Employment Adjustment and Labor Utilization [*]

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Abstract

Standard models that formalize and assess the impact of labor adjustment costs on labor demand suppose that firms can only change the size of their workforce (the extensive margin) and not the number of hours of their existing employees (the intensive margin). I relax this assumption and propose a dynamic general equilibrium model that introduces labor adjustment on both intensive and extensive margins. I calibrate the model to a matched employer-employee panel of Danish firms. I then simulate two labor market policies aimed at promoting job creation: an introduction of hiring subsidies and a reduction in the official workweek.

Keywords: adjustment costs, labor utilization, worker flows, on-the-job search
JEL Classification: J23, J31, J63, J64, L11

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1 Introduction

Labor adjustment costs – such as recruiting costs, costs of screening and training new employees, layoff notice periods, and mandated severance pay – play a prominent role in determining both the timing and the extent of employment variation in response to shocks. As these costs distort firms's employment decisions, they can be detrimental to the reallocation of labor resources from less to more productive firms, and to economic growth in general.[1] Yet, in the short run, firms can respond to productivity fluctuations by varying labor utilization. The fact that firms use the hours margin of adjustment has long been recognized by policymakers, a recent example of it being the short-time working scheme (Kurzarbeit) introduced by Germany in the latest recession.[2] However, standard economic models that are used to formalize and assess the impact of labor adjustment costs only allow firms to change the number of workers they employ (the extensive margin) and not the amount that each worker works (the intensive margin). This paper relaxes that assumption and proposes a dynamic model, which includes both margins of adjustment, and subsequently tests the model using firm-level data.

In this paper, I develop a general equilibrium search model with joint dynamics of the number of workers and hours per worker. The driving force of the model is idiosyncratic profitability shocks, which firms can accommodate by changing the work hours of their existing employees, instead of (or jointly with) varying the size of their labor force. Hours of work and compensation in the model are determined through a bargaining process using a Stole and Zwiebel (1996) bargaining protocol. The presence of frictions in the labor market means that matching workers with vacant jobs takes time and uses resources. On the other hand, raising hours can be done immediately, although at a cost of higher wages. Hence, the firm faces a trade-off between these two channels of adjustment.

In addition, I allow for endogenous and stochastic quits by incorporating on-the-job search into the model. Job-to-job transitions generate different vacancy filling and attrition rates across firms, alleviating the impact of search frictions on the firm's labor demand. In this setup, not only does each firm posts more vacancies in the event of a positive shock, it also finds it easier to fill its vacancies and to keep its current employees. Existing studies on labor adjustment costs typically assume constant quit rates, and thus make no distinction between net and gross employment changes. Yet, the empirical evidence suggests that, by focusing on net employment growth, we disregard a substantial worker turnover at the firm level (see for instance Burgess, Lane and Stevens (2001) and Davis, Faberman and Haltiwanger (2006)).

There is a by-now extensive literature on the effect of adjustment costs on labor demand (see Hamermesh and Pfann (1996) for a comprehensive survey).[3] Most of these papers are based on partial equilibrium models that solve for the optimal labor demand strategy of a firm in isolation from decisions of workers or other firms. In contrast, the search model in this paper accounts for interactions between firms and workers. For example, an increase in hiring costs induces firms to post fewer vacancies, thus creating

[1] More than half of aggregate productivity growth in the US appears to be driven by reallocation (see for instance Foster, Haltiwanger and Krizan (2006)).

[2] Kurzarbeit scheme has been considered key to why Germany had a relatively low unemployment rate in the latest recession. Under this scheme, employers can cut work hours of their employees thus avoiding layoffs, while the government covers most of the resulting shortfall in the workers' earnings. Not only does this program allow employers to reduce their labor costs in the downturn, it also ensures that firms have their experienced workforce on hand when the economy recovers, thus saving on hiring and recruiting costs.

[3] The literature on labor adjustment costs is also closely related to the investment literature (see among others Caballero and Engel (1991; 1992)) and studies on factor demand in general (see Nadiri and Rosen (1973)). Bond and Van Reenen (2007) survey econometric research on adjustment processes for both capital and labor using micro data. Previous research that accounts for labor utilization in the adjustment cost models includes Caballero, Engel and Haltiwanger (1997) and Cooper and Willis (2009).

a positive externality on other firms by reducing congestion in the market. That in turn mitigates the effect of the rise in hiring costs on aggregate employment. I show that this feedback mechanism turns out to be very important when trying to assess the importance of adjustment costs for employment.

This paper uses a unique matched employer-employee panel of Danish administrative firm data that provides high-frequency detailed information on employment changes and work hours of the firm's employees. Based on these data, I show that firms use variation in hours to mitigate changes in the number of workers. In particular, the growth rate of hours per worker and employment growth are negatively correlated at the firm level; while lagged changes in hours are positively correlated with changes in employment. An adjustment cost model provides a natural framework for explaining these empirical facts. The intuition is as follows: Suppose that it takes time to hire a new worker. Then, in the event of a positive shock to the firm's profitability, the firm raises average hours of work instantaneously, while adjustment in employment is sluggish. As the firm's labor force builds up to its new desired level, average work hours fall. Hence, growth in hours and employment are inversely related.

The model is calibrated to fit the Danish firm data and is successful in capturing the overall features of the data. Under the decreasing returns to scale assumption, the model generates an endogenous firm size distribution that mimics closely the empirical distribution. It matches rich patterns of labor dynamics at the firm level, including average hiring and separation rates, job creation and job destruction, as well as the distribution of firms by net employment growth. On-the-job search is a crucial component that enables the model to capture the data characteristics related to worker and job flows. Finally, the model is capable of generating the negative correlation between the growth rates of employment and hours per worker.

Given the calibrated parameter values, I find that the average cost of hiring a new worker is roughly equal to one week of average wages. This value is low compared to the estimates found for other European countries (see among others Rota (2004), Goux, Maurin and Pauchet (2001), and Kramarz and Michaud (2004)), reflecting the fact that the Danish labor market is very mobile, with worker flow rates averaging around 8% per month. In the numerical simulation, I show that doubled vacancy posting costs raise the unemployment rate from 4.8% to 7.5% and reduce the job-finding rate by about 4 percentage points. Moreover, firms substitute towards the intensive margin increasing average hours of work. I perform the same experiment in a partial equilibrium framework and show that doubling the cost parameters, while keeping the vacancy-filling rates and the quit rates at the same level as in the benchmark case, leads to twice the increase in unemployment. This finding suggests that using partial equilibrium models to evaluate the impact of adjustment costs significantly overestimates their effect on aggregate employment.

Finally, I use the model to simulate two types of policy experiments aimed at fostering job creation: (i) introducing a hiring subsidy, similar to the US Hiring Incentives to Restore Employment (HIRE) Act initiated in May 2010, and (ii) imposing an upper limit on the work hours, an example of which was implemented in France in 2000 when the official workweek was reduced from 39 to 35 hours. A hiring subsidy (in the amount of 30 to 70 percent of the average hiring costs) reduces unemployment, though quantitatively the effect is small. The introduction of a shorter workweek, on the other hand, impacts unemployment to a much greater extent. Even though this policy reduces unemployment at the expense of a loss in total welfare due to the inefficient work hours choice, the welfare loss is quantitatively small, thus favoring the introduction of a maximum hours limit over a hiring subsidy.

In many aspects of the methodology, this paper is linked to standard random search models (see for instance Mortensen and Pissarides (1994)) and more recent work that introduces a theory of multi-worker firms into search models (see for instance Coles and Mortensen (2012), Acemoglu and Hawkins (2010), and Moscarini and Postel-Vinay (2013)).[4] The closest paper to this work is Cooper, Haltiwanger and Willis (2007), which examines variation in hours, employment, and vacancies based on micro and macro data. My model differs from theirs by introducing endogenous quits and using a bargaining process to determine hours and compensation.

The paper proceeds as follows. Section 2 presents empirical evidence on employment and hours adjustment using Danish firm data. Section 3 introduces and describes the model. Section 4 shows the calibration of the model and its fit to the data. Section 5 proceeds to demonstrate the impact of changes in adjustment costs on the unemployment rate and the job-finding rate. It then compares the effects of the two labor market policies on unemployment. Section 6 summarizes the findings and outlines directions for future work. The appendix provides details on the data sources used in this paper, as well as on the numerical simulation procedure.

2 Data

A labor adjustment model predicts that firms would eventually adjust their employment to its optimal level after a sufficient period of time; therefore, the trade-off between the number of workers and average work hours exists primarily in the short run. In order to investigate this relationship, we need to observe changes in firm-level hours and employment on a fairly frequent basis. The empirical analysis in this paper is based on a matched employer-employee panel of Danish firms over the period of 1999-2006 (a full description is contained in Appendix A). The dataset contain information on firm-level employment at a monthly basis. Moreover, high-quality longitudinal links enable me to construct monthly hires and separations for each firm.

A work hours series is constructed using data on firms' mandatory pension contributions that are collected on a quarterly basis. In Denmark, firms are required to pay pension contributions for each employee according to her weekly hours of work: (i) zero contribution is paid if working less than 9 hours a week; (ii) one third is paid if working between 9 and 18 hours; (iii) two thirds are paid if working between 18 and 27 hours; and (iv) the full amount is paid for all employees working more than 27 hours a week. Each firm reports the sum of its total contributions over a quarter. Given this step function, I construct a lower bound measure of total hours at the firm level by taking the left boundary of each 9-hour interval. Then, combining it with the data on employment for a given firm, I compute a quarterly hours per worker series. Hence, the empirical moments below refer to the lower bound measure of average hours, unless stated otherwise.[5]

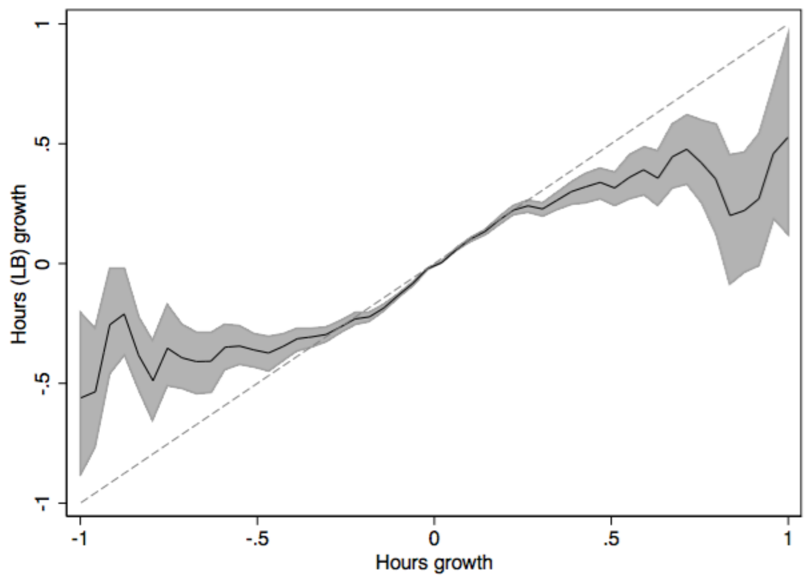
It is important to bear in mind that this measure of labor utilization may mask some of the variation in actual hours. We only observe changes in per-worker hours if an employee moves to a different 9-hour interval. Thus, for instance, if some of a firm's part-time workers start working full time then the

[4] Kaas and Kircher (2011) develop a multi-worker firms model within a competitive search framework that has many similar implications for employment growth at the firm level.

[5] Alternatively, I have constructed an upper bound measure of work hours. Appendix A provides a detailed discussion on the construction of both of these variables. The two measures lead to very similar results for most of the empirical relations presented in this paper, with the exception of the wage-hours relationship, which will be discussed in detail in Section 4.

increase in work hours will be reflected in the data. How well the lower bound measure on hours per worker reflects actual labor utilization can be tested at the annual level using an alternative data source that contains information on actual paid (regular and overtime) work hours for a sample of firms. Figure 1 below compares the annual growth rates of two hours per worker series: the first measure is derived from the pension contributions as described above; while the second measure is the average weekly work hours that are computed by dividing the total amount of hours by the number of employee-weeks in a given year. In general, the two variables move closely together for growth regions below 50% in absolute value.

Figure 1: The relationship between the growth rates of actual and lower bound work hours.



Note: Estimates are based on a uniform kernel with a bandwidth of 0.05. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 2002 - 2006. Sample contains all private firms with more than 10 full-time employees, with the exception of agriculture and fishery.

In addition to work hours and employment records, the data contain information on quarterly payroll costs for each firm over the period of 1999-2006 and purchases and sales records for all VAT-liable businesses over the period of 2002-2006. Wages and value added are measured in Danish Kroner (DKK) and are deflated using a quarterly CPI with 2001 Q1=100. Combining these data with the employment and hours series, I construct hourly wage and labor productivity variables. The empirical analysis is restricted to firms in the private sector.

2.1 Hours and employment

Reallocation of labor from less to more productive firms may well account for a large fraction of both the level and growth rate in aggregate productivity (see among others Foster et al. (2006) for US data and Lentz and Mortensen (2008) for Danish data). The idea of such growth-enhancing reallocation relies on the assumption of the existence of labor market frictions that hinder employment adjustment. Two key questions that this paper addresses are (i) what is the extent to which firms vary work hours of their employees, and (ii) is the observed dynamic interaction between hours and employment consistent with the model of adjustment costs?

I start by looking at the extent of labor reallocation across firms with different productivity. Table 1 contrasts firms across the five quintiles of the productivity distribution in terms of their output and use of labor inputs. More productive firms increase their use of labor resources by both hiring more employees and raising average work hours. Given diminishing returns to labor, more intensive use of labor inputs leads to lower (and eventually negative) future growth rates of productivity. For instance, firms in the top quintile increase their workforce by 4% and their total labor input by 8%; whereas their labor productivity falls by 27%. Firms in the bottom quintile, on the contrary, lose 2.4% of their workers and reduce their total labor resources by 14%, at the same time experiencing a 37% increase in their labor productivity and a 23% increase in output. Overall, the reallocation of labor input is associated with productivity growth and is more prominent when we look at the changes in total labor input as opposed to the changes in the number of workers only.

Table 1: Reallocation of labor resources across firms, by quintiles of the productivity distribution.

	Productivity distribution quintiles				
	1	2	3	4	5
Productivity growth, %	37.2	6.0	-1.4	-8.5	-27.3
Output growth, %	23.1	-3.7	-9.3	-13.1	-21.0
Employment growth, %	-2.4	-1.0	0.1	1.4	3.7
Hours growth, %	-14.1	-9.5	-7.7	-4.0	7.7

Notes: Quintile 1 is the lowest productivity and quintile 5 is the highest. For each year, all variables are computed for firms in a given quintile in that year, then averaged over the period of 2002-2005. Yearly growth rates are expressed as the first differences of log variables. Labor productivity is measured as value added per hour measured based on the pension contributions data. Source: Author's tabulations from the Danish VAT statistics data, 2002-2006.

In the short run, because firms cannot easily change the number of workers they employ, the hours margin becomes even more important. Table 2 presents summary statistics on the cross-sectional variation in the quarterly growth rates of hours per worker and employment, and the relationship between them. First, firms exhibit a significant variation in employment growth. Less than 30% of Danish firms employ the same number of workers in any two consecutive quarters. Much lower magnitudes of employment changes have been reported in empirical studies that look at other European countries. Varejao and Portugal (2007), for instance, find that employment remains unaltered over the course of a quarter for 74.7% of the establishments in a representative sample of Portuguese firms. This finding suggests that the Danish labor market, in contrast to other European countries (especially in continental Europe), is characterized by relatively low adjustment costs.

The second observation is that the standard deviation of hours and employment growth is about the same, suggesting that firms use both the intensive and extensive margins to make adjustments to their labor input. This fact is at odds with the widely known finding in the business cycles literature that hours per workers are mildly procyclical and vary much less than employment in the aggregate data (see, for instance, Cooper et al. (2007)). However, in a recent paper Ohanian and Raffo (2011) claim that this result is characteristic of the US data but need not necessarily hold true for other countries. In fact, they show

Table 2: Variation in the firm-level growth rates of hours per worker and employment.

