Low-energy quasiparticle transport through Andreev levels

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We measure the resistance of a normal mesoscopic sample with two superconducting mirrors and find two regimes with qualitatively different behavior. At temperatures below 90 mK peaks in the conductance were found when the phase difference between the two superconductors is an odd multiple of π. The peak heights increase with decreasing temperature. Above 100 mK the observed peaks give way to dips in the conductance. While the high-temperature behavior can be explained in terms of the thermal effect [Phys. Rev. Lett. **76**, 823 (1996)], we propose that the low-temperature behavior is a manifestation of resonant transmission of low-energy quasiparticles through Andreev states. [S0163-1829(99)11641-2]

Recent experimental and theoretical work on diffusive charge transport in mesoscopic normal superconductor (N/S) samples have revealed a strong energy dependence of an excess quasiparticle contribution to the low-temperature conductance of normal parts in close proximity to superconductors.[1] A characteristic energy is set by the Thouless energy E_{Th} below which a reentrance to normal conduction is seen as the bias voltage or temperature is lowered. In samples with two N/S interfaces the conductance oscillates as a function of a phase difference ϕ between the two superconductors. Conductance maxima occur at even multiples of π; their magnitude peaks at E_{Th} and becomes vanishingly small at low energies. These oscillations have been explained in Ref. 2 as a ''thermal effect.''

In this paper we report on the experimental observation and the theoretical explanation of a low-energy, phase-modulated transport phenomenon in diffusive Andreev interferometers. Several features of the observed conductance oscillations are significantly different from what has been seen previously: (i) as a function of energy (temperature or bias voltage) the oscillation amplitude has not only a maximum around E_{Th} but another at lower energy, (ii) at low temperatures the positions in ϕ of the conductance maxima shift from even to odd multiples of π, (iii) the line shape of the low-energy oscillations strongly differs from being sinusoidal and has a resonant character, and (iv) current-voltage characteristics taken at different ϕ intersect.

The experimental results are explained using the theory of Ref. 3, where it was shown that a strong interference effect due to resonant transmission of quasiparticles through Andreev levels becomes pronounced at low temperatures. Being valid in the diffusive transport regime, the theory is a generalization of a previously developed theory for ballistic electrons.[4] It leads to a ''giant'' resonance effect at temperatures well below E_{Th}, where the thermal effect of Refs. 2 and 5 is less prominent. The transition between the two regimes as the temperature is lowered is accompanied by a phase shift of π in the conductance oscillations as observed.

Our samples consisted of a normal conductor made of silver in the shape of a cross to which a superconducting wire was attached at two points, as shown in Fig. 1. The phase difference ϕ between the N/S interfaces at points C and D was created by applying a magnetic field H perpendicular to the structure. Using the four-terminal method, we measured the resistance of the normal part AB as a function of ϕ using measuring leads I_1, I_2, U_1, and U_2. We performed dc as well as low-frequency ac measurements using lock-in and modulation techniques in the frequency range of 30–300 Hz in magnetic fields of less than 100 G. The measurements were done at temperatures between 20 mK and 1.7 K. The π shift was found in three of four samples. The fourth sample had been kept in air; we believe this led to a degradation of the Ag and the Al/Ag interfaces, which shows the effect to be structure dependent.

The technique used to fabricate normal, insulating, and superconducting layers of the structures was described in Ref. 6. The area S of the N/S interface was about 100

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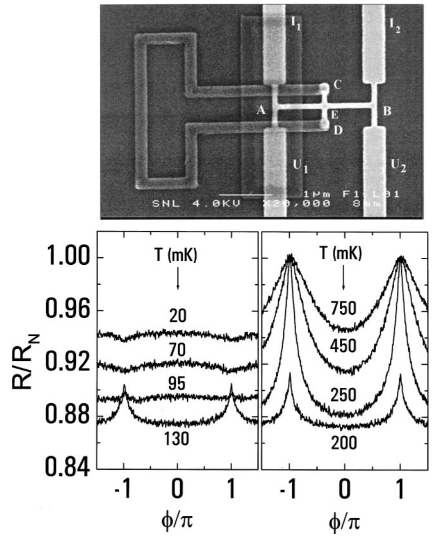

FIG. 1. Zero bias resistance as a function of the superconducting phase difference ϕ between two S/N interfaces. A magnetic flux Φ through the rectangular loop formed by the superconducting part of the mesoscopic sample determines $\phi=2\pi\Phi/\Phi_0$; $\Phi_0=h/2e$.

