The Measurement of Dielectric Constants, with a Note on Preliminary Work on the Location of Adsorption Bands

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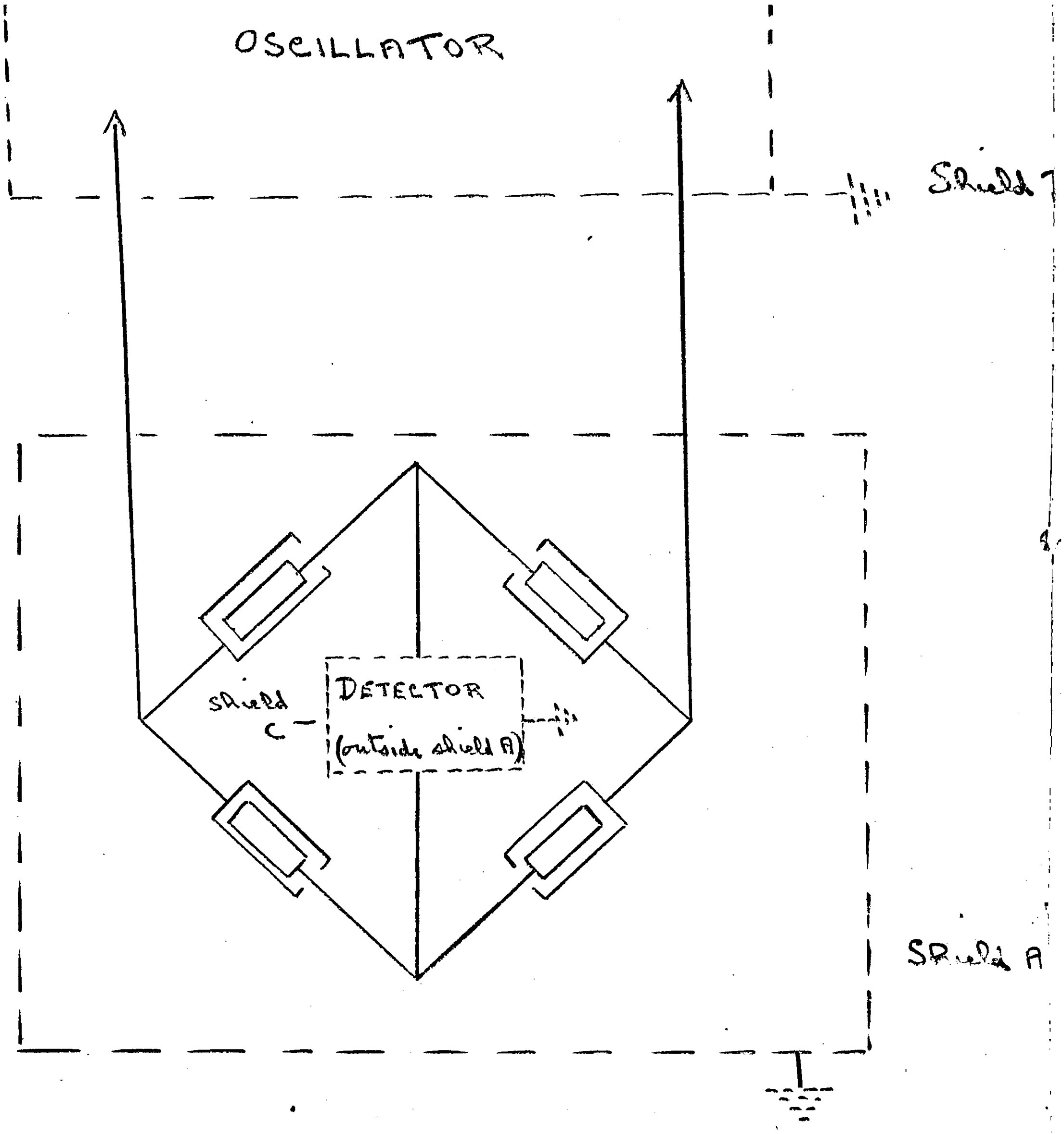
The Measurement of Dielectric Constants, with a Note on Preliminary Work on the Location of Absorption Bands.

Abstract.

A survey is made of methods which have been devised to determine dielectric constants, using alternating currents ranging from audio to kilo-megacycle frequencies. Bridge methods are described which involve the comparison of capacities in various forms of Wheatstone network. Many determinations have been made which depend on the detection of current or voltage resonance, and on the frequency dependance of oscillators on capacity in their circuits. Some free wave methods are described, which lead on to transmission line and co-axial line methods. For wavelengths smaller than 10cm. wave guides and resonators are most suitable. A brief description is given of a new method for detecting absorption regions in liquids.

The Measurement of Dielectric Constants.

The dielectric properties of substances are given by the permittivity or dielectric constant, ϵ, the permeability μ, and the effective conductivity, σ. According to Maxwell's theory, the dielectric constant $\epsilon = n^2$, where n is the refractive index, and many investigations were made into the values of ϵ and n since certain substances followed this rule fairly closely while others deviated widely from it. The theory of Debye suggested that the dielectric properties of all substances depended on distortion of the molecules in an electric field, while for polar molecules it depended also upon orientation of the molecules, and he predicted that sudden changes in the value of the dielectric constant would be found at the resonant frequency of different molecules, causing an ultimate decrease from the static value of the constant ϵ to the square of the refractive index at optical frequencies. As a result, a large number of measurements of dielectric constants and refractive indices were made over the widest available frequency range for investigating the validity of Debye's theory and other theories and modifications which were proposed later. The main methods used were bridge methods, various forms of resonance methods and, more recently, methods using wave guides and resonators. All these methods can be modified to give accurate results over different frequency ranges and under different conditions.



General arrangement of a high frequency bridge.