	Non-weighted	Employment share-weighted	Employment share-weighted, no time effects
Std. dev$(\Delta \log N_t)$	0.277	0.237	0.236
Std. dev$(\Delta \log h_t)$	0.285	0.232	0.231
Corr$(\Delta \log N_t, \Delta \log h_t)$	-0.300	-0.427	-0.432
Corr$(\Delta \log N_t, \Delta \log h_{t-1})$	0.088	0.101	0.108

Source: Author's tabulations from the Danish firm data, 1999-2006.

that in many OECD countries changes in the average hours per worker are about as large as changes in employment also in the aggregate data.

Finally, Table 2 shows that there is a negative association between hours and employment growth rates at the firm level. This empirical finding supports the existence of labor adjustment costs. Consider the following example. Suppose that hiring is impeded by search frictions and the firm is hit by a positive profitability shock. In response to a shock, the average work hours overshoot their optimal level and start falling as the firm's labor force builds up to its new desired level. Hence, hours per worker and employment move in opposite directions. Likewise, a negative shock in combination with an advanced layoff notice period produces an immediate hours response and a more sluggish employment drop, thus generating a negative co-movement between these two variables. Figure 2 shows a non-parametric regression of hours growth on workforce growth.[6] The hours-employment growth relationship is monotone and the negative correlation between the two series is observed for virtually all values of employment growth. Moreover, changes in hours lead changes in employment: the relationship between employment growth this period and hours growth previous period is positive.

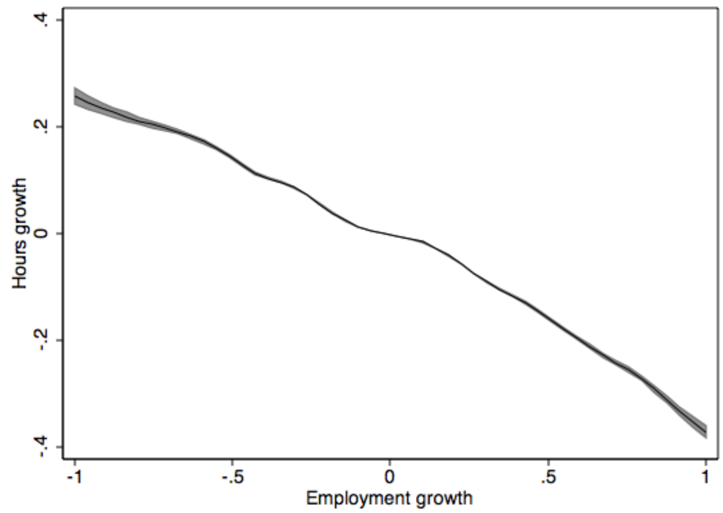
Next, I examine the correlation coefficient between hours and employment growth by broad industry groups. The hours-employment relationship is found to be weaker in Hotels and Restaurants, Fishing and Construction sectors. These industries are associated with relatively low-skilled labor and presumably lower adjustment costs and hence most of the adjustment is done on the workers margin. On the other hand, Real Estate and Business Activities and Transport and Telecommunication demonstrate a stronger association between growth rates of hours and employment. This evidence is again consistent with the labor adjustment costs hypothesis.

2.2 Job and worker flows

Most of the existing models that are used to assess the effect of labor adjustment costs on employment dynamics focus on net employment changes. Consequently, they do not distinguish between firms hiring new workers or devoting more resources to keep their existing employees. Yet, these two employment policies have different implications in terms of recruiting and training costs. Similarly, whether a reduction in the firm's workforce is achieved through quits or layoffs, and therefore whether the firm incurs dismissal costs or not, will have a different impact on its optimal employment decision. In this subsection,

[6] Non-parametric regressions in this paper are based on Nadaraya-Watson estimator and confidence intervals are obtained by bootstrapping (see Simonoff (1996) for theory and applications of kernel-based regressions and Horowitz (1997) for bootstrapping methods).

Figure 2: Non-parametric regression of hours growth on employment growth



Note: Estimates are based on Gaussian kernel with a bandwidth of 0.08. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 1999-2006.

I examine firm-level monthly employment growth patterns in detail, distinguishing explicitly between net and gross employment changes.

Here, I follow the existing literature in constructing and analyzing job flows and worker flows (see for instance Davis et al. (2006), Burgess et al. (2001) and the references therein). Using the matched employer-employee structure of the dataset, I construct monthly hires as the number of individuals that are working in a given firm during month t and not during month $t-1$. Separation flows are equal to the number of workers that are employed in a given firm during month $t-1$ and not during month t. Job flows are defined as the number of jobs created in growing firms (job creation) and the number of jobs destroyed in contracting firms (job destruction) within month t. The corresponding rates are expressed in flows divided by the average employment in month t and $t-1$. This procedure yields growth rates in the interval $[-2, 2]$ with endpoints corresponding to births and deaths (for more details on the properties of this rate measure see Davis, Haltiwanger and Schuh (1996)).

The data at hand indicate that there is a fair amount of job and worker mobility in the Danish labor market (see Table 3). Monthly hiring and separation rates average about 9% of employment. Job destruction and job creation rates are about 5-6% of employment (4% for continuing firms), more than twice the rates in the US labor market (see Davis et al. (2006)). That is, one of every 20 jobs on average gets destroyed every month.

To highlight the difference between job flows and worker flows, I construct a worker churning rate, defined as the sum of the hiring and separation rates less the absolute value of the net growth rate in employment (see Burgess et al. (2001) for more details on this measure). The churning rate refers to worker flows in excess of job flows and it is quite high in the Danish labor market, averaging 7.5% per month. The fact that firms churn workers indicates that contracting businesses still hire workers and workers leave growing firms. On average over the period of 1999-2006, job creation constitutes 32.2% of all (size-weighted) hires; while 30.6% of all separations are associated with job destruction.

Table 3: Average monthly job flow and worker flow rates

	Non-weighted	Employment share-weighted	Employment share-weighted, continuing firms
Hires	0.180	0.097	0.078
Separations	0.152	0.091	0.076
Job Creation	0.158	0.061	0.041
Job Destruction	0.129	0.054	0.039
Net employment change	0.029	0.006	0.002
Churning	0.046	0.074	0.075

Note: Sample includes all private firms and contains more than 10 million firm-month observations.
Source: Author's tabulations from the Danish firm data, 1999-2006.

To sum up, the empirical evidence presented in this section shows that firms vary their labor input on both the extensive and intensive margin. First, the standard deviation of the growth rates in employment and hours per worker are about the same. Second, the movements in hours and employment are inversely related, which is consistent with the idea of adjustment costs causing a fast response of hours to demand shocks and a more sluggish response of employment. Furthermore, hours growth seems to lead employment growth. Finally, I also show that worker flows and job flows are quite distinct. In fact, only about 30% of monthly hires and separations arise in connection with job creation and job destruction, respectively. Different implications of net and gross employment changes in terms of adjustment costs call for a theory that explicitly models hiring and separation decisions of firms. The next section builds up a labor adjustment model that incorporates the empirical facts outlined above.

3 Model

The model is a continuous time matching model of multi-worker firms with heterogenous profitability. I proceed in four steps. I start with an overview and the primitives of the model. Secondly, I show how work hours and wages are determined within each period as a result of bargaining between a firm and its workers. Thirdly, I look at the optimal employment policies of workers and firms in a dynamic context. The employment path within the firm depends on both the firm's and its workers' decisions: the worker's problem determines the quit rate to unemployment and the rate of job-to-job transitions, while the firm's problem defines optimal hiring and firing decisions. Fourthly, I derive steady state conditions for aggregate variables and the distribution of firms across productivity-size types.

3.1 Setup

A final good Y is produced by a continuum of intermediate inputs x and is sold by many suppliers in a competitive output market at price P. Let the final good to be determined by the (Dixit-Stiglitz) CES production function:

$$Y = \left[\int_0^K x(j)^{\frac{\rho-1}{\rho}} dj\right]^{\frac{\rho}{\rho-1}}, \ \rho > 0,$$

where $x(j)$ is the quantity of product j, ρ represents the elasticity of substitution between any two intermediate goods, and K is the total measure of inputs available. The final good is produced by many competitive suppliers; therefore, a profit-maximizing amount of each input is given by

$$x(j) = \left(\frac{P}{p(j)}\right)^{\rho} Y, \ j \in K, \tag{1}$$

where P is the price of the final good, and $p(j)$ is the price of input j. The zero-profit condition for final good producers implies that the aggregate price index is derived as

$$P = \left[\int_0^K p(j)^{1-\rho} dj\right]^{\frac{1}{1-\rho}}.$$

The intermediate good is produced using a linear technology, i.e.

$$x = qhn, \tag{2}$$

where x is the number of units supplied, q is firm's productivity, and hn is total labor input, the product of the number of workers n and the average work hours h. Let the final good be a numeraire with $P = 1$.

Firms differ in their productivity level q, which at any given point in time is subject to a shock that arrives at Poisson rate μ. In the event of a shock, a new productivity level is drawn from distribution $\Phi(\cdot)$, independently of the current productivity level. The corresponding density function is denoted by $\phi(\cdot)$ and is defined on a support $[\underline{q}, \overline{q}]$. Existing firms are subject to the exogenous destruction risk and die at rate δ. At the same time, there is a measure one of potential entrants that generate a new product and enter the market at rate η.

Firms and workers are brought together pairwise through a sequential and random matching process. To recruit, firms post vacancies v at a cost of $c(v)$ per unit of time, where $c(\cdot)$ is strictly increasing and convex. Reflecting search frictions, the offer arrival rate and the vacancy filling rate are exogenous to workers and firms but are determined in equilibrium. A job separation occurs if a worker quits or is laid off. Firing a worker is assumed to be costless.

There is a continuum of infinitely lived identical workers, with a mass normalized to one, that supply labor to intermediate product firms. Individuals derive utility from consuming the aggregate good and incur disutility from working. Worker's utility function is assumed to be separable in aggregate good consumption and work hours:[7]

$$\omega(y, h) = y - g(h),$$

where y is the amount of the aggregate good consumed, h is the number of hours the individual is working, and $g(\cdot)$ is a strictly increasing convex function which takes the following form:

$$g(h) = \chi h^{\xi}, \ \xi > 1, \ \chi > 0.$$

[7]Linear utility in consumption implies risk neutrality; therefore, there is no savings motive in the worker's decisions.

Worker's consumption equals to the real wage, w, when employed and equals to b when unemployed. Here, b can be viewed as unemployment insurance benefits that are indexed by the aggregate price level, or as the value of home production of the aggregate good less utility costs arising from producing it. The work hours and compensation are determined through a bargaining process. Finally, workers search while employed and unemployed.

3.2 Intra-firm wage bargaining

Hours of work and compensation are defined through a bargaining process between the firm and its employees. Given search frictions in the market, it takes time to replace workers and therefore employment is considered to be predetermined at the bargaining stage. Although the production technology is linear in the number of workers, the fact that each firm faces downward-sloping demand leads to a decreasing marginal revenue product. This setup provides a natural environment for an individual bargaining framework within multi-worker firms proposed by Stole and Zwiebel (1996), in which firms engage in pairwise negotiations with their workers.[8]

The key assumption of Stole and Zwiebel's bargaining setup is that firms and their employees cannot commit to long-term employment and wage contracts; therefore, when a worker joins or leaves the firm, wages are renegotiated individually with all workers. Following Hall and Milgrom (2008), I assume that the threat point in bargaining is to delay and not to terminate the employment relationship. Then, the firm's outside option is not to remain idle, but rather producing with one worker less; while the worker's outside option is the value of home production b during any negotiation delay.[9]

The bargaining process can be presented in two stages: firstly, the firm chooses an hours schedule that maximizes joint per-period surplus from production, and secondly, the firm and its workers bargain over wages taken the hours schedule as given. Appendix B.1 shows that the solution to this bargaining problem is equivalent to one, in which there is simultaneous bargaining over wages and hours. This result is intuitive - the optimal hours schedule maximizes the total pie, while wages are used to split it.

Firms face a downward-sloping demand curve and their revenues from selling x units of good are equal to $p(x)x$, where $x = qhn$. The firm chooses a work hours schedule that maximizes total surplus for a given number of workers n and productivity level q, i.e.

$$\max_{h\geq 0} \left\{ p(qhn)qhn - g(h)n \right\}.$$

Assuming an interior solution, the optimal number of hours satisfies the following first order condition:

$$\left(p'(qhn)qhn + p(qhn)\right)q = g'(h). \tag{3}$$

[8]See also Smith (1999), Cahuc and Wasmer (2001), and Ebell and Haefke (2003) for a similar wage bargaining setup, in which firms and their employees bargain over the total match surplus. The paper of Cahuc, Marque and Wasmer (2008) further extends the original Stole and Zwiebel's problem to account for the case when workers differ in their bargaining power parameter.

[9]Shimer (2006) shows that in a model with on-the-job search the Nash bargaining solution over the value of the match is inappropriate because the set of feasible payoffs is non-convex. The no commitment assumption allows me to significancy simplify the bargaining problem and avoid non-convexities of the payoff set as the firm is bargaining with its workers over the current output.

Then, given the optimal hours schedule $h^*(q,n)$, a wage contract solves the following problem:[10]

$$\max_{w(q,n)} \left(\frac{\partial \pi(q,n)}{\partial n}\right)^{1-\beta} \Big(w(q,n) - g(h^*(q,n)) - b\Big)^{\beta} = \tag{4}$$
$$= \max_{w(q,n)} \left(\frac{\partial R(q,n)}{\partial n} - \frac{\partial w(q,n)}{\partial n} n - w(q,n)\right)^{1-\beta} \Big(w(q,n) - g(h^*(q,n)) - b\Big)^{\beta},$$

subject to the participation constraints for both parties in the sense that the continuation value is no less than that of searching for a new partner. Here, β is the bargaining power of workers, and $R(q,n)$ is the firm's revenue given the optimal hours schedule.

Solving for the first order condition of equation (4) leads to a first-order linear differential equation in wages:

$$w(q,n) = \beta \frac{\partial R(q,n)}{\partial n} + (1-\beta)\Big(g\big(h^*(q,n)\big) + b\Big) - \beta \frac{\partial w(q,n)}{\partial n} n, \tag{5}$$

which implies the following equation for the real wage function (see Appendix B.2 for a solution method):

$$w(q,n) = n^{-\frac{1}{\beta}} \int_0^n z^{\frac{1-\beta}{\beta}} \left(\frac{\partial R(q,z)}{\partial z} + \frac{1-\beta}{\beta} g\big(h^*(q,z)\big)\right) dz + (1-\beta)b. \tag{6}$$

From equation (5) it follows that a wage that the worker gets net of disutility of working is lower than her contribution to the total surplus.

If workers and firms bargain over the value of a match then separations are bilaterally efficient. Here, given that bargaining takes place over current output, this is not necessarily true. The firm may choose to fire workers even if the value of employment to a worker is higher than the value of unemployment, and vice versa, the worker may quit even if the firm's value of employing that worker is positive. The implicit assumption of this bargaining process is that if the participation constraint binds for one party then the match is dissolved.

Finally, I assume that firms commit not to match outside offers may an employee receive one.[11] Even though the firm might have an incentive to renegotiate wages in the case of its workers getting a competing offer, there are several reasons for not doing so. In this framework, responding to outside offers will generate a wage dispersion within the firm across otherwise identical workers. This might be perceived as unfair by workers and lead to a lower performance.[12] Moreover, intra-firm wage dispersion among observationally similar workers violates non-discrimination regulations that employers are subject to in many countries. In addition, there are also theoretical models that suggest that offer matching encourages inefficient rent seeking search effort, hence an employer might choose to commit to no-matching policy (see Mortensen (1978) and Postel-Vinay and Robin (2004)). Therefore, I assume in this paper that

[10]Here, the number of workers is assumed to be continuous merely for convenience. It allows me to obtain a closed-form solution for the wage function. While it simplifies the exposition, this is not a crucial assumption. This bargaining problem can be solved for a discrete number of workers in the form of a difference equation, as it was originally done in Stole and Zwiebel (1996). The optimal number of hours is still determined by equation (3) and the wage function has the same qualitative properties. Simulating the model with a discrete n in the wage determination problem produces very similar results but requires recalibrating some of the model's parameters, most noticeably it leads to a lower value of b.