$\times 200\ \mathrm{nm}^2$. The distance $L_N=AB$ between the normal leads was 2000 nm with $AE=EB=L=1000$ nm. The distance between the N/S interfaces, $L_S=CD$, was made much smaller than in structures investigated so far and was 500 nm, with $CE=ED$.[7] The diffusion coefficient D of conduction electrons in silver was, as calculated from the measured value of the resistance, about 80 cm^2/s and the coherence length ξ_N was 100 nm at 1 K. The phase breaking length l_φ of electrons in silver was estimated to be approximately 1500 nm using weak localization measurements in long coevaporated wires. We have found the resistance of the N/S barriers to be of the order of the resistance of the normal wires. With the above estimates for D and L_S, the Thouless temperature $T_{\mathrm{Th}}\equiv E_{\mathrm{Th}}/k_B=\hbar D/k_B L_S^2$ is 200 mK in agreement with our experiments.

Measured values of the zero-bias resistance R are shown in Fig. 1 for temperatures between $0.1T_{\mathrm{Th}}$ and $3.5T_{\mathrm{Th}}$. The deficit resistance ΔR as a function of temperature at zero bias and as a function of bias voltage at about 20 mK is shown in Fig. 2 for $\phi=0$ and $\phi=\pi$. Since both temperature

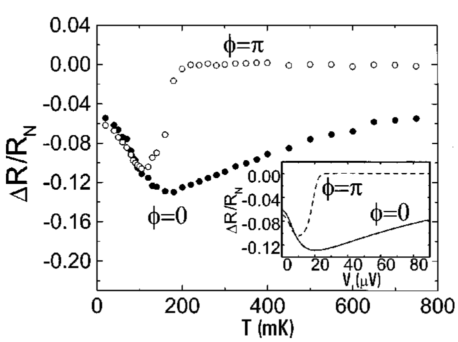

FIG. 2. Temperature dependence of the deficit resistance $\Delta R=R-R_N$ for $\phi=\pi$ and $\phi=0$. The sample is the same as in Fig. 1 and $R_N=[(2e^2/h)N_\perp]^{-1}$, $N_\perp=S/\lambda_F^2$ (λ_F is the Fermi wavelength). The dependence of the deficit resistance on bias voltage is shown in the inset.

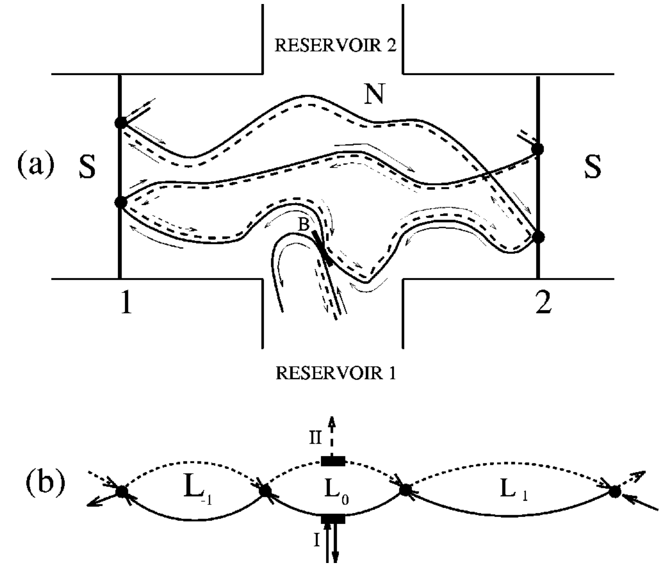

FIG. 3. (a) Semiclassical quasielectron (full lines) and quasihole (dashed lines) trajectories giving the main contribution to the phase sensitive part of the conductance. Thick lines indicate the presence of ''beam splitters'' (see text). (b) The excess conductance (deficit resistance) can be viewed as being due to resonant transmission through localized states formed in a one-dimensional chain of barriers (filled circles).