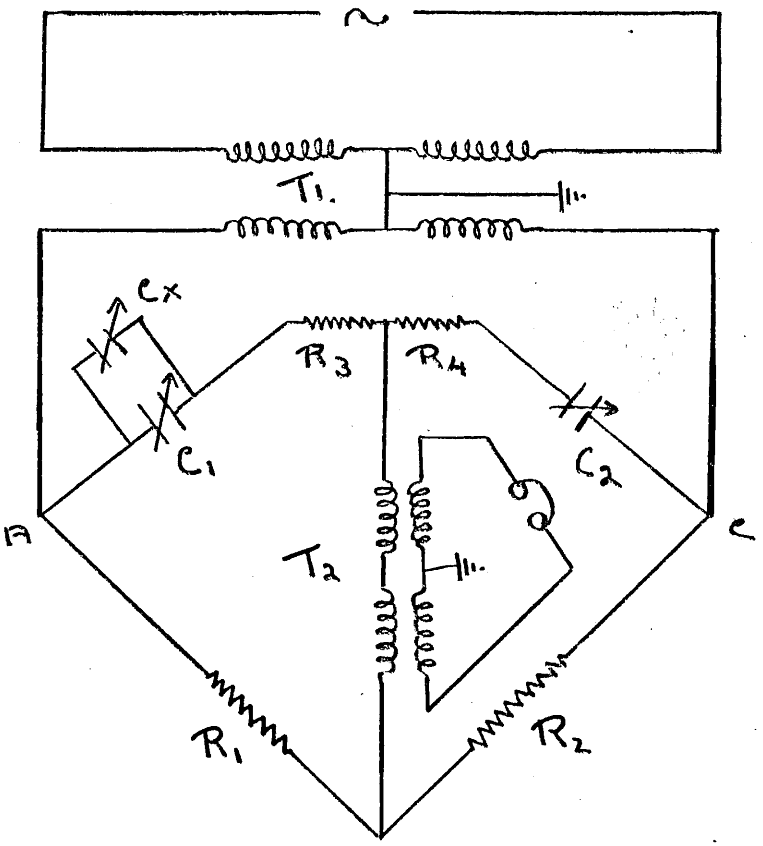
Bridge Methods.

Bridge methods have considerable advantages over other methods and for frequencies up to 5000c.p.s. the difficulties are so easily overcome that they can provide the most accurate measurements of dielectric constants. By means of a capacity bridge the capacity of a suitably designed condenser is measured, first when it is evacuated, C_v and then with the test dielectric between its plates, C_D. Then $\epsilon = \frac{C_D}{C_v}$ when all necessary corrections have been made. Bridge methods are unaffected by current fluctuations which is a very great advantage since the most carefully designed oscillators tend to vary in output unless complex precautions are taken. Providing a sensitive means of detection is used the balance point can be accurately found on a bridge, and being a null method of measurement it is not dependant on the calibration of the detector. In addition, by suitable choice of a bridge the effect of the conductivity of the specdmen may be balanced out.

One particular bridge, however, cannot be used over a very wide frequency range, but must be specially constructed for use within fairly narrow limits. The main difficulties with high frequencies are due to displacement currents. As the frequency is raised the current ceases to pass only through the wire network, and capacitances are formed between the components of the bridge, the detector and any nearby objects including earth, and stray currents flow through these capacities giving an untrue balance. The strength of these currents increases with frequency, with the potential difference between the objectsand with their relative proximity,

The difficulties are largely overcome by shielding. The current generator, the detector and the arms of the bridge should each be enclosed separately in a metal shield of high conductivity and these connected to earth, so that stray currents are eliminated between them and earth. In addition, each arm of the bridge is almost completely enclosed in a metal shield which is connected to the nearest point from the generator as shown, so that any capacitances between the arms merely shunt the generator. All leads are surrounded by earthed metal tubes. Leads to the detector are arranged as symmetrically as possible in order to balance out capacity effects to earth. AW Wagner earthing device could be employed to eliminate all capacities to earth, but the introduction of a more complex network and the lack of symmetry involved make this undesirable.

Another important difficulty in the use of high frequency bridges is the inductance of the leads and mutual inductance of the components. This is overcome by making all leads as short and rigid as possible, by using special non-inductive components and by arranging them so that there are no ~~no~~ loops in the circuit and that the leads from each component are parallel to each other. The simpler and more symmetrical the



L.S.Mc.Dowell's Method.

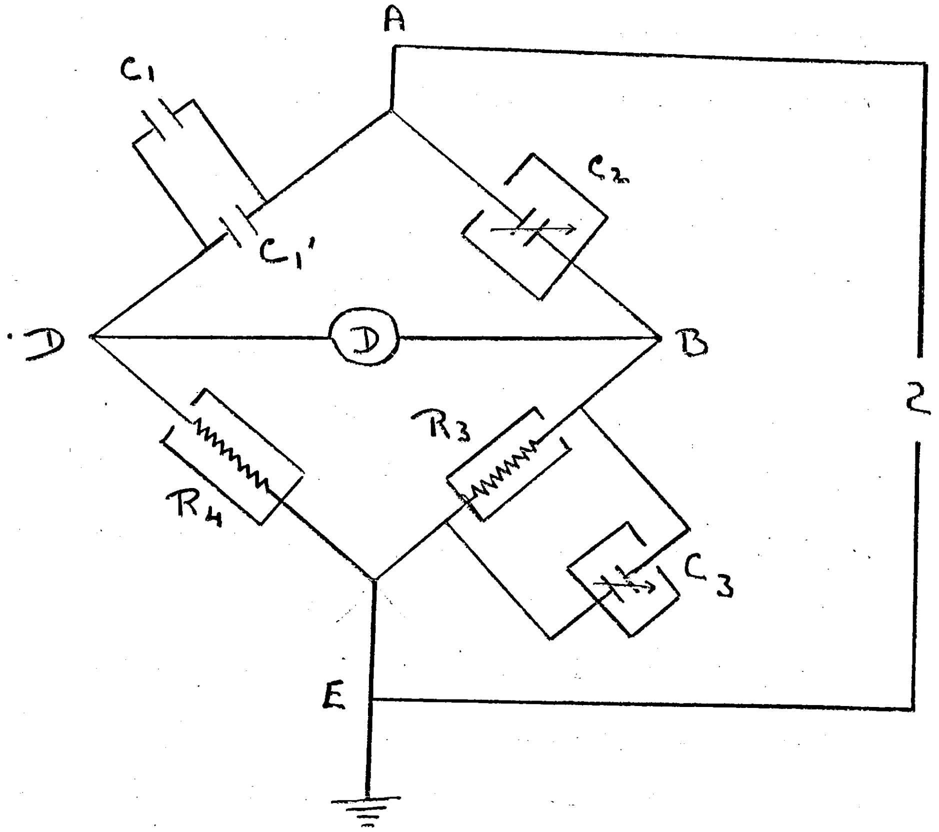
bridge, the smaller the inductance errors become at high frequencies.