[11]See Cahuc, Postel-Vinay and Robin (2006) for an example of wage bargaining with matching of workers' outside offers in a one worker per firm environment.

[12]Fehr, Goette and Zehnder (2008) survey laboratory and field experimental evidence on the role of fairness concerns in established labor market relationships. They show that workers decrease their effort level if they are treated in ways that are perceived to be unfair.

if a worker gets an offer that yields a higher value of employment under the current wage bargaining scheme, she takes the offer and leaves her current employer.

Appendix B.3 provides explicit solutions for the wage, worker's utility and profit functions. The resulting wage function $w(q,n)$ as defined in equation (B9), and similarly the utility function $\omega(q,n)$ given by equation (B11), is increasing in productivity q and decreasing in the number of workers n. Equation (B10) shows that the profit function $\pi(q,n)$ is increasing in productivity q and is bounded from above by $\pi(q,n^*(q)) = \overline{\pi}(q)$, where $n^*(q)$ is the level of employment that maximizes the firm's profits for a given level of productivity q. In Stole and Zwiebel's original problem, n^* is the optimal number of workers that the firm would employ. This employment level is inefficient since the firm has an incentive to hire additional workers to decrease their bargaining power down to the point where the marginal profit is zero and workers are paid their reservation wage. However, Mortensen (2010) shows that in a model with on-the-job search the presence of tougher competition among firms counterbalances the incentive to over employ and leads to lower employment overall.

Returning to the motivation for this exercise, consider the trade-off between the number of workers and the number of hours that the firm faces. The driving force in this model is idiosyncratic shocks to firm's productivity q.[13] The hours schedule, as defined in equation (B7), guarantees that a positive shock to productivity q produces an increase in the average hours if there is no (or little) change in employment. As the number of workers starts growing, average hours decline. Hence, the model can capture the negative relationship between hours and employment growth observed in the data if the response of employment to shocks is slow enough. The employment decisions of firms and workers are discussed in detail in the following two subsections.

3.3 Worker's decision problem

In this subsection, I describe the worker's problem taking as given all equilibrium objects that are outside of the worker's control, such as labor market tightness and the distribution of offers across firm types, as well as employment decisions of firms. Although the value of unemployment and the value of working at a firm with productivity q and employment n depend on the aggregate variables, they are not listed as arguments for notational simplicity.

When unemployed, the worker obtains consumption flow b by means of home production, and she has an option of finding a job. Hence, the value of unemployment expressed in terms of final output, U, solves the continuous time Bellman equation:

$$rU = b + \lambda(\theta) \int (\max\{W, U\} - U) \mathrm{d}F(W), \tag{7}$$

where r is the common firms' and workers' discount rate; $\lambda(\theta)$ is the job arrival rate, and θ is market tightness; $F(W)$ is the cumulative distribution function of job vacancies posted by firms that provide

[13] Note that I refer to a shock to q as a productivity shock. However, it can be thought of as a firm-specific demand shock or, more generally, as a profitability shock. For instance, consider an alternative specification where the aggregate demand function is defined as

$$Y = \left[\int_0^K \alpha(j) x(j)^{\frac{\rho-1}{\rho}} dj \right]^{\frac{\rho}{\rho-1}},$$

where $\alpha(j)$ is a firm-specific demand shock, and production technology for the intermediate good is $x = hn$. This specification is equivalent to the current formulation of the production side of the market with $q = \alpha^{\frac{\rho}{\rho-1}}$.

workers with the value of employment of at most W.

The job arrival rate λ is derived from a matching function that is assumed to be increasing, concave, and homogenous of degree one in both arguments, vacancies and job seekers.[14] Given the matching function properties, $\lambda(\theta)$ is increasing and concave in market tightness θ, which is the ratio of the aggregate number of vacancies posted to individuals searching for a job, the variable that is determined endogenously in equilibrium.

The value of employment at a firm with n workers and productivity q, $W_n(q)$, satisfies the following Bellman equation:

$$rW_n(q) = \tag{8}$$
$$= \left\{ \begin{array}{c} \omega_n(q) + (\delta + s_0)(U - W_n(q)) + \lambda(\theta)\kappa \int (\max\{W', W_n(q)\} - W_n(q))\mathrm{d}F(W') \\ +H_n(q)(\max\{W_{n+1}(q), U\} - W_n(q)) + s_n(q)(n-1)(\max\{W_{n-1}(q), U\} - W_n(q)) \\ +\mu \int\limits_{\underline{q}}^{\bar{q}} \left(\begin{array}{c} \mathbf{1}[f_n(q') > 0] \left(f_n(q')U + (1 - f_n(q')) \max\left\{U, W_{\overline{n}^F(q')}(q')\right\}\right) \\ +\mathbf{1}[f_n(q') = 0] \max\{U, W_n(q')\} - W_n(q) \end{array} \right) \phi(q')\mathrm{d}q' \end{array} \right\},$$

where $\omega_n(q)$ is the utility flow expressed in terms of final output as defined in equation (B11). The worker becomes unemployed at constant Poisson rate $\delta + s_0$, where s_0 represents the exogenous component of the quit rate and δ refers to the destruction shock. The worker receives an alternative job offer at rate $\lambda(\theta)\kappa$, where $\kappa \geq 0$ represents search intensity when employed relative to search intensity when unemployed (if $\kappa = 1$ then workers search with the same intensity regardless of their employment status; $\kappa = 0$ implies no on-the-job search). Hence, the next term on the right-hand side of equation (8) is attributed to the option value of moving to a better employment position.

The following two terms are related to the expected change in the value of employment when the firm adjusts its labor force. In particular, at rate $H_n(q)$ the firm hires another worker, and at rate $s_n(q)(n-1)$ one of the other $(n-1)$ workers separates from the firm. These rates are determined endogenously in equilibrium and are defined in the firm's problem below.

The last term on the right-hand side of equation (8) embodies the expected change in the value attributable to a shock to the firm's productivity q that arrives at rate μ. A new productivity is drawn from distribution $\Phi(\cdot)$, with a corresponding density $\phi(\cdot)$. Recall that separations are not necessarily mutually efficient; therefore, the value function accounts for a possibility that, when hit by a productivity shock, the firm may find it optimal to fire workers. Denote by $\overline{n}^F(q)$ the maximum labor force size that the firm is willing to employ given its current productivity level q. Then the firing probability $f_n(q)$ is equal to $\frac{n-\overline{n}^F(q)}{n}$ if the firm's labor force exceeds its maximum size, i.e. $n > \overline{n}^F(q)$, and equal to zero otherwise.

Proposition 1. A unique continuous solution for the value of employment at a firm with n workers and productivity level q, $W_n(q)$, and for the value of unemployment, U, exists.

Proof: $W_n(q)$ and U represent a fixed point of the contracting mapping $T[W_n(q), U]$ defined by equations (7) and (8). Given that $\omega_n(q)$ is positive and bounded from above by $\omega_1(q)$, T maps the set of non-negative, continuous, and bounded from above functions into itself. Given this set is compact under the sup norm, one can apply Blackwell's sufficient conditions to show that T is a contraction mapping

[14] See Pissarides (2000) and Petrongolo and Pissarides (2001) for details on the concept of a matching function.

(see Stokey and Lucas (1989)). The mapping is monotone; furthermore, T discounts, i.e.

$$T\begin{bmatrix} W_n(q)+z \\ U+z \end{bmatrix} = T\begin{bmatrix} W_n(q) \\ U \end{bmatrix} + \begin{bmatrix} \frac{\delta+s_0+\lambda(\theta)\kappa+H_n(q)+s_n(q)(n-1)+\mu}{r+\delta+s_0+\lambda(\theta)\kappa+H_n(q)+s_n(q)(n-1)+\mu} \\ \frac{\lambda(\theta)}{r+\lambda(\theta)} \end{bmatrix} z,$$

which completes the proof that T is indeed a contraction. Then, by Contraction Mapping Theorem, there exists a unique continuous solution $W_n(q)$ and U. ■

Define $\overline{n}^W(q)$ as the lowest employment level, beyond which worker's participation constraint binds, i.e. $W_n(q) \leq U$ for all $n \geq \overline{n}^W(q)$. If search on the job is at least as efficient as when unemployed, i.e. $\kappa \geq 1$, then the participation constraint never binds for $n \leq n^*(q)$, where $n^*(q)$ is employment that maximizes the firm's per-period profits. To see it, subtract equation (7) from equation (8) to get

$$W_n(q) - U = \frac{\left\{\begin{array}{c} \omega_n(q) - b + \lambda(\theta)\int\left(\kappa\max\{W'-U, W_n(q)-U\} - \max\{W'-U,0\}\right)\mathrm{d}F(W') \\ +H_n(q)\max\{W_{n+1}(q)-U,0\} + s_n(q)(n-1)\max\{W_{n-1}(q)-U,0\} \\ +\mu\int\limits_{\underline{q}}^{\overline{q}}\left(\begin{array}{c} \mathbf{1}[n>\overline{n}^F(q')]\frac{\overline{n}^F(q')}{n}\max\{W_{\overline{n}^F(q')}(q')-U,0\}+ \\ \mathbf{1}[n\leq\overline{n}^F(q')]\max\{W_n(q')-U,0\} \end{array}\right)\phi(q')\mathrm{d}q' \end{array}\right\}}{r+\delta+s_0+\lambda(\theta)\kappa+H_n(q)+s_n(q)(n-1)+\mu}$$

which is nonnegative for all $\omega_n(q) \geq b$ due to the option value of the firm getting a positive productivity shock and/or adjusting its labor force. This result implies that, as long as the worker's utility is higher than or equal to b, she will never quit to unemployment. However, for all $n > n^*(q)$, as well as for other n when $\kappa < 1$, I need to verify that worker's participation constraint $W_n(q) - U \geq 0$ is satisfied.

Note that the value of working at a given firm is not necessarily monotone in the firm's workforce, even though wages and utility are monotonically decreasing in employment for a given value of productivity. The intuition for this result resides in the difference between the effect that a rise in the number of workers has on current versus future wages. An increase in employment unambiguously lowers current wages. On the other hand, it raises the prospects that some of the existing workers separate from the firm and that fewer workers are hired in the future, thus lowering the probability of a further decrease in wages. Therefore, contrary to the standard search theory, this model can potentially generate job-to-job transitions that are associated with wage cuts since a change in the value of employment may still be positive even if a change in wages is not. This prediction of the model is similar in spirit to Postel-Vinay and Robin (2002) - based on a different wage-setting mechanism, they show that a worker is willing to accept a wage cut to trade a lower share of the total rent today for a larger share tomorrow.

3.4 Firm's decision problem

This subsection provides details on employment dynamics within a firm. Firms can adjust their labor force by recruiting and firing workers. To hire a worker, the firm needs to post vacancies that are then randomly matched with job seekers. In addition, the employment decision of the firm is affected by the separation rate at which existing workers quit to unemployment or move to a better job. Both hiring and separation rates depend on aggregate market tightness, unemployment, and overall distribution of vacancies and workers across firm types. As before, for notational simplicity the aggregate variables are omitted from the arguments in firm's value function, as well as in the hiring and separation rates.

For the firm's problem, it is useful to write the hiring and separation rates explicitly. The rate, at which each worker separates from a firm with productivity q and employment n, is the sum of the exogenous quit rate into unemployment and the job-to-job transition rate, i.e.

$$s_n(q) = s_0 + \lambda(\theta)\kappa\big[1 - F\big(W_n(q)\big)\big], \tag{9}$$

where $F\big(W_n(q)\big)$ is the fraction of vacancies posted by firms that provide workers with the value of employment of at most $W_n(q)$.

The probability that any offer is acceptable to a randomly contacted worker is

$$a_n(q) = \begin{cases} \frac{u+(1-u)\kappa G\left(W_n(q)\right)}{u+(1-u)\kappa} & \text{if } W_n(q) > U \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

where u is the fraction of unemployed workers, and $G\big(W_n(q)\big)$ is the fraction of employed workers who gain the value of employment of at most $W_n(q)$. Employed job seekers are weighted by their search intensity, κ. If the worker's participation constraint is binding then no worker will accept the firm's offer.

Then, the hiring rate is equal to $H_n(q) = a_n(q)v_n(q)\nu(\theta)$, where $\nu(\theta)$ is the rate, at which vacancies are matched with workers, and $v_n(q)$ is the number of vacancies posted by a firm with n employees and productivity level q. The rate $\nu(\theta)$ is exogenous to the firm and is derived in equilibrium from the matching function.

The value of a firm with productivity q and employment n, $V_n(q)$, expressed in final output terms, solves the following Bellman equation:

$$\begin{aligned} &(r+\delta)V_n(q) \\ &= \max\left\{\left(\begin{array}{c} \pi_n(q) + \max\limits_{v\geq 0}\left\{a_n(q)\nu(\theta)v\Big(V_{n+1}(q) - V_n(q)\Big) - c(v)\right\} \\ +s_n(q)n\Big(V_{n-1}(q) - V_n(q)\Big) + \mu\int\limits_{\underline{q}}^{\overline{q}}\Big(V_n(q') - V_n(q)\Big)\phi(q')\mathrm{d}q' \end{array}\right), (r+\delta)V_{n-1}(q)\right\}, \end{aligned} \tag{11}$$

under the assumption that firing a worker is costless and the worker's participation constraint is satisfied. If the worker's participation constraint binds then workers quit randomly until $n = \overline{n}^W(q)$ and $V_n(q) = V_{\overline{n}^W(q)}(q)$.

The first term on the right-hand side of equation (11) is the firm's profit flow expressed in final good terms, as defined in equation (B10). The second term refers to the capital gain that is obtained from the possibility of hiring an additional worker, given the optimally chosen vacancy posting decision. The third term is the expected capital loss related to the possibility that any worker quits. The last term accounts for the expected change in the value of the firm caused by a shock to firm's productivity q.

Here, the advantage of modeling in continuous time, as opposed to discrete time, becomes evident. Discrete time models require a careful specification of the timing of events. Given that the hiring and separation rates depend on the number of workers, the order in which existing workers quit and new workers get hired matters for the optimal employment policy of a firm. By building the model in continuous time I avoid making any (arbitrary) assumption on the timing of events.[15]

[15]Furthermore, the timing of profitability shocks and that of employment adjustment matters for replicating the negative correla-

Proposition 2. Equation (11) has a unique continuous solution, $V_n(q)$, if $c(v)$ is strictly increasing, convex and $c(0) = c'(0) = 0$.

