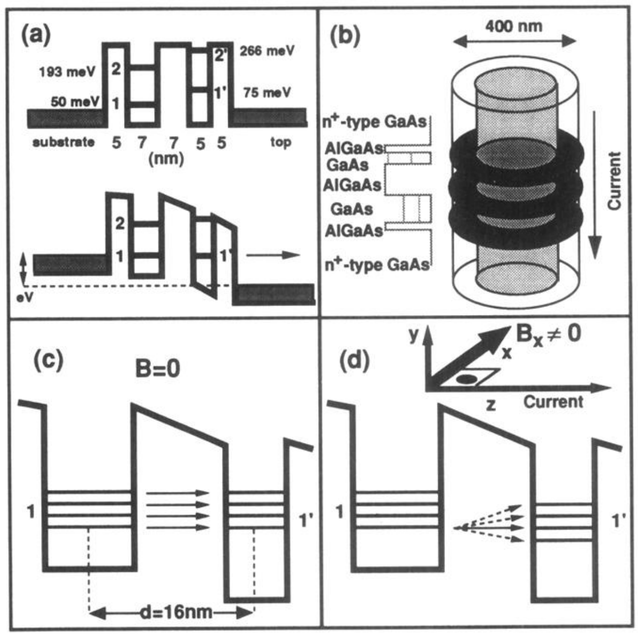


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