The output of the generator used must be independant of the load in order to make determination of the balance point easy. The detector may be a thermo-couple, a valve-voltmeter or a heterodyne receiver with headphones to detect the balance as the audible beat-note disappears.

In an investigation of power loss in condensers with liquids as dielectric, (Phys. Rev. 1924 P.507)L.S.Mc. Dowell carried out measurements by means of Mc.Leod's bridge. The circuit was as shown, the oscillator and input transformer being in a room adjoining the experimental room, and primary and secondary were tuned to th same frequency. R_1 and R_2 were fixed non-inductive resistances, R_3 and R_4 variable non-inductive resistances, C_1 and C_2 variable precision condensers and C_x the test condenser. C_x was a heavily nickel-plated, conical-shaped condenser, the inner cone being supported by a pyrex rod and the distance between the cones being variable. It could be taken to pieces, washed and re-assembled between readings. C_x was filled with the oil under test and left for 24 hrs. to reach room temperature. C_x was then placed in parallel with C_1 and the bridge was balanced, varying C_2 and R_4 with suitable values of C_1 and R_3 At balance,

$$R_2R_3 = R_1R_4 \quad \text{and} \quad \frac{R_2}{C_1+C_x} = \frac{R_1}{\cancel{R_3}C_2}$$

C_x was then replaced by a precision condenser and the



Shering and Semm's Bridge used by

H. Harris.

bridge balanced again, the value of C_x being found by direct substition. By measuring R_4 at balance with no condenser in parallel with C_1 the series resistance R_x of C_x could be found since the series resistance of C_x and C_1 in parallel, R_s, was given by

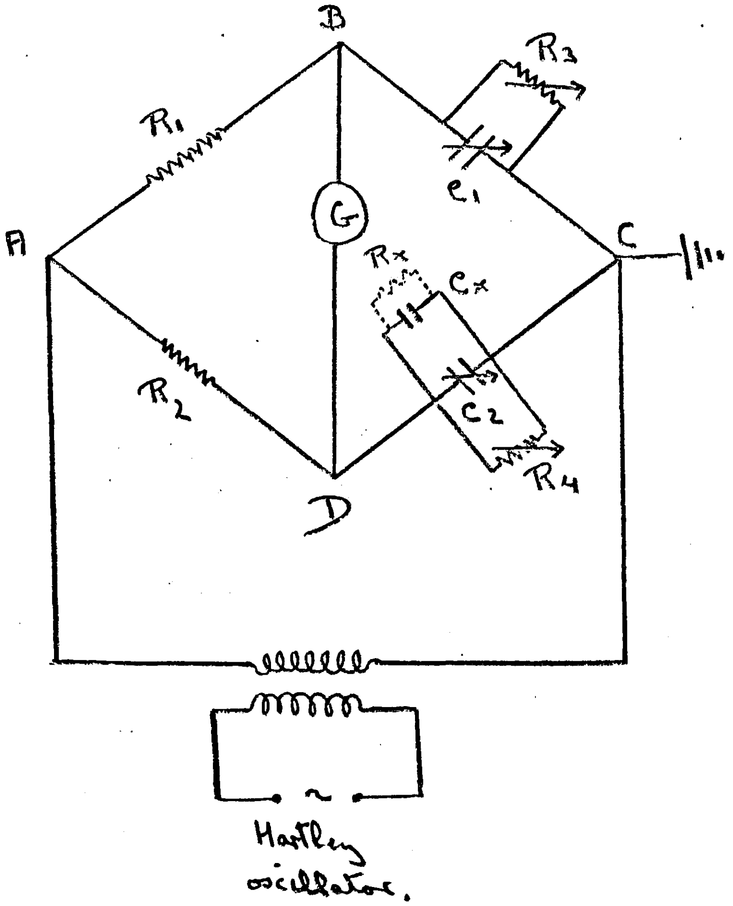
$$R_x = \frac{R_s(C_x+C_1)^2}{C_x^2}$$

The dielectric constants of liquids could be compared by comparing the values of C_x when immersed in the respective liquids. It was important that R_1 R_2 and R_4 should be pure non-inductive resistances, Any small deviations from this condition, and errors due to stray capacity effects were largely eliminated by the substitution procedure. Loose coupling to the oscillator and tuning of the bridge to resonance before making an accurate balance ensured the absence of harmonics.

H. Harris used a Schering bridge for the measurement of dielectric constants of a number of liquids. (Journ. Chem. Soc. 1925 P.1049) The generator used gave a pure sine wave of frequency variable between 500 and 5000c.p.s. and it was screened to prevent interaction with the bridge. R_3 and R_4 were screened 1000ohms resistances, C_1 was the test condenser which consisted of two brass cylinders with end-pieces of compressed fibre. It was fixed in a glass container which held the liquid dielectric and great care was taken to remove air-bubbles. C_2 and C_3 were variable precision condensers, C_2 being used to give the value of C_1 and C_3 to balance out the conductivity effect of the dielectric. The generator and bridge telephone detector were 14ft. apart and several feet from the bridge to reduce inductive effects. Connections were made by means of mercury cups and all unscreened apparatus was supported on ebonite stands to reduce stray capacities. At balance, if ρ_1 was the equivalent series resistance of C_1

$$\rho_1 = \frac{C_3}{C_2} \cdot R_4 \quad \text{and} \quad \frac{C_1}{C_2} = \frac{R_3}{R_4}$$