Proof: Equation (11) has a unique solution that is a fixed point of the mapping

$$[TV](q,n) = \max \left\{ \max_{v\geq 0} \frac{\left(\begin{array}{c} \pi_n(q) + a_n(q)v\nu(\theta)V_{n+1}(q) - c(v) \\ +s_n(q)nV_{n-1}(q) + \mu \int_{\underline{q}}^{\overline{q}} V_n(q')\phi(q')\mathrm{d}q' \end{array} \right)}{r + \delta + a_n(q)v\nu(\theta) + s_n(q)n + \mu}, \quad V_{n-1}(q) \right\}. \tag{12}$$

Given that (i) the profit function is bounded from above by $\overline{\pi}(q)$, (ii) $c(v)$ is strictly increasing and convex and $c(0) = c'(0) = 0$, and (iii) firing a worker is costless, T maps the set of non-negative continuous functions bounded from above by $\overline{V}(q) = \frac{\overline{\pi}(q)}{r+\delta+\mu} + \frac{\mu}{r+\delta+\mu} \int_{\underline{q}}^{\overline{q}} \frac{\overline{\pi}(q')}{r+\delta} \phi(q')\mathrm{d}q'$ into itself. As this set is compact under the sup norm, I need only to confirm that the map satisfies Blackwell's sufficient conditions for a contraction. First, note that T is monotone. Moreover,

$$T[V_n(q) + z] = T[V_n(q)] + \frac{a_n(q)v_n(q)\nu(\theta) + s_n(q)n + \mu}{r + \delta + a_n(q)v_n(q)\nu(\theta) + s_n(q)n + \mu} z,$$

where $v_n(q) = \arg\max_{v\geq 0} \left\{ a_n(q)\nu(\theta)v\Big(V_{n+1}(q) - V_n(q)\Big) - c(v) \right\}$. The map T discounts if $v_n(q) < \infty$, a condition that holds under the assumption that $c(v)$ is strictly increasing and convex, and the fact that $0 \leq V_{n+1}(q) - V_n(q) \leq \overline{V}(q)$ due to boundedness and non-negativity of V function. Hence, T is indeed a contraction mapping. Therefore by Contraction Mapping Theorem, there exists a unique solution to equation (12). ■

Since profit falls without a bound as the firm's labor force increases, there exists an upper limit on employment, $\overline{n}^F(q)$, beyond which the firm fires workers. That is, $\overline{n}^F(q)$ is the lowest number of workers, for which $V_{\overline{n}^F(q)+1}(q) = V_{\overline{n}^F(q)}(q)$. If the worker's participation constraint is binding and $\overline{n}^W(q) < \overline{n}^F(q)$ then the maximum labor force size $\overline{n}(q)$ is determined by the worker's problem. Therefore, the maximum employment is defined as the minimum of the two threshold values, i.e.

$$\overline{n}(q) = \min \left\{ \overline{n}^F(q), \overline{n}^W(q) \right\}.$$

Note that $\overline{n}(q)$ is also the lowest level of employment, for which firms post zero vacancies.

Proposition 3. $V_n(q) - V_{n-1}(q)$ is strictly positive for all $n \leq n^*(q)$, that is $\overline{n}^F(q) \geq n^*(q)$.[16]

Proof: Note that costless firing implies $V_n(q) - V_{n-1}(q) \geq 0$. To show that the difference is strictly

tion between the growth rates in hours and in the number of workers in the simulation. Intuitively, if a shock arrives in the middle of a period (as opposed to the beginning of a period) the observed change in hours is smaller because we average the before-shock and after-shock levels of work hours. Discrete time labor adjustment models implicitly synchronize the timing of a shock with the change in labor resources. This assumption, however, is not innocuous and tends to generate a more negative relationship between the extensive and intensive margins of adjustment.

[16] In the proof of Proposition 3, I implicitly assume that $n^*(q) \leq \overline{n}^W(q)$. However, the same logic applies if $n^*(q) > \overline{n}^W(q)$, then $V_n(q) - V_{n-1}(q) > 0$ for all $n \leq \overline{n}^W(q)$.

positive for $n \leq n^*(q)$, I apply proof by induction. Differencing equation (11) leads to

$$\Big(r + \delta + \mu + a_{n-1}(q)\nu(\theta)v_{n-1} + s_n(q)n\Big)\Big(V_n(q) - V_{n-1}(q)\Big) = \tag{13}$$
$$= \max\left\{ \left(\begin{array}{c} \pi_n(q) - \pi_{n-1}(q) + c(v_{n-1}) + \max\limits_{v \geq 0}\Big\{a_n(q)\nu(\theta)v\Big(V_{n+1}(q) - V_n(q)\Big) - c(v)\Big\} \\ +s_{n-1}(q)(n-1)\Big(V_{n-1}(q) - V_{n-2}(q)\Big) + \mu\int\limits_{\underline{q}}^{\overline{q}}\Big(V_n(q') - V_{n-1}(q')\Big)\phi(q')\mathrm{d}q' \end{array} \right), \quad 0 \right\}.$$

First, note that the assertion holds for $n = 1$; that is, $V_1(q) - V_0(q) > 0$ since $c(\cdot)$ is increasing and $c(0) = 0$, the benefit from posting vacancies is non-negative, and $\pi_1(q) > 0$. Then, under the assumption that $V_{n-1}(q) - V_{n-2}(q) > 0$, also $V_n(q) - V_{n-1}(q) > 0$ given that profit is increasing in n for all $n \leq n^*(q)$. That completes the induction proof. ■

Proposition 3 shows that, similarly to the original work of Stole and Zwiebel (1996), firms have an incentive to recruit workers up to the level of employment that maximizes per period profit flow. Contrary to their work, however, labor hoarding effect may arise in this model in the sense of $\overline{n}(q) > n^*(q)$. Intuitively, in the economy with search frictions, the firm might choose to hire workers above the profit-maximizing employment level in anticipation of a future positive productivity shock. However, the presence of on-the-job search makes it increasingly harder for firms to hire workers, as well as to retain their current employees, when employment increases. In the model simulation the effect of competition seems to dominate the labor hoarding motive so that the average firm size is far below $n^*(q)$.

3.5 Steady state conditions

3.5.1 Size distribution

Firms are identical ex ante and their type is revealed upon entry. Productivity of potential entrants is assumed to be distributed according to the same distribution function, $\Phi(\cdot)$. Then, under the assumption that shocks to q are drawn unconditionally from distribution $\Phi(\cdot)$, the productivity distribution at entry is preserved among existing firms. The steady state number of firms conditional on their productivity is derived by equating market entry and exit

$$\delta K(q) = \eta\phi(q), \tag{14}$$

where η is the exogenous entry rate, $\phi(q)$ is the density of entrants of type q, and δ is the proportion of existing firms that become obsolete and exit the market.

Denote by $K_n(q)$ the aggregate number of products supplied by the set of firms of type q with employment n. Then steady state mass of firms conditional on the firm's type is derived by equating inflows into and outflows from $K_n(q)$. First, for all $n \in [1, \overline{n}(q) - 1]$ the following relationship must hold:

$$H_{n-1}(q)K_{n-1}(q) + s_{n+1}(q)(n+1)K_{n+1}(q) + \phi(q)\mu\int\limits_{\underline{q}}^{\overline{q}} K_n(q')\mathrm{d}q' \tag{15}$$
$$= H_n(q)K_n(q) + s_n(q)nK_n(q) + \delta K_n(q) + \mu K_n(q),$$

where the first two terms on the left-hand side represent the expected hiring and separation flows; whereas, the next term is the average proportion of all firms, which are hit by an idiosyncratic shock and which become q-type firms. The outflow consists of the transition flows of firms with n workers to $n+1$ due to new hires, firms with n workers to $n-1$ due to quits, the firm destruction, $\delta K_n(q)$, and a change in productivity type, $\mu K_n(q)$.

For $n = 0$, equation (15) includes an additional term that accounts for the entry of new firms of type q, $\eta\phi(q)$. Thus, the steady state relationship for $n = 0$ reads

$$\begin{aligned} s_1(q)K_1(q) + \phi(q)\mu \int_{\underline{q}}^{\overline{q}} K_0(q')\mathrm{d}q' + \eta\phi(q) \\ = H_0(q)K_0(q) + \delta K_0(q) + \mu K_0(q). \end{aligned} \tag{16}$$

FInally, equation (15) has to be modified for $n = \overline{n}(q)$ to incorporate the possibility of a firm firing workers in the event of an adverse productivity shock, i.e.

$$\begin{aligned} H_{\overline{n}(q)-1}(q)K_{\overline{n}(q)-1}(q) + \phi(q)\mu \int_{\underline{q}}^{\overline{q}} \sum_{n=\overline{n}(q)}^{\overline{n}(q')} K_n(q')\mathrm{d}q' \\ = s_{\overline{n}(q)}(q)\overline{n}(q)K_{\overline{n}(q)}(q) + \delta K_{\overline{n}(q)}(q) + \mu K_{\overline{n}(q)}(q). \end{aligned} \tag{17}$$

Note that the last equation uses the fact that $v_{\overline{n}(q)}(q) = 0$ and $K_n(q) = 0$ for all $n > \overline{n}(q)$ since there are no firms with employment that exceeds $\overline{n}(q)$. Therefore, $K_n(q) = 0$ for all $n > \overline{n}(q)$ serves as the boundary condition for a second order difference equation, defined in equations (15) - (17).

The steady state distribution of firms of type q is

$$K(q) = \sum_{n=0}^{\overline{n}(q)} K_n(q).$$

3.5.2 Aggregate unemployment, market tightness, and output

The unemployment rate can be derived from the labor market clearing condition, which states that in equilibrium labor supplied to the market should be equal to total employment across all firms:

$$1 - u = \int_{\underline{q}}^{\overline{q}} \sum_{n=1}^{\overline{n}(q)} nK_n(q)\mathrm{d}q. \tag{18}$$

Aggregate market tightness is defined as the ratio of vacancies to job seekers weighted by their search intensity, i.e.

$$\theta = \frac{v}{u + (1-u)\,\kappa}\,, \tag{19}$$

where the total number of vacancies posted by all firms is

$$v = \int_{\underline{q}}^{\overline{q}} \sum_{n=0}^{\overline{n}(q)} v_n(q) K_n(q) \mathrm{d}q. \tag{20}$$

The matching function is assumed to take a Cobb-Douglas form:

$$M(v, u + (1-u)\kappa) = m v^{\zeta} (u + (1-u)\kappa)^{1-\zeta}, \tag{21}$$

with $0 < \zeta < 1$ and a scaling parameter $m > 0$. The parameter m represents the efficiency of a matching process. The job-finding rate, defined as the ratio of matches to job seekers, can be expressed as a function of market tightness alone, i.e. $\lambda(\theta) = m\theta^{\zeta}$. Similarly, the worker meeting rate, defined as the ratio of matches to posted vacancies, can be written as $\nu(\theta) = m\theta^{\zeta-1}$.

Finally, equilibrium in the goods market is achieved when total output produced by all intermediate firms is equal to the aggregate demand for the final good, i.e.

$$\begin{aligned} Y &= \left[\int_{\underline{q}}^{\overline{q}} \sum_{n=1}^{\overline{n}(q)} \Big(q h_n(q) n \Big)^{\frac{\rho-1}{\rho}} K_n(q) \mathrm{d}q \right]^{\frac{\rho}{\rho-1}} \\ &= \left(\frac{\rho - 1}{\chi \xi \rho} \right)^{\frac{1}{\xi-1}} \left[\int_{\underline{q}}^{\overline{q}} q^{\frac{\xi(\rho-1)}{(\xi-1)\rho+1}} \sum_{n=1}^{\overline{n}(q)} n^{\frac{(\xi-1)(\rho-1)}{(\xi-1)\rho+1}} K_n(q) \mathrm{d}q \right]^{\frac{(\xi-1)\rho+1}{(\xi-1)(\rho-1)}}. \end{aligned} \tag{22}$$

3.5.3 F and G distribution functions

The job offer distribution $F(W)$ is merely a fraction of aggregate vacancies that are posted by firms, which provide workers with the value of employment of at most W, i.e.

$$F(W) = \frac{\int_{\underline{q}}^{\overline{q}} \sum_{n=0}^{\overline{n}(q)} \mathbf{1}\big[W_n(q) \leq W\big] v_n(q) K_n(q) \mathrm{d}q}{v}. \tag{23}$$

Similarly, the steady state distribution of workers, $G(W)$, refers to the fraction of total workforce employed at jobs, which guarantee workers the value of employment of at most W, i.e.

$$G(W) = \frac{\int_{\underline{q}}^{\overline{q}} \sum_{n=1}^{\overline{n}(q)} \mathbf{1}\big[W_n(q) \leq W\big] n K_n(q) \mathrm{d}q}{\int_{\underline{q}}^{\overline{q}} \sum_{n=1}^{\overline{n}(q)} n K_n(q) \mathrm{d}q}. \tag{24}$$

Combining equations (15) - (17) with the definitions of $F(W)$ and $G(W)$, after some algebraic manipulations leads to a familiar condition for the steady state unemployment rate that equates flows into and out of unemployment. That is, the steady state unemployment rate solves the following equation:

$$\lambda(\theta)u = (1-u)(\delta + s_0 + \mu l). \tag{25}$$

The left-hand side of this equation refers to the outflow from unemployment due to finding a job. The right-hand side includes the inflow into the pool of unemployed workers due to three reasons: the destruction shock, the exogenous quit, and the layoff in the event of an adverse productivity shock.[17] The last term is the product of the shock arrival rate, μ, and the average proportion of workers that are laid off when the firm's productivity changes, l, which in turn is computed as

$$l = \int_{\underline{q}}^{\bar{q}} \frac{\int_{\underline{q}}^{\bar{q}} \sum_{n=\overline{n}(q)}^{\overline{n}(q')} (n - \overline{n}(q)) K_n(q') \mathrm{d}q'}{\int_{\underline{q}}^{\bar{q}} \sum_{n=1}^{\overline{n}(q')} n K_n(q') \mathrm{d}q'} \phi(q) \mathrm{d}q.$$

3.6 Equilibrium

Definition: A steady state market equilibrium is a set of numbers (θ, u, U, Y), a set of distribution functions $\big(G(W), F(W)\big) : \mathbb{R}_+ \to [0,1]$, a set of functions defined on a state space $\big(W_n(q), V_n(q), v_n(q), K_n(q)\big) : [\underline{q}, \overline{q}] \times \mathbb{I}_+ \to \mathbb{R}_+$, a set of functions defined on firm productivity types $K(q) : [\underline{q}, \overline{q}] \to \mathbb{R}_+$ and $\overline{n}(q) : [\underline{q}, \overline{q}] \to \mathbb{I}_+$, that satisfy equations (7), (8), (11), (14) - (24).

To find a steady state equilibrium, I look for a fixed point of the mapping where the worker's and firm's problems are solved given aggregate market tightness, unemployment, aggregate demand, and distribution functions of vacancies and workers across firm types. Then, the aggregate variables and steady state distributions are updated using the optimal employment decisions of firms. Appendix C.1 provides details on a steady state equilibrium solution algorithm used in the numerical simulation procedure described below.

4 Calibration

The model is simulated under the assumption that the economy is in steady state. I solve for the type-conditional equilibrium hiring and separation rates, $H_n(q)$ and $s_n(q)$, respectively, as well as the maximum labor force size, $\overline{n}(q)$. Then, under the assumption of a Poisson arrival rate of firm-specific shocks, I can simulate firms' employment histories (further details can be found in Appendix C.2). The continuous-time nature of the model eliminates the need to make any arbitrary assumptions on the timing of the events. Worker flows are simulated at a monthly frequency; while hours, wages, and value added are aggregated into quarterly series to mimic the reporting frequency of the administrative data. Most importantly, to ensure that the simulated moments are comparable to empirical moments the simulated hours measure is constructed based on the 9-hour intervals following the pension contribution payment rule, in parallel with the data.

[17] Here, the term 'layoff' is used loosely since maximum labor force size may be determined by the worker's problem, i.e. if $\overline{n}(q) = \overline{n}^W(q)$ then workers quit.

In the following subsections, I start by discussing the parameter choice for the simulation and implied values of adjustment costs. I then present main predictions of the model and compare them with the Danish firm data.

4.1 Parameter choice

Table 4 presents the pre-determined parameters of the model. The monthly interest rate r is equal to 0.4%, which is equivalent to a yearly interest rate of about 5%. The destruction rate parameter δ is set to be equal to the average (size-weighted) firm exit rate, which is found to be 0.52% per month over the period of 1999-2006. Without the data on vacancies, the matching function parameters m and ζ cannot be identified separately. For the purpose of this simulation, the elasticity of the matching function with respect to vacancies, ζ, is set to 0.5.

The elasticity of substitution between intermediate goods, ρ, determines the degree of decreasing returns to scale of the revenue function. Recall that the revenue function, expressed in final output terms, is given by

$$R = Y^{\frac{1}{\rho}} x^{\frac{\rho-1}{\rho}},$$

where R is revenues, Y is aggregate demand for the final good, and x is the amount of the intermediate good produced. The returns to scale parameter is equal to $\frac{\rho-1}{\rho}$. There is a wide range of estimates of the revenue returns to scale in the literature: for instance, Khan and Thomas (2003) use the estimate of 0.905, Cooper and Haltiwanger (2006) report an estimate of 0.592, while Gourio and Kashyap (2007) choose a calibrated value of 0.6. I set ρ to 2.3, which yields a somewhat lower value of the returns to scale parameter of 0.56. However, I find that smaller ρ reduces the incentive to hire new workers in the event of a positive shock by making profits less sensitive to productivity fluctuations, which helps the model to replicate a negative correlation between hours and employment growth.