and bias voltage control the characteristic quasiparticle energy E, the temperature and voltage dependence are essentially the same. Below we will discuss the temperature dependence.

The temperature and ϕ dependence of the resistance above $\sim$150 mK are in good agreement with predictions based on the thermal effect;[2,5] $\Delta R_{\phi=0}$ has a minimum at $T_{\mathrm{Th}}\sim 200$ mK, while, as shown in Fig. 2, $\Delta R_{\phi=\pi}$ does not depend much on temperature in this regime; above $\sim$150 mK, R has broad minima around $\phi=0$ (even multiples of π) with a maximum depth when $T\sim T_{\mathrm{Th}}$, while as shown in Fig. 1 the peak heights when $\phi=\pm\pi$ (odd multiples of π) are temperature independent.

As we lower the temperature, we observe, starting from $\sim$140 mK, a drop in $\Delta R_{\phi=\pi}$ to a rather deep minimum at 100 mK (Fig. 2). This contradicts the theory,[2] which does not predict any minimum in $\Delta R_{\phi=\pi}(T)$. A related unexpected drop of the peak heights in R at $\phi=\pm\pi$ can be seen in Fig. 1. It is interesting to note also that the width of these peaks first decreases when the temperature is lowered in agreement with theory[2] but then it eventually saturates at $\sim 0.2\pi$ around 120 mK. We believe that this behavior is due to a variation of the condensate phase ϕ along the N/S boundaries.[8] One can estimate from Fig. 3 of Ref. 2 that an uncertainty in ϕ of order 0.2π would result in a resistance minimum in $\Delta R_{\phi=\pi}(T)$ consistent with our experiment. The effect of an uncertainty in ϕ on $\Delta R_{\phi=0}$ is very small since ΔR does not vary much with ϕ in the vicinity of even multiples of π.

Below 140 mK the height of the peaks in R at $\phi=\pm\pi$ continues to decrease as the temperature is lowered. When $T\sim 90$ mK they have disappeared completely and the ϕ oscillations of R have vanished. With a further decrease of temperature $\Delta R_{\phi=\pi}(T)$ dips below $\Delta R_{\phi=0}(T)$ (Fig. 2) and *minima* in R develop at $\phi=\pm\pi$ (Fig. 1). In terms of conductance we hence observe a π shift of the maxima in the conductance oscillations with ϕ from even to odd multiples of π as we decrease the temperature.[9] The amplitude of the π-shifted conductance oscillations increases from zero at $T = 90$ mK and seems to saturate at the lowest temperatures (see Fig. 2).[10]

The π shift of the oscillations and the change in the temperature dependence of the oscillation amplitude indicate different physical origins of the oscillation phenomena observed below and above $T \sim 90$ mK. While the thermal effect clearly can explain the high-temperature results we believe that resonant quasiparticle transport through Andreev levels is responsible for the observed low-temperature behavior of the conductance oscillations.[11] For the case of an N part separated from the reservoirs by low-transparency potential barriers, such resonant transmission has indeed been predicted[3] to lead to a π shift (recently observed[12] although its temperature dependence below T_{Th} was not measured). Below we show that the potential barriers of Ref. 3 may not be necessary for this effect to occur if isolated, extended defects such as grain boundaries or twin boundaries are present. Such defects serve as ''beam splitters''[13] in the sense that they split the semiclassical quasiparticle trajectory by ''quantum'' scattering providing low transparency, $\epsilon_r \ll 1$, for trajectories oriented nearly parallel to the defects.[14] A low concentration of such splitters will lead to a coexistence of the two mechanisms for conductance oscillations discussed in Refs. 2 and 5 and 4 and 3 and, hence, to a temperature-induced π shift.