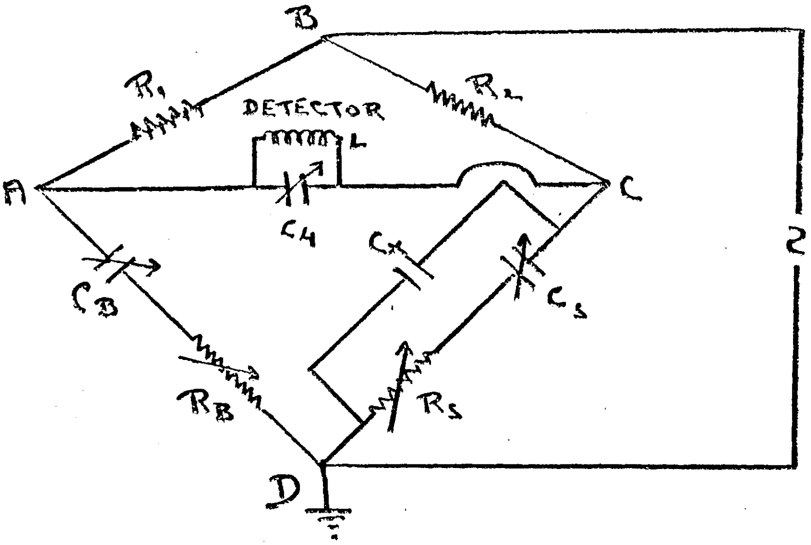
Byvarying C_2 and C_3 alternately a balance was obtained first with air between the condenser plates and then with different liquids between them. The dielectric constant could thus be found from the ratio of the capacities. Corrections were applied for the effect of the fibre insulating end-pieces in the test condenser. The earthed screens and the fact that the arms DE and EB were of low impedance compared with DA and AB and hence that D and B were nearly at earth potential eliminated stray capacities. By connecting the shield and leads in without the test condenser a correction was made for the lead capacity. In order to measure high dielectric constants and still have fairly large capacities to measure, series of dielectric constants were determined, not absolutely with respect to air or a vacuum, but with respect to each other using test condensers of different sizes e.g. benzene with respect to air, chloroform with respect to ~~to~~ benzene etc. with increasing values of dielectric constant, so that the smallest capacity that had to be measured

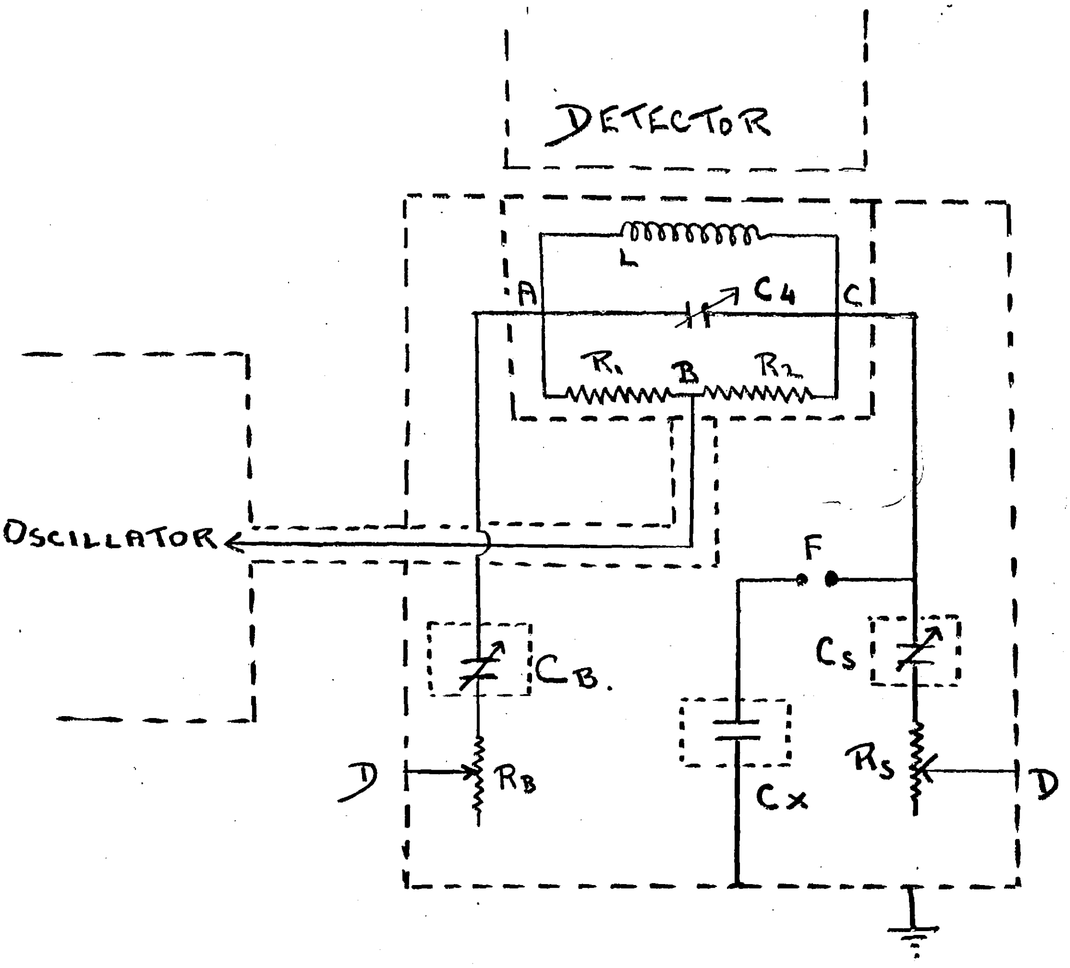


Bridge used by Krchma and Williams.

was 400p.f. The disadvantage of this procedure was that it was indirect and the errors in each measurement were added together in the final result.