Table 4: Pre-determined parameter values

Parameter	**Value**
Monthly interest rate, r	0.4%
Destruction rate, δ	0.52%
Curvature of matching, ζ	0.5
Elasticity of substitution, ρ	2.3

Table 5 presents the remaining parameter values calibrated to match empirical moments. While it is not possible to associate individual parameters with individual moments, the results of numerical simulation help to identify particular moments that play key roles in identifying structural parameters. Unemployment benefit b, or alternatively the value of home production, is set to match the unemployment rate of 4.8%, the average unemployment rate during the period of 1999-2006 (OECD Economic Outlook 2007). The implied replacement ratio, defined as the ratio of unemployment benefit to the average monthly wage, is about 60%. Worker's bargaining power parameter, β, affects the amount of rent-sharing in the model and is set to 0.1 to match the average labor share in value added in the data. This value is consistent with the estimates reported in Cahuc et al. (2006).

The scale parameter χ on the disutility from working is chosen to reproduce the average weekly work hours in the model. The curvature parameter of the utility cost function ξ plays a prominent role for the hours-employment trade-off through its effect on the cost of varying hours of work. I set $\xi = 2.2$ that is within the range of estimates found by Cooper and Willis (2009) and Bloom (2009). This value ensures that the correlation between hourly wages and hours predicted by the model is close to that observed in the data. Under this parameterization of the utility cost function, the implied disutility from working one hour of overtime, i.e. one hour above 37 hours a week, amounts to about 600 DKK per month.

Table 5: Calibrated parameter values

Parameter	**Value**	**Description**
b	10,410	Value of home production, DKK
χ	0.16	Scale parameter, utility cost function, DKK
ξ	2.2	Convexity, utility cost function
β	0.1	Workers' bargaining power
c_0	1,430	Scale parameter, vacancy cost function, DKK
c_1	1.4	Convexity, vacancy cost function
κ	0.9	Relative search intensity of employed workers
s_0	0.004	Exogenous quit rate
m	0.4	Efficiency of matching
μ	0.04	Shock arrival rate
η	0.00164	Entry rate

For the numerical simulation, I use the following specification of the vacancy posting costs: $c(v) = c_0 v^{c_1}$, with $c_0 > 0$ and $c_1 > 1$. The scale parameter c_0 is chosen such that the job-finding rate is equal to 0.2, which corresponds to mean unemployment duration of 5 months.[18] The curvature of the vacancy cost function, c_1, reduces variation in the hiring rate: a more convex cost function means that firms make smaller and slower adjustments to their workforce. The degree of convexity $c_1 = 1.4$ may seem low compared to some estimates found in previous studies – for example, Mertz and Yashiv (2007) report that labor adjustment costs are approximately cubic. However, these values are sensitive to both time and cross-section aggregation. The observed labor variation patterns look smoother in the aggregate data and hence produce higher estimates of the costs function convexity.[19]. Thus, while Mertz and Yashiv (2007) find that a cubic specification for adjustment costs fits the data well on a quarterly basis, $c_1 = 1.4$ is a reasonable value for monthly worker flows.

Given a low rate of unemployment yet relatively long unemployment duration in the Danish labor market, the implied values for the job finding rate λ and the separation rate into unemployment are also low. Thus, job-to-job transitions are a necessary component that allows the mode to match the observed magnitudes of hiring and separation rates of about 8 percent per month. The relative search intensity is set at $\kappa = 0.9$ to fit the average monthly separation rate. The exogenous quit rate parameter, s_0, is set to match the standard deviation of the separation rate. The idea behind it is that a larger share of exoge-

[18]The average distribution of unemployed workers by duration during the period of 1999-2006 was the following: 23.1% of workers were unemployed for less than one month; 18.4% - for 1 to 3 months; 19.5% - for 3 to 6 months; 17.6% - for 6 to 12 months, and 21.4% were unemployed for more than a year (OECD Economic Outlook 2007). Median duration is between 3 and 6 months. I choose mean duration of 5 months to be a reasonable target, which corresponds to the monthly job-finding rate of 0.2.

[19]This point was first noted by Hamermesh (1989) by demonstrating a stark difference between aggregate and individual employment adjustment patterns when aggregation was done over just seven plants. Bloom (2009) illustrates how adjustment costs estimates change depending on time aggregation and within-firm aggregation over different production units

nous quits in total separations reduces their responsiveness to productivity shocks and hence lowers the volatility of the separation rate. Also, a higher value of s_0 increases the correlation between the separation rate and employment. Intuitively, under the assumption of an exogenous and constant quit rate, the separation rate is independent of employment (or positively related since the probability of layoffs rises with the firm's workforce). In the data, however, this relationship is slightly negative with the size-weighted correlation coefficient of -0.03. Thus, bringing in endogenous quits into the model ensures that the correlation coefficient between the separation rate and employment is also negative and equal to -0.04 in the simulated data.

The underlying productivity is assumed to follow a Generalized Pareto distribution and its parameters are chosen to fit the size distribution of firms (the following subsection provides a thorough discussion on that). The arrival rate of productivity shocks, μ, determines the persistence of labor productivity at the firm level.[20] Table 6 summarizes the empirical moments and the corresponding simulated moments that are used to calibrate the parameters of the model.

Table 6: Data-based versus simulated moments

Statistics	Data	Model
Unemployment rate, u	0.048	0.048
Job-finding rate, λ	0.200	0.197
Average separation rate^a, $E(SR_t)$	0.076	0.075
St. dev. of separation rate^a, $sd(SR_t)$	0.152	0.142
St. dev. of hiring rate^a, $sd(HR_t)$	0.153	0.130
Employment-hours growth relation, $corr(\Delta \log N_t, \Delta \log h_t)$	-0.300	-0.232
Average weekly hours^b, $E(h_t)$	33.7	33.8
Wage-hours relation^c, $corr\left(\frac{W_t}{N_t h_t}, h_t\right)$	0.166	0.260
Labor share^d, $E(W_t/R_t)$	0.51	0.55
Productivity autocorrelation, $corr\left(\frac{R_{t-1}}{N_{t-1}}, \frac{R_t}{N_t}\right)$	0.698	0.617
Mean employment^e, $E(N_t)$	9.2	8.7
Median employment^e, $Med(N_t)$	4	5.7
Standard deviation of employment^e, $sd(N_t)$	15.4	10.4

Notes: ^{a} - Continuing firms only, moments are weighted by employment share. ^{b} - Based on annual hours reported by OECD Economic outlook (2007) over the period of 1999-2006. To get weekly hours, I divide annual hours by 46 weeks assuming there are 6 weeks of vacation. ^{c} - Hourly wages here are based on the upper bound measure of hours (for more details see Section 4.2.4.); moments are weighted by employment share. ^{d} - Estimated on firms with non-negative profits, i.e. $R_t > W_t$. ^{e} - Top one percent of firms (with more than 150 employees) are excluded from the sample.

4.2 Labor adjustment costs

In this paper, I focus on labor adjustment costs that are associated with hiring frictions. In general, it is not easy to obtain information on various sources and sizes of adjustment costs. Recent works of Abowd and Kramarz (2003) and Kramarz and Michaud (2004) estimate employment adjustment costs directly based on survey data for a representative sample of French firms. They find considerable magnitudes of

[20]This Poisson arrival shock process is equivalent to a discrete time mean-reverting AR(1) process with autocorrelation coefficient of $e^{-\mu}$. Thus, a lower value of μ implies higher persistence of the underlying productivity process.

both hiring and separation costs, with the latter exceeding the former. Many of these costs, however, are implicit – such as forgone production when existing workers spend their time to train new hires – and thus are not reported. On the other hand, imputing adjustment costs indirectly from the firm-level labor dynamics captures all, including implicit, components of these costs.

The average cost of hiring a worker is computed as the flow cost of sustaining an open vacancy ($c_0 v^{c_1}$) multiplied by expected duration of that vacancy, or the inverse of the hiring rate. Given the vacancy cost parameters and the implied steady state worker meeting and offer acceptance rates, the average cost of hiring a new worker is found to be DKK 4,750, which is equivalent to about one week of wages. Other European labor markets are characterized by a higher degree of employment protection; hence, the estimates of adjustment costs found from data sources featuring other European countries in general are much larger. For instance, Rota (2004), based on annual firm level data from the Italian manufacturing industry, reports an estimate of fixed adjustment costs of 15 months of labor costs. For the US labor market, there is no consensus in the literature on the magnitudes of adjustment costs. Similarly to the results found in this paper, Shimer (2005) calibrates the hiring costs of two weeks of wages.[21] However, Mertz and Yashiv (2007) find that a marginal cost of hiring is roughly equivalent to two quarters of wage payments - twelve times higher than my estimates. Note, however, that many of these estimates of labor adjustment costs pertain to net employment changes and therefore are likely to be higher than the estimates based on gross worker flows data.

4.3 Model fit

4.3.1 Employment distribution

A standard search theory typically models single-worker firms, or more generally, it assumes a constant returns to scale production function, and hence does not have a meaningful definition of firm size. In this paper, under the assumption of diminishing returns to labor, the model produces an endogenous steady state size distribution, which then can be compared to its empirical counterpart.

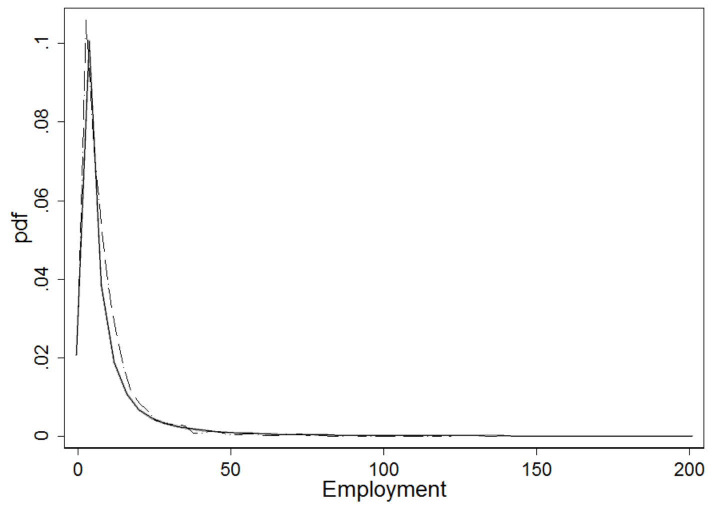
In order for the model to be able to generate large firms, while keeping the state space manageable, I assume that firms act as a collection of product lines and that each product faces its own hiring and separation process. It is natural to think of large firms as multi-product entities. Lentz and Mortensen (2008) develop a model, in which the number of product lines for each firm is a result of a costly innovation process and product destruction. In their model, more productive firms innovate more frequently and in steady state supply a larger share of product varieties. Here, I assume for simplicity that the distribution of products across firms is exogenous and independent of the firm's productivity q. In that case, all steady state equilibrium conditions hold and one can think of $K_n(q)$ as a mass of products instead of firms. The number of product lines is drawn randomly for each firm from a Poisson distribution with a mean of two and is kept constant over the firm's lifetime.

A well-established fact in the existing literature is that the size distribution of firms is highly skewed to the right with a very long right tail (see Figure 3). That is, most of the firms in the data are small with a few firms that have much larger than average workforce. These features of the data put restrictions on the shape of the firm-specific productivity distribution $\Phi(\cdot)$ in the model. I use a highly skewed Generalized

[21] In his model, the cost of hiring a worker is equal to the flow cost of sustaining an open vacancy (0.213) multiplied by an average duration of a vacancy (1/1.35 of a quarter), which corresponds to about two weeks of wages.

Pareto distribution for q with its parameters set to match the observed firm size dispersion and the median to mean ratio (the exact parameter values can be found in Appendix C.2). The ratio of the entry rate to the exit rate, η/δ, determines the total mass of products, and through that, the average number of workers per product line.

Figure 3: Size distribution in the data (solid line) and in the model (dashed line)



Note: Density estimation is based on Gaussian kernel with a bandwidth of 1. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 1999-2006.

Under these assumptions, the model is able to replicate the overall shape of the observed employment distribution (see Figure 3). The simulated size distribution shows a lower dispersion and a higher median than the actual distribution; nevertheless, it successfully captures the fact that there is a significant size dispersion and that the average firm employs about twice as many workers as the median firm.

4.3.2 Hours and employment adjustment

Table 7 compares the summary statistics for employment and hours growth in the data and in the simulation. The model captures the variation in employment growth very well, but underestimates the variation in hours growth. This result partly reflects the way in which hours are measured in the data. Recall that changes in actual hours are registered in the data when at least one worker moves between the 9-hour intervals. For the variation in labor utilization to be captured in the simulation, all workers at a given firm have to move to a different 9-hour interval because workers are identical in the model. Therefore, worker heterogeneity might be one of the reasons for why the variation in hours growth differs between the data and the model. On top of that, the model predicts that in the absence of search frictions each firm would hire $n^*(q)$ workers and would employ them for the same number of hours, regardless of the firm's productivity level q. That is, $h(n^*(q), q)$ as defined in equation (B7) is independent of q. Therefore, the variation in hours in the model is driven only by the deviations of employment from its optimal level.

In the model, the firm responds to a positive shock in profitability by increasing labor utilization and

Table 7: Hours and employment growth rates in the model and in the data.

	Data	Model
Std.dev $(\Delta \log N_t)$	0.277	0.307
Std.dev $(\Delta \log h_t)$	0.285	0.121
Corr $(\Delta \log N_t, \Delta \log h_t)$	-0.300	-0.232
Corr $(\Delta \log N_t, \Delta \log h_{t-1})$	0.089	0.108

Source: Author's tabulation from the Danish firm data over the period of 1999-2006 and the simulated data.

posting more vacancies. Given search frictions in the labor market, it takes time to recruit new workers; therefore, as vacancies start filling up, hours of work begin to fall. The mechanism is slightly different in the event of a negative shock. In the aftermath of the shock, the firm reduces work hours of its existing employees and, if the firm's productivity falls too low, fires some of its workers. The initial cut in employment happens immediately; however, the firm now faces a higher attrition rate so that its workforce continues to decline further down. The average hours per worker, on the contrary, start rising. Hence, endogenous quits are key in letting the model to capture the trade-off between hours and employment in the case of a negative profitability shock.

There are a few key parameters that are important for matching the relationship between employment and hours adjustment. First, a more persistent shock process in terms of a lower arrival rate of profitability shocks, μ, strengthens the dynamic interaction between hours per worker and the number of workers. The firm has a stronger incentive to respond to changes in profitability by adjusting its labor force size if shocks last longer. On the contrary, if shocks are white noise then the firm will be more likely to keep its workforce at the same level and adjust labor input mostly through the hours margin. I chose the value of μ to match the quarterly autocorrelation in labor productivity at the firm level. A monthly arrival rate of 0.04 per product line, together with mean of two products per firm, implies that on average a shock to firm productivity arrives about once a year.

Second, a higher vacancy cost parameter, c_0, induces firms to create fewer vacancies and therefore slows down the recruiting process. However, it has a countervailing effect of increasing the vacancy filling rate and thus raising the return on vacancies. Therefore, the effect of higher vacancy posting costs on the extent of employment adjustment may be non-monotone. In the simulation, I pin down vacancy posting costs to match the job-finding rate and use the matching efficiency parameter m to essentially make the hiring process more sluggish. The worker contact rate declines in m for a fixed job-finding rate, thus lowering the vacancy posting rate of firms.

Table 7 shows that overall the model is successful in reproducing the trade-off between the extensive and intensive margins of adjustment. The correlation coefficient between hours and workers growth is -0.232, compared to -0.300 in the data. Changes in hours lead changes in employment: the correlation coefficient between employment growth and lagged hours growth is 0.108, slightly higher than in the data.