If the mean free path (l), with respect to scattering by the beam splitters, exceeds the linear sample size L ($L \sim L_S$) the thermal effect[2,5] controls the phase and temperature dependence of the conductance to zero order in $L/l \ll 1$. To first order in L/l the correction to the conductance is due to the quasiparticle scattering at only a single beam-splitting extended defect. As will be shown this correction, being of a resonant character, gives the dominant contribution to the phase-sensitive part of the conductance at low temperatures $T \ll T_{\mathrm{Th}}$, where the thermal effect vanishes.[2,5]

A typical classical trajectory relevant to our problem is presented in Fig. 3. It starts in one reservoir as an electron trajectory (solid line), then crosses the splitter B before being either normally reflected or Andrev reflected a number of times at N/S boundaries and finally tunneling through the splitter a second time ending up in the reservoir as a hole trajectory (dashed line). The probability for an electron to be reflected as a hole in this way determines the above-mentioned contribution to the phase-sensitive conductance. We shall calculate the amplitude of the electron-hole reflection in the semiclassical approximation (assuming λ_F to be the shortest length in the problem in between beam splitters). Within such an approach one can find the wave function of the scattered quasiparticles by propagating the incident wave along classical paths determining its phase from the classical action $S = \int p\,dl$ along the path. The trajectory in Fig. 3(a) determines a number of different paths corresponding to different numbers of successive Andreev reflections. If $E \ll E_{\mathrm{Th}}$ and ϕ close to an odd multiple of π, all different paths interfere constructively[15] leading to resonant transmission through Andreev levels.

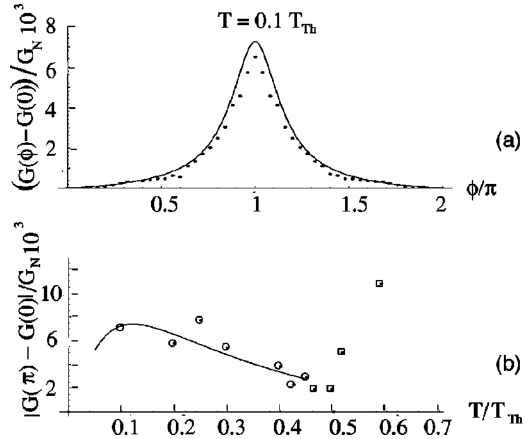

FIG. 4. (a) Phase dependence of the conductance at $T = 0.1 T_{\mathrm{Th}} = 20$ mK. Fitting was done using Eq. (1) with $|r_N^{(1)}| = 0.4$, $|r_N^{(2)}| = 0.1$, and $\rho = 1/120$. (b) Temperature dependence of the oscillation amplitude. Experimental data [circles for $G(\phi = \pi) > G(\phi = 0)$, and squares for $G(\phi = \pi) < G(\phi = 0)$] are compared with theory (solid line).