I. J. Krchma and J. W. Williams (Journ. Am. Chem. Soc. 1927 P.2408) measured the dielectric constants of a number of binary mixtures at frequencies of the order 1m.c.s. The resonance method they used was unsuitable for conducting liquids so they devised a bridge method and checked their results by measurements on pure non-conducting liquids by both methods. R_1 and R_2 were non-inductive 1000ohms resistances. C_1 was a variable air condenser of maximum capacity 2000p.f. which was shunted by a variable resistance R_3. C_2 was a standard variable condenser in parallel with the test condenser C_x which had an equivalent parallel resistance R_x due to leakage. C_2 was shunted by the variable non-inductive liquid resistance R_4. G was a two-stage amplifier which could be adjusted to produce audible beats with the bridge oscillations which disappeared when the bridge was balanced. The balance of this bridge depended on the frequency, and variations in impedance of any components of the bridge would have varied the frequency, so that any variation in C_x had to be compensated by C_2 and R_4 for balance. C_x was a variable ~~pl~~ plate condenser contained in a Pyrex vessel and having a perforated copper container bolted to a thermostat, thus allowing water to circulate around the Pyrex





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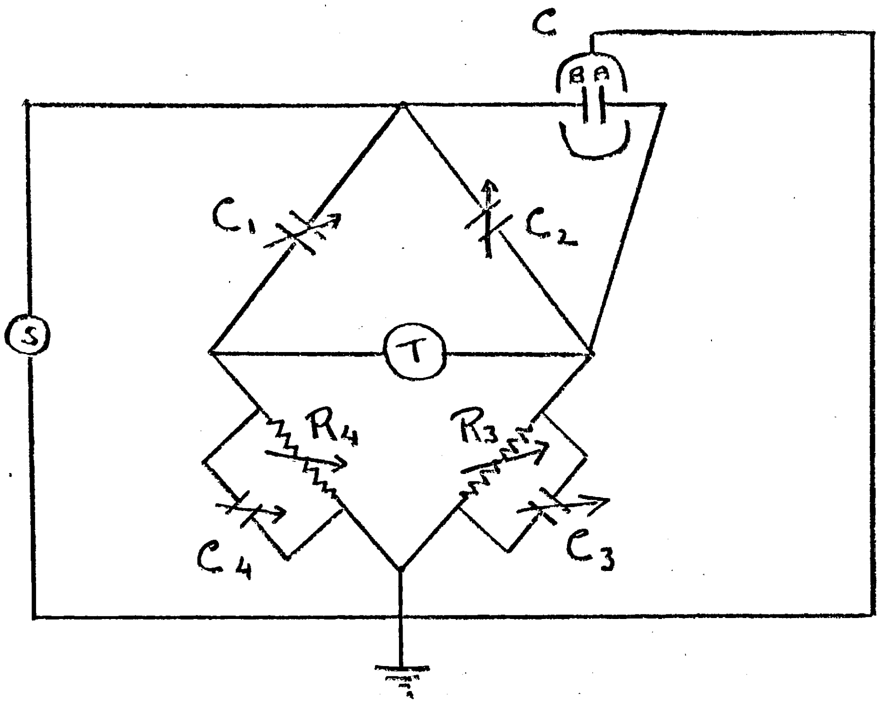
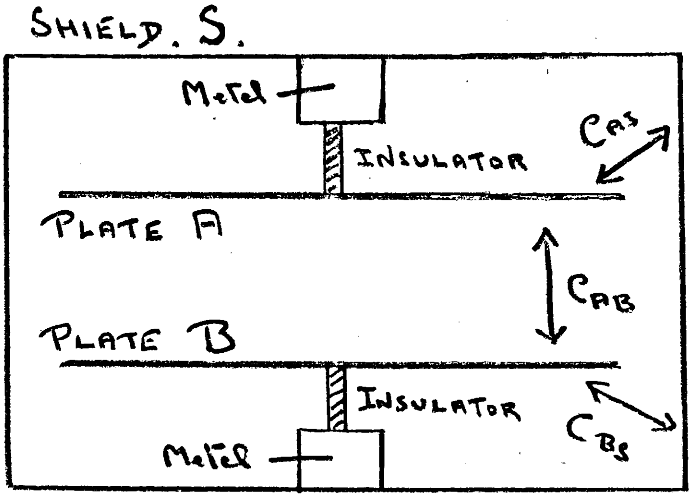
Method of Smyth, Morgan and Boyce.

vessel yet still protecting the condenser from stray capacities. The change in capacity of the condenser as its plates were moved from position AA to position B had already been found by the resonance method used previously, and the bridge was brought to balance with the plates in these two positions with liquid in the condenser. R_4 was adjusted to balance out the conductivity effect. The dielectric constant was then given by the difference in the two values of C_2 divided by the known change of C_x in air. This method gave good agreement with the resonance method for pure liquids and by means of it a number of binary systems were investigated.

In a paper on the dielectric polarisation of liquids, C.P.Smyth, S.O.Morgan and J.C.Boyce, (Journ. Am. Chem. Soc. 1928 P.1536) described a bridge method of measuring dielectric constants at 500k.c.s. The circuit diagram and symmetrical arrangement of the apparatus were as shown, the detector, oscillator and bridge being shielded from each other as completely as possible. The heavy shielded lead from the oscillator was brought up underneath the bridge to the point B. R_1 and R_2 were wound non-inductively on the same tube and were shielded. C_B was a shielded air condenser with a variable resistance R_B in series and C_S was a precision variable condenser with resistance R_S in series mainly to preserve the symmetry of the system. The

Method of Hartshorn and Oliver.