4.3.3 Job and worker flows

On-the-job search is a necessary component that enables the model to capture the characteristics of the data related to worker and job flows. Allowing for workers to search while employed means that both the quit rate and the offer acceptance rate depend on the firm's type: more productive firms face lower attrition rates and are able to attract workers faster than their less productive counterparts. These features of the model are consistent with empirical evidence reported in earlier studies: Davis, Faberman and Haltiwanger (2010), for instance, document that the vacancy yield (the number of hires per vacancy) increases in employment growth.[22] The model also predicts that a sizable workforce reduction can be brought about through quits in the case of an adverse profitability shock. Similarly, Davis et al. (2006) find that quits account for a bigger portion of separations than layoffs for firms that shrink by less than 12% during the month; furthermore, the quit rate is higher in contracting firms than in growing firms.

Table 8 summarizes the empirical and simulated moments concerning worker and job flows. For these results, I restrict the sample to continuing firms because the model does not have a rich theory of entry and exit. The average monthly job and worker flow rates in the data are matched closely by the model. In line with the data, worker flows are about twice the size of job creation and job destruction. The model can produce a relatively high churning rate, which means that also in the simulation contracting firms are still hiring workers, while growing firms lose workers.

Table 8: Monthly job and worker flow rates in the data and in the model.

	Data	Model
Hires	0.078	0.076
Separations	0.076	0.075
Job Creation	0.040	0.044
Job Destruction	0.038	0.043
Net employment change	0.002	0.001
Churning	0.075	0.064

Note: Empirical moments are size-weighted and refer to continuing firms only. Source: Author's tabulations from the Danish firm data (1999-2006) and simulated data.

Table 9 shows the relationship between monthly worker flows and net employment adjustment, size-weighted by employment share. Firms are split into five groups according to their net employment growth rate. Contracting firms reduce their labor force mostly through separations; while growing firms increase their employment mostly through hiring. However, even contracting firms are hiring at a 4.5% rate. These results appear to be qualitatively similar to those found for the US and Dutch labor markets (see Davis et al. (2006) and Hamermesh, Hassink and van Ours (1996), respectively), but are in contrast to the behavior of French firms reported by Abowd, Corbel and Kramarz (1999). The latter paper finds that employment variation in France is made predominantly through the hiring margin; that is, establishments are changing their labor force primarily by reducing entry and not by varying their separation rates.

The model captures these employment growth patterns very well. The empirical finding that con-

[22] Kaas and Kircher (2011) provide an alternative explanation within the competitive search framework. In their model, firms offer long-term wage contracts and compete to attract workers. Firms that want to hire faster raise the attractiveness of their offers, which results in a higher probability of hiring and hence a higher vacancy yield.

Table 9: Average monthly hiring and separation rates, by net employment growth rate.

	Data				Model			
Net Emp. Growth	Hires	Sep.	Net	Empl. Share, %	Hires	Sep.	Net	Empl. Share, %
Less than -0.10	0.046	0.496	-0.450	10.1	0.023	0.337	-0.314	12.9
-0.10 to -0.025	0.040	0.093	-0.053	13.7	0.032	0.092	-0.060	13.3
-0.025 to 0.025	0.035	0.035	0.000	50.3	0.037	0.037	0.000	45.0
0.025 to 0.10	0.094	0.040	0.054	14.8	0.092	0.032	0.060	14.7
More than 0.10	0.503	0.044	0.459	11.1	0.273	0.025	0.248	14.1

Note: Data moments are based on all private firms. Both data and simulated moments are weighted by employment.

tracting firms reduce their labor force mostly through separations, while growing firms increase their employment mostly through hiring, is consistent with the model's predictions. Furthermore, in the model contracting firms still exhibit positive hiring rates, albeit lower than those observed in the data. In general, the model performs well matching firms with employment adjustment between -10% and 10%, but underestimates worker turnover in firms that grow or contract by more than 10%.

In the Danish labor market, about a half of all firms have zero monthly net employment change and they represent about one fifth of total employment. A common claim in the literature is that non-convex adjustment costs are necessary to match the large region of inactivity observed in the data (see for instance Cooper and Willis (2009)). This model is capable of generating a significant share of firms with zero employment growth with convex adjustment costs: on average, 55% of simulated firms have zero net monthly employment change and they employ about 40% of total workforce. The reason for that is the presence of search frictions in the market – although firms post vacancies continuously, they may have zero hires if these vacancies are not matched with workers.

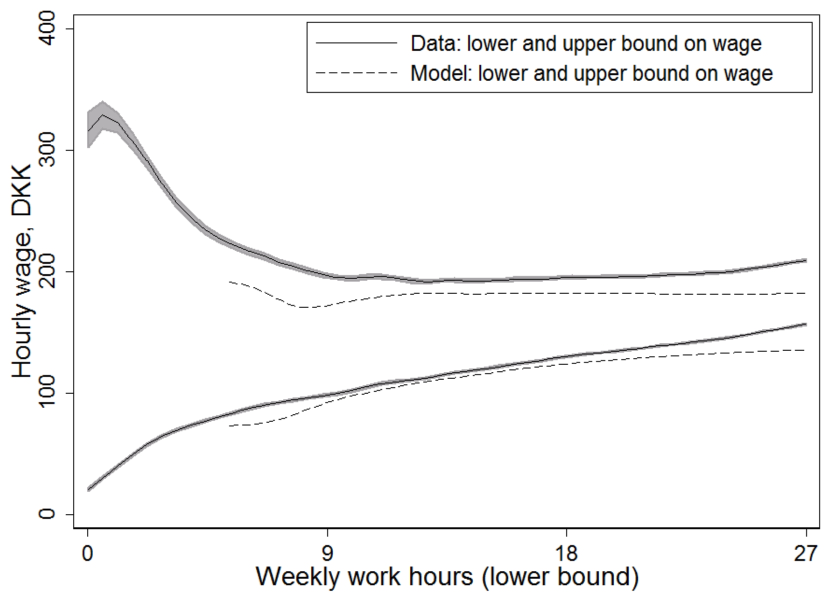
Earlier studies (see for instance Christensen, Lentz, Mortensen, Neumann and Werwatz (2005)) have reported that the separation rate is higher in low-pay jobs. Although in this paper hourly wage per se is not a sufficient statistics for the separation rate, the model admits a negative association between firm-level wages and separations: the size-weighted correlation coefficient between the quarterly (cumulative) separation rate and (upper bound) hourly wages is -0.23 in the model and -0.14 in the data.

4.3.4 Wages and hours

The trade-off between changes in hours and employment comes from two sources: (i) hiring costs that halt the adjustment in the number of workers and (ii) costs of changing hours of work. If variations in hours are inexpensive then firms would make all the adjustment on the intensive margin alone. In the model, it is the convexity of disutility arising from working that generates the increasing marginal cost of employing the worker for an extra hour. The existing studies support the claim that variations in hours are expensive in the data. For example, ample empirical evidence for a negative part-time/full-time wage premium has been documented in the literature (see Blank (1990) for a review). Hence, this subsection aims to test whether hourly wages are increasing also in the Danish data.

An hourly wage series is constructed as the ratio of total payroll costs paid in a given quarter to the total number of work hours, where hours are measured at their lower bound as before. This measure of wages represents an upper bound on cenactual wages. The correlation between the two series – wages and work hours – turns out to be slightly negative in the data. One explanation for this counterintuitive finding is mis-measurement of wages: if wages are overestimated relatively more for low values of hours then we expect to see a decline in hourly wages as hours rise. For that reason, I construct an alternative (upper bound) measure of hours per worker that assumes the right boundary of each 9-hour interval for all employees with positive pension contributions; moreover, it assigns 9 hours to workers with zero contributions (see Appendix A for more details on how this variable is constructed). The hourly wage series constructed in this manner represents a lower bound on actual wages. The difference between the two wage measures is more prominent for low values of hours, mainly due to fact that the latter measure includes employees that work less than 9 hours.

Figure 4: Non-parametric regression of hourly wages on hours per worker in the data (solid line) and in the model (dashed line).



Note: Estimates are based on Gaussian kernel with a bandwidth of 0.5. Shaded area is 90% pointwise bootstrap confidence interval around upper and lower bound on hourly wage (clustered by firm ID). Hourly wages less than 80 DKK per hour are removed from the analysis (this figure is regarded as an estimate of the effective legal minimum wage), as well as the top one percent of the observed wage distribution. Source: Author's calculations from the Danish firm data, 2006.

Figure 4 displays the relationship between two measures of wages and work hours. The upper bound on wages displays a marked drop in wages for low values of hours. On the contrary, the lower bound is undoubtedly increasing in work hours. Hence, the negative association between wages and hours can be attributed to a large extent to mismeasurement of hours. Most importantly, the model is capable of reproducing the observed empirical relationship between wages and hours: the upper bound on wages (constructed in the same way as in the data) is non-monotone for low values of hours; while the lower bound on wages in increasing for the whole range of hours.

In the data, average wages appear to be increasing in firm size - the finding that has been well-documented in many other studies (see for instance Oi and Idson (1999)). Despite the fact that Stole-

Zwiebel bargaining implies that a rise in the workforce pushes wages downwards, the model generates a positive relationship between wages and employment due to the composition effect. That is, larger firms tend to be more productive on average and thus offer higher wages.

5 Counterfactual experiments

Given the parameter values above, I use the model to analyze how changes in the adjustment costs affect employment policies of firms. I simulate an introduction of firing costs and an increase in the vacancy posting costs, and quantify their impact on unemployment, the job finding rate, and average work hours. I compare the effects obtained in a general equilibrium framework to that of partial equilibrium, in which vacancy-filling rates and quit rates are kept unchanged after the change in costs. Then, I perform two policy experiments – (i) an introduction of hiring subsidies, and (ii) imposing an upper limit on work hours – and show their effects on aggregate employment and total welfare. Note that I do not consider transition dynamics after the changes are introduced, instead these experiments should be thought of as a comparison between two steady state economies - the baseline versus a counterfactual economy with alternative parameter values.

5.1 Firing costs

A significant part of labor adjustment costs is related to various forms of labor market regulations, including dismissal costs. In fact, it is the current consensus in this literature that legal impediments to firing workers are responsible for the relatively sluggish employment growth and high levels of unemployment in many European countries in recent decades (see for instance Goux et al. (2001) and Kramarz and Michaud (2004) for empirical studies, Bentolila and Bertola (1990) for a theoretical model). That has led economists to promote labor market reforms aimed at lowering the cost of dismissals, a policy change that many European countries have considered or implemented. In Denmark, firing costs are considered to be virtually non-existent; therefore, they are assumed to be absent in the benchmark model (see Appendix A.1. for details on collective bargaining rules on dismissal costs). As a counterfactual experiment, I introduce into the model fixed (lump-sum) firing costs to reflect collective dismissals.

Compared to the baseline case, the introduction of firing costs of three months of wages raises unemployment by 2.5 percentage points.[23] The effect of firing costs on unemployment is mitigated by the fact that most of the reduction in the workforce in the model is achieved through quits.[24] This result relies on a fairly high job-finding rate in Denmark, so that workers prefer to leave troubled firms to move to other businesses or to quit to unemployment. One can imagine that in countries with limited worker mobility, as well as in economic downturns (as they are associated with lower quit rates - see for instance Nagypal (2008)), workers will be less willing to quit the firm in the event of a negative shock, thus making firing constraints binding for employers. Also note that wages in this model provide another margin of adjust-

[23]Perhaps surprisingly, I find that the job finding rate also rises by 4 percentage points. Even though firms now post fewer vacancies for a given productivity and employment level, the inclusion of firing costs leads to a different size distribution, in particular, firms tend to be smaller on average. Since the number of vacancies falls with firm size, the aggregate vacancies increase.

[24]Under the assumption that a match is dissolved if either the worker's or firm's participation constraint is binding, the distinction between layoffs and quits (and, correspondingly, whether the firm is sustaining firing costs) is somewhat arbitrary. In fact, given the parameter values used in this simulation, maximum employment is determined primarily by the worker's problem. Therefore, in the event of an adverse shock workers prefer to leave the firm, making firing costs non-binding for their employer.

ment, in that they can be costlessly re-negotiated in the event of a shock. In the case of downward wage rigidity, for example, the effects of dismissal costs on aggregate employment are expected to be higher.

5.2 An increase in the vacancy costs

In order to quantify the effect of hiring costs on aggregate employment, I simulate the model with a doubled vacancy creation cost parameter; that is, I set c_0 to DKK 2,860 vis-à-vis the benchmark case of DKK 1,430. The results presented below are relevant for marginal employment subsidies programs that are aimed at reducing recruiting costs. The first two columns of Table 10 compare the results of this experiment with the baseline model. Doubling the costs increases the unemployment rate from 4.8% to about 7.5% and reduces the job-finding rate by about 4 percentage points. The average hours per person rise compared to the benchmark case; that is, higher hiring costs induce firms to substitute towards the intensive margin of adjustment.

Table 10: The effect of doubling the vacancy posting costs

	Benchmark	Doubled vacancy cost	
	Model	General Equilibrium	Partial Equilibrium
Unemployment rate	4.8%	7.5%	11.1%
Job-finding rate	0.197	0.159	0.168
Average hours per person	33.8	34.5	34.5

Most of the existing models that analyze the impact of labor adjustment costs on firms labor demand use a partial equilibrium framework (see for instance Bentolila and Bertola (1990)). In a general equilibrium search model, an increase in vacancy costs leads to a lower number of of aggregate vacancies in the market and hence less congestion in the matching process. These effects turn out to be quantitatively very important. To show this, I perform the above experiment of doubling c_0 parameter, while keeping the quit rates and the vacancy filling rates constant at the same level as in the benchmark model. The results of this experiment are presented in the last column of Table 10. Eliminating general equilibrium channels generates a much higher unemployment rate: unemployment rises to 11%. These results demonstrate how partial equilibrium models of labor demand might significantly overstate the effect of adjustment costs on the unemployment rate.

Finally, I examine the importance of the hours margin of adjustment when evaluating the impact of hiring costs on employment. In order to do so, I shut down the intensive margin by setting work hours of all employees at their average level of 33.7 hours per week. In that case, keeping all other parameters unchanged, doubling vacancy costs leads to a 65% increase in the unemployment rate compared to a 55% increase with flexible hours; whereas, the job finding rate falls by about 20% in both cases. Ignoring the intensive margin of adjustment leads to overestimation of the effects of adjustment costs on aggregate employment but the difference is small in magnitude, perhaps reflecting the fact that the model produces insufficient variation in hours.

5.3 A hiring subsidy

One of the main concerns of policymakers during recessions is to find an effective way to stimulate job creation. Among proposed solutions is a new jobs tax credit. A recent example of this policy is the US Hiring Incentives to Restore Employment (HIRE) Act, enacted in March 2010, that provides tax incentives for businesses that hire previously unemployed workers. To examine the impact of such a policy, I simulate the model with an employment subsidy received by firms for every new hire (although without distinguishing whether this hire comes from the pool of employed or unemployed workers). To finance this policy, I assume that all firms pay a fixed lump-sum tax. Table 11 compares the results of the experiment with the baseline model. I consider three values of the hiring subsidy: 1500, 2500 and 3500 Danish kroner, which corresponds to roughly 30, 50 and 70 percent of the average hiring costs.