There is an analogy between the above problem and the one presented in Fig. 3(b). The latter corresponds to the transmission of a quasiparticle (injected as an electron at point I and exiting as a hole at point II) through a one-dimensional (1D) chain. Each scattering at an N/S boundary is represented in the chain by a dot where two-channel (Andreev and normal) scattering takes place. L_i is the length of the quasiparticle path between successive scatterings at N/S boundaries. Different sections of the chain have different lengths; the chain of Fig. 3(b) is a 1D system with randomly distributed scattering centers. For $E = 0$ quasiparticle localization does not take place because of the complete compensation of the electron and hole phase gains between the scatterers. When $E \neq 0$ this compensation is not complete and the problem reduces to the conventional one of a particle with energy E moving in a disordered chain, where localization does occur. Below we shall consider the limit of weak normal scattering $r_N^{(1,2)} \ll 1$ ($r_N^{(1,2)}$ are the probability amplitudes for normal reflection at N/S boundaries 1 and 2), which allows a sharp resonant transmission from point I to point II through the discrete (Andreev) energy levels corresponding to quasiparticle states in a disordered 1D chain localized around the section of injection [Fig. 3(b)]. Solving the problem of Fig. 3(b) we have found the probability of electron-hole resonant transmission through an energy level E_α (Ref. 16) to be of the Breit-Wigner form, $T(E,\alpha) \propto \epsilon_r^2 / \{[(E - E_\alpha)\tau_0]^2/\hbar^2 + \epsilon_r^2 \times \mathrm{const}\}$, where τ_0 is the propagation time in the section of injection.

In order to find the total electron-hole transmission $T_{eh}(E)$ one has to sum $T(E,\alpha)$ with respect to the starting points of the semiclassical trajectories inside the reservoir, which cross all relevant splitters. Classical paths separated by a distance greater than λ_F meet different ''random'' sets of impurities, and hence their path lengths L_n as well as the corresponding propagation times τ_n are randomly distributed. Therefore, the summation over starting points is equivalent to averaging the transmission probability with respect to realizations of the times τ_n. The distribution of propagation times τ_n depends on details of the disordered potential in the mesoscopic normal region. It is natural to assume that propagation times along different sections of the trajectory are uncorrelated. Under this assumption it can be shown that such an averaging is equivalent to summing $T(E,\alpha)$ over the resonant energy E_α. After such a summation only the density of localized states remains. For the configuration of Fig. 3(b), the summation can be carried out exactly if we choose a Lorentzian distribution for the propagation times, $\overline{P}(\tau)=\gamma/\pi[(\tau-\overline{\tau})^2+\gamma^2]$, where $\overline{\tau}=L_S^2/D$. Choosing $\gamma\approx\overline{\tau}$ and using the Landauer-Lambert formula[17] we find that the resonant part of the conductance can be expressed as

$$G=G_N\frac{\rho}{\overline{T}\sqrt{2}}\int_0^\infty \frac{x}{\cosh^2(x/2\overline{T})}\times\left\{\frac{\sqrt{(4x^4+\epsilon_a^4)(4x^4+\epsilon_b^4)}+\epsilon_a^2\epsilon_b^2-4x^4}{(4x^4+\epsilon_a^4)(4x^4+\epsilon_b^4)}\right\}^{1/2}dx, \quad (1)$$

where $G_N=(2e^2/h)N_\perp$ and $\rho=\epsilon_r N/N_\perp$ is the parameter that characterizes the transport properties of the splitters involved in Andreev scatterings,[20] $\epsilon_{a,b}=[\delta\phi^2+(|r_N^{(1)}|\pm|r_N^{(2)}|)^2]^{1/2}$, $\delta\phi$ is the minimal value of $|\phi-\pi(2l+1)|$, $l=0,\pm1,\ldots,$ and $\overline{T}=T/T_{\rm Th}$. Equation (1) is valid for $\epsilon_r\ll(|r_N^{(1)}|+|r_N^{(2)}|)/2\ll1$.

Comparing theory and experiment we use three free parameters $|r_N^{(1)}|$, $|r_N^{(2)}|$, and ρ, which enable the ϕ dependence of the conductance and the temperature dependence of the oscillation amplitude to be fit quite well (see Fig. 4). The decrease in amplitude at low temperatures shown in Fig. 4 is due to an asymmetry in the normal scattering, $r_N^{(1)}\neq r_N^{(2)}$, required to reproduce the observed saturation of the amplitude at 20 mK.[18]

In conclusion, we have observed conductance oscillations in an N/S sample of the Andreev interferometer type; as the temperature is lowered below the Thouless temperature a π shift of conductance oscillations is observed whose maxima occur when the superconducting phase difference ϕ is an *odd* rather than an *even* multiple of π. We explain the low-temperature oscillations as resonant transmission of low-energy quasiparticles through Andreev levels.