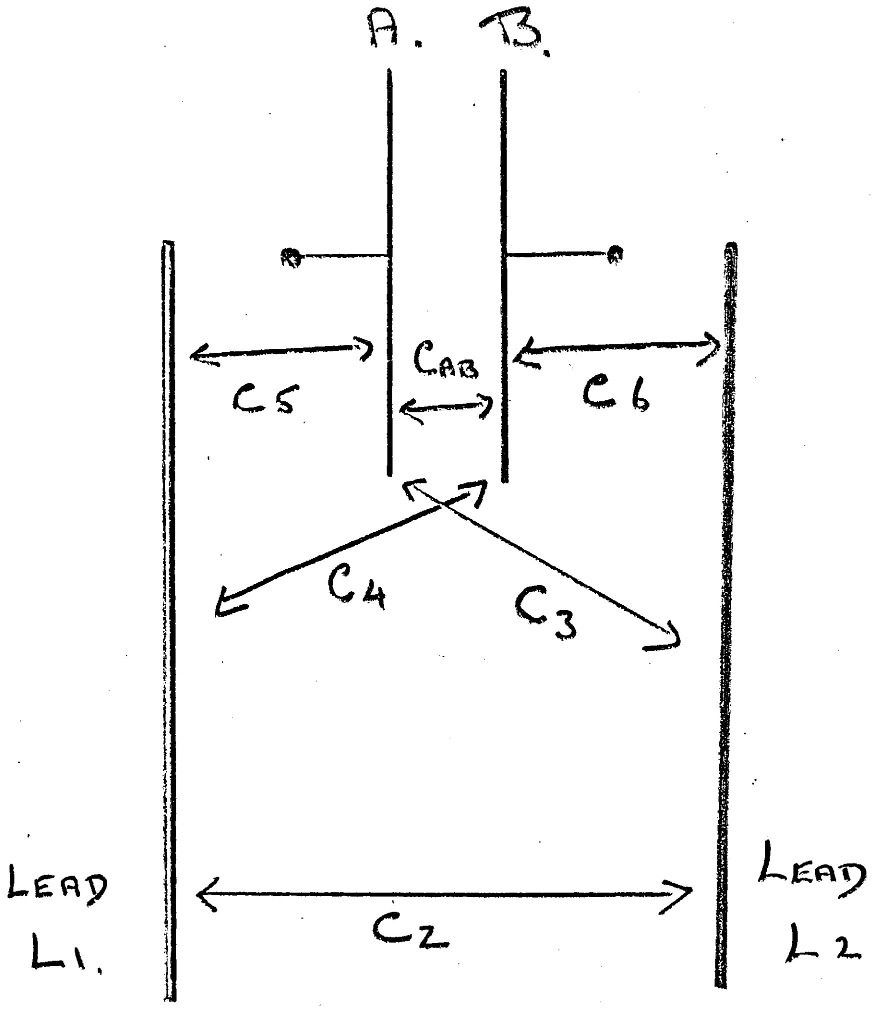
DIAGRAMMATIC FORM OF TEST CONDENSER.

test condenser C_x consisted of two concentric gold-plated brass cylinders insulated from each other by quarz discs at the ends. The bridge was balanced with C_x disconnected by varying C_S and R_B and rebalanced in the same way with C_x connected at F. C_x had a fixed capacity due to the leads and ends containing insulator, C_F, and the effect of this was eliminated by measuring its capacity filled with air first and then with benzene. Assuming a value for the dielectric constant of benzene, the variable portion of the capacity of the condenser, C_v, could be calculated and the dielectric constant of other liquids determined by measuring the capacity of the condenser containing them.

$$\epsilon = \frac{C_{x\,\text{liquid}} - C_F}{C_v}$$

In a later modification, the cell used consisted of three concentric platinum cylinders open at the ends and enclosed between the walls of two concentric glass tubes. It was to reduce end effects and lead capacities to a minimum (4%) so that the correction described above was very small.

Hartshorn and Oliver investigated the discrepancies between results obtained by different methods of measuring dielectric constants, and devised a bridge for measuring accurately the dielectric constant of benzene, (Proc. Roy. Soc. 1929 P.664). They took special precautions with the purity of the samples used and made measurements mainly at 1000c.p.s. at which frequency stray capacities and inductances were small. They

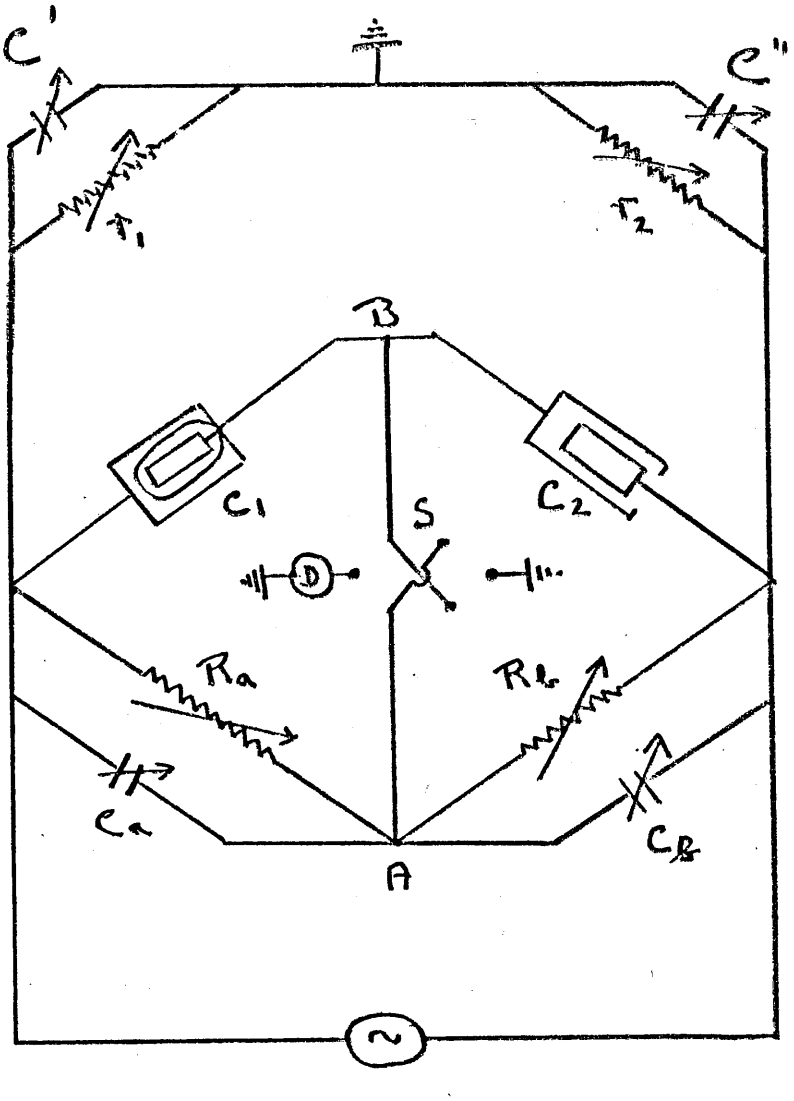

A.
B.
CAB
C5
C6
C4
C3
LEAD
L1.
C2
LEAD
L2

also examined the variation of dielectric constant with frequency, but did not regard the higher freqaency values as standard. Thenetwork used was as shown, the condensers $C_1\, C_2\, C_3\, C_4$ being variable standards, R_3 and R_4 non-inductively wound resistances and C_{AB} the test condenser. The conditions for balance were

$$\frac{R_3}{C_1} = \frac{R_4}{C_2+C_{AB}} \quad \text{and} \quad \frac{C_3}{C_1} = \frac{C_4}{C_2+C_{AB}}$$