Table 11: The effect of a hiring subsidy

	Benchmark	Hiring subsidy		
	Model	DKK 1500	DKK 2500	DKK 3500
Unemployment rate, %	4.82	4.58	4.50	4.47
Job-finding rate	0.197	0.204	0.209	0.213
Percentage difference relative to benchmark				
total firm value		-2.6	-4.3	-6.1
total workers' welfare		0.0	0.0	0.0
total welfare		-0.9	-1.5	-2.1

The introduction of the hiring subsidy reduces the unemployment rate from 4.8 to 4.4-4.5 percent and increases the job finding rate by about 1 to 2 percentage points. Given the structure of the model, it is possible to evaluate the impact this policy has on workers' and firms' welfare. Let us first consider the effect on workers' welfare. An increase in the job-finding rate has a positive effect on workers' well-being. However, higher average employment means lower wages for employed workers, as a consequence of the bargaining process. These two effects seem to cancel each other out and the overall change in workers' welfare is virtually zero. In terms of the firm's value, the hiring subsidy decreases the costs of employment adjustment and thus increases the value of a firm by about 0.2-0.5%. However, the fact that firms have to pay a tax to finance employment subsidies affects profits negatively. The latter effect prevails and the average firm's value falls by 3 to 6 percent. In sum, total welfare drops by 1-2% as a result of the subsidy. The welfare analysis shows that, even though an employment subsidy is effective in reducing the unemployment rate, it creates a loss in total welfare.[25]

5.4 An upper limit on hours

In this subsection, I examine the effect of introducing a shorter workweek on aggregate employment and welfare. This change in working time regulations is often viewed as a cheaper alternative to a reduction in hiring costs. The idea behind this policy is that firms would need to hire more workers to

[25] Another way of financing this policy, such as a differential tax depending on the firm's size and productivity, might be more efficient in terms of overall welfare. For example, one alternative would be to impose a tax on sales. However, it has not been done here since the tax on sales distorts firms' hiring decisions, while a lump-sum tax allows to identify the effect of hiring subsidies on the unemployment rate in a clean way.

sustain their total labor input as they cannot increase work hours of their employees beyond a certain level. However, this policy comes at a loss of flexibility in firms' choices, which in turn lowers their profits and therefore might negatively affect firms' labor demand. The empirical evidence on the efficacy of this policy in fostering employment growth, however, is mixed. Crepon and Kramarz (2002), for example, find that a reduction in the official workweek from 40 to 39 hours in 1982 in France led to employment losses of 2 to 4 percent. On the other hand, Chemin and Wasmer (2009) show that France's switch from a 39-hour to a 35-hour workweek in 2000 had no significant impact on employment.

Here, I consider a similar experiment by imposing an upper limit on weekly work hours. Suppose that firms are allowed to employ their workers for at most h_{max} hours. In particular, I consider three different threshold values for h_{max}: 35, 37, and 39 hours a week. If the maximum hours constraint is binding – i.e. when the optimal number of hours as determined by equation (B7) exceeds the upper limit – then the firm sets the work hours of its employees at the maximum and bargain over wages only. In particular, the wage equation then becomes

$$w(q,n) = \beta \frac{\rho - 1}{\rho - \beta} \left(\frac{Y(qh_{max})^{\rho-1}}{n} \right)^{\frac{1}{\rho}} + (1-\beta)(b + \chi h_{max}^{\xi}).$$

The results of this experiment are shown in Table 12. Given the model parameters, the unemployment rate decreases by about 0.4, 0.5 and 1.2 percentage points if maximum hours are set to 39, 37 and 35 hours, respectively. The job finding rate gradually increases with a shorter workweek. Recall that work hours from the bargaining problem are set optimally in order to maximize total per-period surplus; therefore, restricting the hours choice affects negatively firm profits, which leads to a decrease in the total firm value. Similarly, if hours constraint is binding wages will be lower than in the benchmark case. However, the overall effect of this policy on workers' utility might be positive since imposing an upper limit on hours reduces disutility arising from working longer hours and hence improves workers' welfare. Given the parameters of the model, the overall workers' welfare increases by 0.1-0.7 percentage points. The total welfare decreases by less than one percentage point.

Table 12: Introducing an upper limit on work hours.

	Benchmark	Maximum hours per week		
	Model	$h_{max} = 35$	$h_{max} = 37$	$h_{max} = 39$
Unemployment rate, %	4.8	3.6	4.3	4.4
Job-finding rate	0.197	0.217	0.209	0.202
Percentage difference relative to benchmark				
total firm value		-2.3	-1.1	-0.4
total workers' welfare		0.7	0.4	0.1
total welfare		-0.3	-0.1	-0.05
Fraction of workers employed for more than h_{max} in the baseline model, %		42.0	12.6	5.7

6 Conclusion

This study is motivated by the observation that firms use variation in work hours of their employees to adjust their labor demand in response to shocks. In particular, I use a matched employer-employee panel of Danish firms to document firm-level employment and hours growth patterns. I find that the standard deviation of quarterly growth rates in the number of workers and hours per worker is about the same. Moreover, the data exhibit a strong negative relationship between these series, which is consistent with a hypothesis that hours respond to shocks immediately, while changing the workforce takes time. These empirical facts call for a model that allows for both intensive and extensive margins of labor adjustment.

In this paper, I build a general equilibrium theory of heterogeneous multi-worker firms that choose their hiring and firing policies optimally in an economy with search frictions. The driving force of the model is idiosyncratic profitability shocks that firms can accommodate by varying work hours of their existing employees and/or adjusting their employment. Wages and hours are determined through a bargaining process. In addition, allowing for on-the-job search delivers a rich theory of quits that enables the model to capture most of the features of the data regarding worker flows.

The model is calibrated to assess its fit to the Danish firm data and appears to be successful in capturing the overall characteristics of the data. The numerical simulation does an outstanding job of reproducing employment variation at the firm level. It matches closely hiring and separation rates, job creation and job destruction rates, and the distribution of firms by net employment growth. In addition, the model is capable of generating the negative correlation between the growth rates of employment and hours per worker.

In the process of matching the model to the data, I obtain an indirect estimate of the average hiring costs. I find that to hire a new worker, the firm has to bear a cost in the amount of one week of wages, on average. This value is low compared to the estimates of the adjustment costs found for other European countries and it is more similar to the values reported for the US labor market. High magnitude of worker flows found in the Danish labor market is at the heart of this result. Using the numerical simulation, I show that doubled vacancy posting costs raise the unemployment rate from 4.8% to 7.5% and reduce the job-finding rate by about 4 percentage points. The increase in the unemployment rate would be much higher (up to 11%) in a partial equilibrium framework if the quit rates and vacancy filling rates were to remain unchanged. I then use the model to simulate two policy experiments – introducing of a hiring subsidy versus imposing an upper limit on weekly work hours – and estimate their effects on aggregate employment. I show that introduction of a shorter workweek can lower the unemployment rate and, even though this policy creates production inefficiencies, the resulting loss in total welfare is quantitatively small.

While beyond the scope of this paper, there are two obvious directions in which this model can be extended. The first is introducing aggregate shocks into the model. Intuitively, aggregate shocks will have different implications for firms' employment strategies than idiosyncratic shocks do. In the current setup, in addition to posting more vacancies, the firm finds it easier to retain workers in the event of a positive shock. If all firms experience an increase in their profitability at the same time, i.e. the shock is aggregate, the attrition rate will remain unchanged; hence, firms have to post more vacancies to reach their desired level of employment. Similarly, in the event of a negative shock workers will be less willing to quit the firm because the value of unemployment is lower in recessions due to a lower job-finding rate.

That would make firing costs binding for employers. Moreover, extending the model to include aggregate shocks would allow for evaluation of such policies as Kurzarbeit scheme that was introduced in Germany and Austria in the latest recession.

The second possible extension of this model is to allow for worker heterogeneity. The model in its current form does not produce enough variation in hours because the marginal cost of an extra hour is determined by workers' disutility function and is the same for all firms. However, empirically workers have different preferences about the length of their workweek, as well as about its variability. Introducing differences in workers' disutility of hours would help the model to fit the data on hours variation.

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Appendix

A. Data

A1. Danish labor market

The Danish labor market is regulated mostly by collective bargaining agreements between trade unions and employer organizations: about 80% of all employees are unionized.[26] Collective agreements regulate wages and main issues concerning work conditions, such as overtime, paid leave, etc. The recent tendency in the labor market is for the unions to play the role of a coordinating institution, whereas wages are negotiated at the firm level.[27]

There is no statutory protection against dismissals, as there is no statutory minimum wage. Collective agreements' rules on individual dismissals are particularly flexible, which makes the Danish labor market one of the least rigid by international standards. [28] Long-term employees receive an average of one month's wage compensation upon dismissal. For comparison, a severance pay in Portugal is three months of wages for short-term employees and up to twenty months for long-term employees; up to four months of wages in France, up to nine months of wages in Netherlands and up to a year in Spain.[29] Regulations on dismissals may differ across industries. For instance, average notice periods vary from three days in construction to one month for industrial workers and up to three months for salaried workers depending on their seniority in the firm. The reason for short notices is that employees in turn have flexibility to switch jobs: workers are required to notify their employers eight days in advance if they want to quit. In addition, there has been an increase in the use of temporary contracts and there are no longer limitations on how often these temporary contracts can be renewed.

Working time has always been one of the primary issues in collective bargaining in Denmark; however, until the 1980s the total length of the workweek has been a dominant concern. In the mid-1990s the focus shifted to the variability of working time allowing for additional flexibility of work hours. For instance, in the manufacturing sector the collective agreement was introduced in 1998 that specified that working time could vary over a twelve-month period as long as the average weekly hours amounted to 37 hours a week, provided that an agreement between management and a union representative is reached locally. Further changes were made in 2004, which stated that specific organization of the working time could be agreed directly with an individual employee or a group of employees.[30]

To sum up, the Danish labor market is one of the most flexible in Europe permitting firms to adjust their workforce under a minimal set of regulations. Lax dismissal rules in combination with a generous unemployment insurance scheme result in a very mobile labor market. Moreover, recent trends towards

[26] See Danish Confederation of Trade Unions website (as downloaded on May 20, 2010): http://lo.dk/Englishversion/About%20LO/TheDanishLabourMarket.aspx

[27] Source: Danish Flexicurity Model: The Role of the Collective Bargaining System (2005).

[28] Here, I refer to measures of labor market flexibility developed by Botero, Djankov, Porta, de Silanes and Shleifer (2004). Their original data have been extended by the World Bank and are available at http://www.doingbusiness.org/ExploreTopics/EmployingWorkers/. Difficulty of firing index, which includes requirements for grounds for dismissal, dismissal procedures, severance pay and terms of notice, is 0 out of 100 in Denmark compared to, for instance, 30 in France, 40 in Italy, and an average of 22.6 for OECD countries. Overall rigidity of employment index, which refers to legal requirements concerning minimum pay, working time, paid holidays, use of part-time and fixed-time contracts, and dismissal procedures, is reported to be 7 out of 100 for Denmark compared to 26.4 OECD average (as downloaded on May 10, 2010).

[29] Source:"The flexible labour market needs strong social partners.The European discussion on the Danish Labour Market: Flexicurity". Published by the Danish Confederation of Trade Unions on January 2008: http://lo.dk/Englishversion/˜ /media/LO/English/FinalFexicurity.ashx (as downloaded on May 10, 2010).

[30] Source: Danish Flexicurity Model: The Role of the Collective Bargaining System (2005).

decentralization of bargaining allow for more flexibility in determining both wages and working time because many conditions can now be negotiated at the firm level.

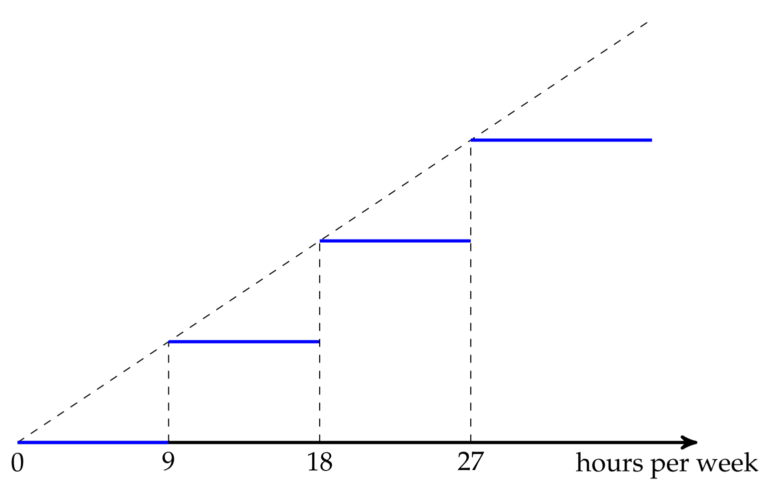
A2. Data sources

The empirical analysis in this paper is based on Danish firm data drawn from administrative records for the period of 1999-2006. They come from four major sources. First, the detailed information on employment changes is obtained from a matched employer-employee panel that includes all individuals that have paid employment in a given month. Monthly employment is constructed as a head count of all individuals employed in a given firm. Quarterly number of employees is derived as an average of three months employment for firms that have positive employment in all three months of a given quarter. The fact that this dataset has time-consistent identifiers for both firms and workers makes it possible to construct hires and separations series for each firm.

Second, a work hours series is derived from the firms' mandatory pension contribution data collected on a quarterly basis. In Denmark, firms are required to pay pension contributions for each 16-66 year old employee according to her weekly hours of work. The rule for the pension contribution (depicted in Figure 6) is as follows:

- full amount of contribution (670.95 DKK in 1999-2005 and 731.70 DKK in 2006 per quarter) is paid for an employee working more than 27 hours a week;
- 2/3 of the full amount is paid for an employee working between 18 and 27 hours a week;
- 1/3 of the full amount is paid for an employee working between 9 and 18 hours a week;
- zero contribution is paid for all employees working less than 9 hours a week.

Figure 5: Mandatory pension contribution scheme



The available data contain the sum of pension contributions paid by the firm for all of its employees in a given quarter. Then, a full-time equivalent (FTE) measure reported by the Danish Central Statistical Office is constructed as the total amount of quarterly pension contributions divided by the payment norm for a full-time employee (where full-time refers to working more than 27 hours per week). Given proportionality of the schedule, the average hours per worker can be derived by dividing the total FTE measure, N^*, by the number of employees, N, and multiplying by 27 hours a week, i.e.

$$H_{LB} = 27\frac{N^*}{N}. \tag{LB}$$

This approach implicitly assumes the left boundary of each 9-hour interval for all employees and therefore represents the lower bound on the weekly hours of work.

In order to construct the upper bound measure of work hours, I consider the right boundary point for each of the 9-hour intervals in Figure 6. The right boundary of the upper interval is assumed to be 36 hours a week. This assumption, albeit not very realistic, preserves the proportionality of the hours schedule. Also, recall that the FTE measure excludes employees that work less than 9 hours per week. Therefore, if the number of workers in a given firm is higher than the number of full-time employees, I allocate 9 hours of work to those extra workers. In sum, the upper bound on work hours per employee is defined as

$$H_{UB} = \frac{36N^* + 9(N - N^*)\mathbf{1}[N > N^*]}{N}. \tag{UB}$$

One can argue that the firm have an incentive to adjust hours only within (and not between) the 9-hour intervals in order to minimize its pension contributions. However, the level of pension contributions is relatively low compared to other labor costs, such as wages, income and social security taxes: a pension contribution for a full-time employee amounts to about 1% of average wages. It is unlikely that firms have economically significant incentives to "bunch" workers at the right boundary point of each interval. To test this hypothesis, I use another dataset that has hours information for each employee, in particular, the Earnings survey. It contains all firms in the private sector with more than 10 full-time employees, excluding agriculture and fishery. This survey collects information on paid hours for each employee on a yearly basis (or for the length of a job spell if it was shorter than a year) and allows me to distinguish between salaried and pay-rate workers. Average work hours of salaried workers (that comprise about 55% of workers in the data) show little variation: more than 60% of them works about 37 hours a week. On the other hand, average hours of workers that are paid on the pay-rate basis are distributed across all levels of hours, as can be seen in Figure 6. The figure below shows no evidence of "bunching", that would imply a higher mass of workers at the right boundary points of the pension scheme intervals.

The third dataset is drawn from the VAT statistics for the period of 2002-2006. It provides information on purchases and sales of all VAT-liable businesses on a quarterly basis, measured in Danish kroner (DKK). In Denmark a business enterprise must register for VAT if its annual turnover is expected to exceed 50,000 DKK. The VAT declaration frequency depends on the annual turnover: firms report monthly if their annual turnover exceeds 15 million DKK, quarterly if their turnover is between 1 million DKK and 15 million DKK, and semi-annually if it is below 1 million DKK. Hence, the empirical moments on value added and labor productivity in this paper refer to businesses with annual turnover above 1 million DKK (in total, 13.9% of firm-quarter observations are excluded due to missing quarterly information). Lastly, I use only data on firms with positive value added.