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[1] For a recent review see C. J. Lambert and R. Raimondi, J. Phys.: Condens. Matter **10**, 901 (1998).
[2] Yu. V. Nazarov and T. H. Stoof, Phys. Rev. Lett. **76**, 823 (1996).
[3] H. A. Blom *et al.*, Phys. Rev. B **57**, 9995 (1998).
[4] A. Kadigrobov *et al.*, Phys. Rev. B **52**, R8662 (1995).
[5] A. F. Volkov *et al.*, J. Phys.: Condens. Matter **8**, L45 (1996).
[6] V. T. Petrashov *et al.*, Pis'ma Zh. Éksp. Teor. Fiz. **67** 489 (1998) [JETP Lett. **67**, 513 (1998)].
[7] Different arm lengths, $EC\neq ED$, do not give a π shift; see V. T. Petrashov *et al.*, Phys. Rev. B **58**, 15 088 (1998).
[8] The phase gradient induced by the magnetic flux gives a change in ϕ of about 0.05π across the N/S interface for the geometry of our experiment.
[9] A similar π shift with decreasing bias was seen by E. Toyoda and H. Takayanagi, Physica B **249-251**, 472 (1998).
[10] In all our previous experiments the distance between the S contacts was up to four times larger than here; hence $T_{\rm Th}$ was up to one order of magnitude lower. Therefore the temperature was not far enough below $T_{\rm Th}$ for the π shift to be observable.
[11] Conductance oscillations have been observed and discussed for ballistic conductors (Ref. 19) without any mechanism for the quasiparticle confinement necessary for proper Andreev levels and resonant effects to appear. Indeed, the observed conductance does not have localized peak typical for resonant phenomena. The same effect in the diffusive regime was discussed by A. F. Volkov and A. V. Zaitsev, Phys. Rev. B **53**, 9267 (1993).
[12] V. N. Antonov *et al.*, cond-mat/9803339 (unpublished).
[13] See Ref. 3 for a description of the role of beam splitters and disorder and for an explanation of the difference between the thermal and resonant transmission effects.
[14] J. M. Ziman, *Electrons and Phonons* (Oxford Univiversity Press, London, 1963), p. 244.
[15] This requires $H\ll H_c=\Phi_0 l_i^2/L_S^4$. In our experiment $H\approx0.5H_c$ and the orbital effect of the magnetic field may be neglected.
[16] The barrier transparencies are low and the dispersion in the distribution of propagation times inside the wells is of the order of the mean value $\overline{\tau}$. Hence, the localization radius is of order $\overline{L}=v_F\overline{\tau}$, and for a given time configuration there is only one resonant level in the energy range of interest $E\sim|r_N|E_{\rm Th}$.
[17] C. J. Lambert, J. Phys.: Condens. Matter **3**, 6579 (1991); **5**, 707 (1993).
[18] We do seem to observe a decrease of the oscillation amplitude below $0.1T_{\rm Th}$, but the data (not shown) are inconclusive; the asymmetry used, $r_N^{(1)}-r_N^{(2)}=0.3$, is $10-20\,\%$ larger than the experimentally measured asymmetry that arises due to imperfect alignment.
[19] A. F. Morpurgo *et al.*, Phys. Rev. Lett. **79**, 4010 (1997).
[20] The number N of trajectories that cross one grain boundary at sufficiently low angles to produce the required small value of ϵ_r, while also reaching both S/N interfaces, can be estimated using a simple model for boundary scattering. If a typical grain size is 100 nm one finds that $N/N_\perp\sim0.3$. Since from the width of the peaks in Fig. 1, ϵ_r can be at most ~0.1, the value of $\rho=1/120$ chosen to fit the experiments is reasonable.