With C_{AB} disconnected, and R_3 R_4 and C_3 having suitable values, C_1 and C_4 were adjusted to balance. This was repeated with C_{AB} connected when it had air and when it had benzene as dielectric. The test condenser was designed as shown so that few lines of force passed through the dielectric. ~~The test condenser~~ Stray capacities due to leads etc. were as shown in the diagram. By use of a shield S further external capacities were eliminated, leaving C_{AS} and C_{BS}. Since S was earthed, C_{BS} merely shunted the source and C_{AS} affectedonly the value of C_4. The lead capacities c_1---c_6 could be calculated and allowed for by interchanging the connections of the condenser plates, leads and shield S. All the bridge components were shielded in metal cases, and allowance was made for reactance in R_3 and R_4 and for the imperfect insulation of the condensers, but this was found to be negligble. The dielectric constant of ~~b~~e benzene at 20° C and 1000c.p.s. was given as 2.282_{55}.

L.M.Heil (Phys. Rev. 1932 P666) used a similar network to determine the electric polarisation of certain organic liquids. A 476k.c.s. quartz crystal oscillator was inductively coupled to the bridge, and the detector, also inductively coupled to the bridge, was a tuned circuit, also adjusted to resonance at 476k.c.s., in conjunction with a carborundum crystal rectifier and a highly sensitive galvanometer. The ascillator, detector and bridge were all separately shielded. The resistances were both 1000ohms. The test condenser consisted of three brass cylinders, the outer two being connected and insulated from the inner one by glass capillary. Low temperature readings were taken by immersing the condenser in solid carbon dioxide and alcohol and warming it by a heating coil wound round the condenser. This may have had some effect on the capacity reading. The temperature was measured by means of a copper-constantan thermocouple. C_2 was a variable precision condenser, and the bridge was balanced by varying this and C_3 or C_4. The bridge was balanced with the condenser connected in parallel with C_2 and also with it disconnected. Correction was made for the leads and spacers by calibrating the condenser with benzene, the dielectric constant of which was assumed. The bridge was then balanced with the condenser filled with hexane, p-xylene, o-xylene, m-xylene and with solutions of brombenzene and iodo-benzene in them as solvents at different temperatures. The values of dielectric constants obtained for the hydrocarbons agreed to within rather more than 1% with a number of other



Method of Jordan, Broxon and Walz.

experimental results quoted.

Jordan, Broxon and Walz (Phys. Rev. 1934 P.66) examined the dependance of the dielectric constant of air upon pressure and frequency, using pressures up to 170atmos. and frequencies up to 70k.c.s. Previous measurements on gases were highly discordant owing to the fact that the dielectric constants varied only slightly from unity. Considerable error could have been introduced by the instability of high frequency sources, and this wasovercome by using a bridge method. Inaccuracies of calibration of variable condensers used were a further source of possible error which was avoided by using variable resistances R_a and R_b as the main impedances. At 70k.c.s. these ceased to behave as pure resistances, and limited the frequency range of the apparatus. r_1 and r_2 were decade resistances of 10,000ohms, c' and c'' were variable air condensers and C_a and C_b special low loss variable condensers. The oscillator was carefully shielded from the rest of the apparatus and all leads and bridge components were individually guarded. The null indicator D was an amplifier feeding into a dynatron circuit and producing an audible beat note in head-phones with the circuit through BA. C', r_1, C'' and r_2, constituted a Wagner earthing device to eliminate stray capacities. By means of the switch S, A was earthed and B connected to D, and R_1, C', R_2 and C'' were adjusted until B was also at earth. The switch was then reversed, B being earthed and A connected to D, and R_a, C_a, R_b and C_b were adjusted for balance. This process was repeated until A and B were both permanently at earth potential. The equation for balance was

$$\frac{C_1}{C_2} = \frac{R_b}{R_a}\left[\frac{1 + \frac{R_a}{r_a} - \frac{1}{\omega^2 C_a C_b r_a r_b}}{1 + \frac{R_b}{r_b} - \frac{1}{\omega^2 C_a C_b r_a r_b}}\right]$$

where r_a and r_b were the equivalent series resistance of C_a and C_b respectively. With the apparatus used, the quantity in the bracket was proved to be negligible varying little more than 10^{-4} from unity, and hence at balance