Figure 6: Average weekly hours of work for pay-rate workers

Note: The vertical axis show the fraction of workers in each hours category. Source: Author's calculations based on the Danish firm data, 2006.

The fourth dataset contains information on total payroll costs that firms pay in a given quarter. Wages and output are measured in Danish Kroner (DKK) and are deflated using quarterly CPI with 2001 Q1=100. The empirical analysis is carried out based on private firms data. The resulting dataset has about 3 million firm-quarter observations.

B. Wage determination

B.1 Bargaining over an hours schedule

Here, I show that in Stole-Zwiebel's framework simultaneous bargaining over hours and wages is equivalent to bargaining over wages given the hours schedule is chosen by the employer to maximize total surplus. Consider the bargaining problem for an individual i. Her wage and work hours have to maximize the following equation (note that I have suppressed q in all equations below):

$$\max_{w_i,h_i} \left\{ \begin{array}{c} \Big(R\big(\overline{h}(n+\Delta n)n + h_i\Delta n\big) - \overline{w}(n+\Delta n)n - w_i\Delta n - R\big(\overline{h}(n)n\big) + \overline{w}(n)n\Big)^{1-\beta} \times \\ \Big(\big(w_i - g(h_i) - b\big)\Delta n\Big)^{\beta} \end{array} \right\}, \tag{B1}$$

where $\overline{h}(n)$ is the average work hours chosen by other workers in the firm with employment n, and $\overline{w}(n)$ are wages paid to other employees if total employment is n. The first order condition with respect to wage w_i reads

$$\begin{aligned} &\beta\Big(R\big(\overline{h}(n+\Delta n)n + h_i\Delta n\big) - \overline{w}(n+\Delta n)n - w_i\Delta n - R\big(\overline{h}(n)n\big) + \overline{w}(n)n\Big) \\ &= (1-\beta)\big(w_i - g(h_i) - b\big)\Delta n. \end{aligned} \tag{B2}$$

The optimal hours choice of employee i maximizes the problem above, taking the number of hours of other employees as given. The first order condition with respect to h_i is:

$$R'\left(\overline{h}(n+\Delta n)n + h_i \Delta n\right) = g'(h_i), \tag{B3}$$

where $R'(\cdot)$ refers to the derivative of total revenues with respect to total labor input, hn. Equation (B3) is equivalent to equation (3), given the symmetry of the bargaining problem for all workers.

Next, note that the following limit can be rewritten as

$$\lim_{\Delta n \to 0} \frac{R\left(\overline{h}(n+\Delta n)n + h_i(n+\Delta n)\Delta n\right) - R\left(\overline{h}(n)n\right)}{\Delta n} =$$

$$= \left(\begin{array}{c} \lim_{\Delta n \to 0} \frac{R\left(\overline{h}(n+\Delta n)n + h_i(n+\Delta n)\Delta n\right) - R\left(\overline{h}(n+\Delta n)n\right)}{h_i(n+\Delta n)\Delta n} h_i(n+\Delta n) \\ + \lim_{\Delta n \to 0} \frac{R\left(\overline{h}(n+\Delta n)n\right) - R\left(\overline{h}(n)n\right)}{\overline{h}(n+\Delta n)n - \overline{h}(n)n} \lim_{\Delta n \to 0} \frac{\overline{h}(n+\Delta n) - \overline{h}(n)}{\Delta n} n \end{array} \right)$$

$$= R'\left(h(n)n\right)\left(h(n) + \frac{\partial h(n)}{n} n\right),$$

again, under the symmetry assumption. Likewise,

$$\lim_{\Delta n \to 0} \frac{w(n+\Delta n)(n+\Delta n) - w(n)n}{\Delta n} = w'(n)n + w(n).$$

Dividing equation (B2) by Δn and taking limits as $\Delta n \to 0$, I obtain the outcome of the bargaining problem that is identical to equation (4).

B.2 Solution to the bargaining problem

Solving for the first order condition of equation (4) leads to a first-order linear differential equation in wage

$$w(n) = \beta R'(n) + (1-\beta)\Big(g\big(h(n)\big) + b\Big) - \beta w'(n)n. \tag{B4}$$

The solution of the homogenous equation $w'(n) + \frac{w(n)}{n\beta} = 0$ is equal to

$$w(n) = An^{-\frac{1}{\beta}}, \tag{B5}$$

where A is a constant of integration of the homogenous equation. Assuming that A is a function of n and substituting (B5) into (B4) I get

$$A'(n) = R'(n)n^{\frac{1-\beta}{\beta}} + \frac{1-\beta}{\beta}\Big(g\big(h(n)\big) + b\Big)n^{\frac{1-\beta}{\beta}},$$

or, by integration

$$A(n) = \int_0^n z^{\frac{1-\beta}{\beta}} \left(R'(z) + \frac{(1-\beta)}{\beta} g\big(h(z)\big)\right) dz + (1-\beta)bn^{\frac{1}{\beta}} + B,$$

where B is a constant of integration.

The last equation implies that the solution to (B4) is

$$w(n) = n^{-\frac{1}{\beta}} \int_0^n z^{\frac{1-\beta}{\beta}} \left(R'(z) + \frac{(1-\beta)}{\beta} g\big(h(z)\big) \right) dz + (1-\beta)b, \tag{B6}$$

where to pin down B, I assumed as in Stole and Zwiebel (1996) that wage is finite when $n \to 0$ which implies $B = 0$.[31]

B.3 Hours, wages and profits

Here, I derive the optimal hours choice, worker's wages, and firm's profit stemming from the bargaining problem above. Given the demand for the firm's product and the disutility of working function $g(h) = \chi h^{\xi}$, the optimal number of hours that solves equation (3) is equal to

$$h(q,n) = \left[\left(\frac{\rho-1}{\chi\xi\rho} \right)^{\rho} Y \frac{q^{\rho-1}}{n} \right]^{\frac{1}{(\xi-1)\rho+1}}, \tag{B7}$$

which is increasing in productivity q and decreasing in the number of employees n.

The firm's revenue at the optimal number of hours, expressed in units of the aggregate good, reads

$$R(q,n) = \left(\frac{\rho-1}{\chi\xi\rho} \right)^{\frac{\rho-1}{(\xi-1)\rho+1}} \left(Y q^{\rho-1} \right)^{\frac{\xi}{(\xi-1)\rho+1}} n^{\frac{(\xi-1)(\rho-1)}{(\xi-1)\rho+1}}. \tag{B8}$$

Revenue rises with productivity and with the number of workers if the elasticity of substitution between any too intermediate goods ρ is higher than one. Hence, the condition $\rho > 1$ is imposed in this paper.

Using the expressions for the revenue function above, and disutility from working evaluated at the optimal hours choice (B7), the solution to the bargaining problem defined in equation (6) leads to the following real wage equation:

$$w(q,n) = \chi \left(1 + \beta(\xi-1) \frac{(\xi-1)\rho+1}{(\xi-1)\rho+1-\beta\xi} \right) \left[\left(\frac{\rho-1}{\chi\xi\rho} \right)^{\rho} Y \frac{q^{\rho-1}}{n} \right]^{\frac{\xi}{(\xi-1)\rho+1}} + (1-\beta)b. \tag{B9}$$

Notice that $\frac{\partial w(q,n)}{\partial n}$ is negative: as the number of workers per firm increases, the bargained wage declines.

Given the wage curve equation above, the firm's profit is derived as

$$\begin{aligned} \pi(q,n) &= \Big(R(q,n) - w(q,n)n \Big) \qquad \text{(B10)} \\ &= (1-\beta) \left[\frac{\chi\big((\xi-1)\rho+1\big)^2}{(\rho-1)\big((\xi-1)\rho+1-\beta\xi\big)} \left(\left(\frac{\rho-1}{\chi\xi\rho} \right)^{\rho} Y \frac{q^{\rho-1}}{n} \right)^{\frac{\xi}{(\xi-1)\rho+1}} - b \right] n, \end{aligned}$$

which is increasing in productivity q and is bounded from above.

[31] Also, I assume as in Stole and Zwiebel (1996) that the conditions for the existence of the integral in (B6) are satisfied.

Finally, the worker's utility

$$\omega(q,n) = \Big(w(q,n) - g\big(h(q,n)\big)\Big) \tag{B11}$$
$$= \beta(\xi-1)\frac{\chi\big((\xi-1)\rho+1\big)}{(\xi-1)\rho+1-\beta\xi}\left(\left(\frac{\rho-1}{\chi\xi\rho}\right)^{\rho} Y\frac{q^{\rho-1}}{n}\right)^{\frac{\xi}{(\xi-1)\rho+1}} + (1-\beta)b,$$

is decreasing in employment and increasing in productivity.

C. Simulation

C.1 Solving for an equilibrium

Given the model parameters r, β, ρ, μ, δ, η, χ, ξ, b, s_0, m, ζ, κ, $c(\cdot)$, $\Phi(\cdot)$, the solution algorithm can be described as a fixed point search of equilibrium variables (θ, u, Y), and distribution functions $F(W)$ and $G(W)$ through the mapping, which is defined in equations (7), (8), (11), (14) - (24). I solve for the equilibrium numerically applying iteration on this mapping. The iteration procedure turned out to be more stable than looking for a fixed point using minimum distance routines.

Note that the value functions and steady state equilibrium equations can be rewritten in terms of $\hat{q} = Y^{\frac{1}{\rho-1}}q$. In this way, I reduce the number of equilibrium variables that I iterate over. Thus, I start with the distribution of $\hat{q}$ and solve for an equilibrium. Then I derive the aggregate output as

$$Y = \left(\frac{\rho-1}{\chi\xi\rho}\right)^{\frac{\rho-1}{(\xi-1)\rho+1}} \int \hat{q}^{\frac{\xi(\rho-1)}{(\xi-1)\rho+1}} \sum_{n=0}^{\overline{n}(\hat{q})} n^{\frac{(\xi-1)(\rho-1)}{(\xi-1)\rho+1}} K_n(\hat{q})d\hat{q},$$

and recover the underlying productivity q from $q = \hat{q}Y^{\frac{1}{1-\rho}}$.

I discretize the state space in terms of productivity and use Gaussian (Gauss-Laguerre) quadrature method to approximate the expected value of any function of $\hat{q}$ (see Judd (1998) for details). I use ten nodes for the productivity distribution. Here, I assume that productivity $\hat{q}$ follows Generalized Pareto Distribution with the density

$$\frac{1}{\sigma}\left(1 + k\frac{\hat{q}-\underline{\hat{q}}}{\sigma}\right)^{-\frac{1}{k}-1},$$

where k is a shape parameter, σ is a scale parameter, and $\underline{\hat{q}}$ is a location parameter. The mean of the distribution is $\underline{\hat{q}} + \frac{\sigma}{1-k}$ for $k < 1$ and variance is $\frac{\sigma^2}{(1-k)^2(1-2k)}$ for $k < 1/2$.

First, I compute the firm's profit and the worker's utility from equations (B10) and (B11). Then, given the initial guess for the distribution functions $F(W)$ and $G(W)$, market tightness θ, and the unemployment rate u, I construct the separation and offer acceptance rates. I apply the value function iteration procedure to find the firm's value $V_n(\hat{q})$, the value of employment $W_n(\hat{q})$, and the value of unemployment U. Note that at each step we need to verify that the worker's and firm's participation constraints are satisfied.

The optimal vacancy posting rate $v_n(q)$ and the maximum labor force size $\overline{n}(q)$, derived from the firm's problem, are then used to find a steady state distribution of products across types, $K_n(\hat{q})$. Given the state space is discretized in the numerical solution algorithm, the equations (15) - (17) represent a linear programming problem that can be solved for directly. However, due to numerical approximation imprecisions, the iteration over the product distribution turned out to perform better. Using equations (19) - (24), I update the initial guess for distribution functions $F(W)$ and $G(W)$, the unemployment rate u, and market tightness θ. I then repeat the procedure until the convergence of equilibrium objects is achieved.

C.2 Simulation

The equilibrium hiring and separation rates, $H_n(\hat{q})$ and $s_n(\hat{q})$, as well as the maximum labor force size $\overline{n}(q)$, are the key variables that determine employment dynamics at the firm level. Given Poisson arrival rates, the waiting time until the next occurrence of any shock is distributed exponentially with parameter $\vartheta = \mu + \delta + H_n(\hat{q}) + s_n(\hat{q})n$. Thus, I generate a time path for each of the simulated firms as a random draw from an exponential distribution. Whether it is a destruction shock, a productivity shock, a new hire, or a separation is decided according to the relative probability of each event.

I simulate 1000 firms for 120 months and discard first 30 months under the assumption that the economy will converge to a steady state equilibrium within the first 30 periods. The simulated monthly employment series includes all workers who were employed in a given month. Quarterly employment is an average of three month employment. The average hours series is constructed according the pension contribution schedule to copy the lower bound hours measure reported in the data. Wage, revenue, and hours variables are aggregated over three months to generate the corresponding quarterly series. In the data, I construct quarterly series of work hours, wages, and value added only for those firms which have employment in all three months of a quarter to avoid spurious negative correlation between the growth rates of hours and employment. I apply the same sample selection to the simulation.

To fit the empirical size distribution of firms, in particular, a long right tail of the distribution, I assume that firms act as a collection of product lines and that each product faces its own hiring and separation process. The number of products per firm is exogenous and independent of the firm's productivity level q. In that case, all steady state equilibrium equations hold, and we can think of $K_n(q)$ as the distribution of product lines. The number of product lines for each firm is drawn from a Poisson distribution with the average number of products of two at the start of the simulation and kept constant thereafter.

Firm's underlying productivity $\hat{q}$ is drawn from a Generalized Pareto Distribution. To ensure that the model is able to capture a positive association between wages and employment, the productivity distribution needs to have a long right tail. The reason for that is that the positive type-composition effect of larger firms being on average more productive has to offset the negative effect of employment on wages. Thus, a Pareto distribution is preferred over, for instance, a lognormal distribution. The scale and shape parameters are set to match the observed dispersion and the mean to median ratio of firm size. A higher value of the shape parameter, k, means that the size distribution has a longer right tail.[32] For a given draw of productivity, to obtain the corresponding hiring and separation rates, I use a linear interpolation between $\hat{q}$ nodes.

The destruction rate parameter δ can be identified directly from the exit rate of firms. Monthly exit

[32] Note that if the following parameters - $\underline{q}$, σ, b, c_0, and χ - are all increased by the same factor then the resulting equilibrium does not change. Hence, I need to normalize one of the parameters. I use a Generalized Pareto Distribution with parameters $\underline{\hat{q}}$, $\sigma = 1.25\underline{\hat{q}}$, and $k = 0.45$ for $\hat{q} = Y^{\frac{1}{\rho-1}}q$ as described in Appendix C.1. I then solve for steady state equilibrium aggregate output, Y ,and use it to recover underlying productivity q. I scale up the above parameters to match average monthly wages in the data. Hence, the resulting productivity distribution has a mean of DKK 587 and a standard deviation of DKK 1290. For comparison, the hourly wage rate is between DKK 130 and DKK 207 depending on whether it is computed using the upper or lower bound measure of hours.

rate is defined as a ratio of exiting businesses to all businesses, where exiting firms are the ones with positive employment in month t and zero employment in month $t+1$. Under this definition, the average (employment-weighted) monthly exit rate in the data over the period of 1999-2006 is 0.8%. However, many (small) firms 're-enter' the market, that is, they have zero workers during one month and then positive employment the next month. Note that this can also happen in the model: the firm may have zero employment during one month if all of its workers have separated and positive employment next month as it hires new workers. Thus, to distinguish between the destruction rate δ and other separations in the data, I refine the definition of exit as a fraction of firms that exit the labor market and do not re-appear over the period of 1999-2006. In that case, the monthly exit rate is 0.52%, which is the value for the destruction rate δ used in the simulation.