$$\frac{C_1}{C_2} = \frac{R_b}{R_a}$$

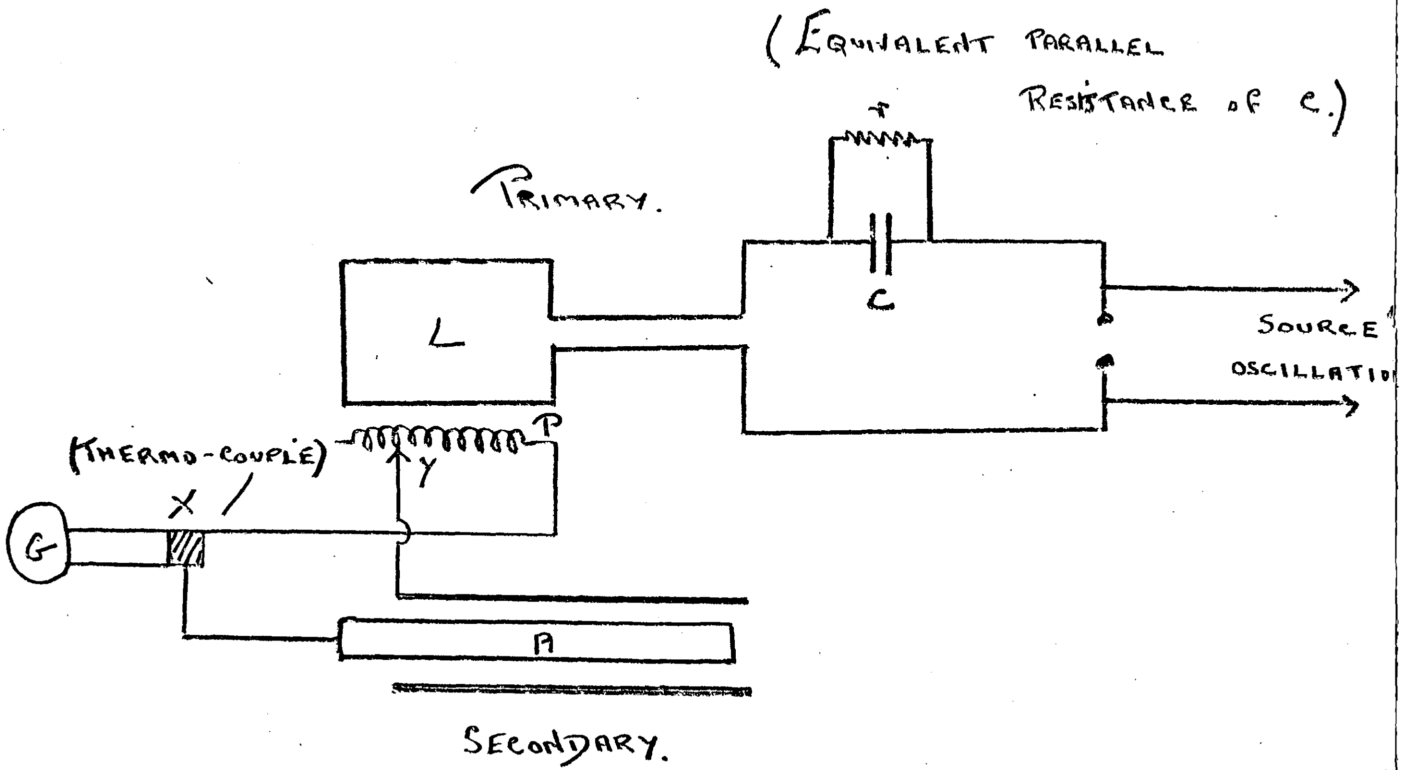
C_2 was a fixed concentric cylinder condenser closed at one end and with solid insulation at the other end shielded from the electric field. The inner cylinder was connected to the earthed shield. C_1 the test condenser was similar to C_2 but was contained in a high pressure chamber so constructed that the cylinders were subjected to equal pressures inside and out, and the only mechanical strain was directly upon the solid metal walls and could not distort the cylinders. The capacity of C_1 was measured as described above when it was evacuated and when filled with air at different pressures, and hence the dielectric constant was determined. Itwas found that this increased linearly with pressure. Slight inconsistencies at higher pressures were thought to be due to mechanical distortion of the axis of the test condenser. The dielectric constant was found to be independant of frequency, values above 50k.c.s. being less trustworthy owing to the appearance of reactance in R_a and R_b.

Capacity bridges have been constructed for use at frequencies up to 100m.c.s. The main requirements are complex shielding systems and adequate corrections or compensation for inductances introduced (Bell System Tech. Journ.)- It is possible to use these bridges for the determination of dielectric constants, but the corrections involved and the fact that more suitable methods are available at these frequencies make such determinations less practicable.

Resonance Methods.

Resonance methods have been the most widely used of all in the measurement of dielectric constants. For accurate results it is necessary to have a continuous source of oscillations stable in current and frequency output, so that a very important part of every determination is the design or choice of a suitable oscillator. Resonance methods involve the measurement of the capacity of a test condenser when empty and when filled with the dielectric under test, and consequently the previous remarks in the section on bridge methods regarding the shielding of the condenser and necessity of correction for stray capacitances and inductances still apply. In addition correction often has to be made for the conductivity of the dielectric, and if this is high it usually reduces the sensitivity of the resonance determination. The threemain methods involve the detection of current or voltage resonance in a secondary circuit or of minimum effect of the secondary on the frequency of the oscillator at resonance. Of the first two methods, voltage resonance is generally preferable since current measuring methods do not detect the current passing through the self capacitance of circuit components. Moreover, current and voltage resonance do not coincide if the test condenser has any conductance, since this affects the conditions for current resonance but, in the case of voltage resonance merely acts as a shunt across the voltmeter, reducing its sensitivity but not affecting the position of its peak reading. The third method, depending on the reaction of the oscillator is the most accurate method of determining resonance. A second stabilised oscillator is used as a frequency standard and is kept at the original frequency of the first oscillator or at some known frequency difference. The secondary circuit is then tuned until the heterodyne beat between the two oscillators is reduced to zero or to the frequency difference arranged, and the tuning can be made very critical. In all resonance methods great care must be taken to prevent the interaction of different circuits and consequent pulling of the frequency. Thus the coupling of the secondary to the oscillator in current and voltage resonance measurements must be very loose, and in the heterodyne beat method the two oscillators must be screened and placed well apart. At very high frequencies resonance methods become less accurate owing to stray capacities and lead inductances and to the difficulty of finding suitable stable oscillators.

C.Niven (Proc. Roy. Soc. A 1912 P. 139) measured the dielectric constants and their variation with temperature of water and alcohol. The test condenser C was joined in series with the inductance L, a single wire rectangle in the primary circuit, oscillations of the order of 1m.c.s. being set up by a spark discharge.



C. Niven's Method.

In the secondary circuit, P was a coupling inductance and A a cylindrical condenser, these both being varied as the point Y was moved, and remaining in approximately the same ratio. X was a coil containing a thermo-junction connected to a galvanometer, and was used to detect current resonance. The coupling between the primary and secondary circuits was fairly loose, and it was assumed that the current in the secondary circuit had no effect on the primary. Lead capacitances were ignored. The test condenser used consisted of two concentric spheres of radii 4.75 and 3.75cms. The outer sphere had taps top and bottom with a spiral at the bottom so that the whole could be immersed in a water bath and liquid kept circulating through the condenser. This was joined in the primary circuit with a battery of Leyden jars in series, which enabled the condenser to stand up to the discharge without affecting the capacity of the circuit. The secondary circuit was brought to resonance and C was then replaced by an air condenser formed of nine glass plates coated with tin foil and separated by small pieces of glass, the capacity of which was calculated from its dimensions, end effects and the effect of the glass insulators being ignored. The secondary circuit was brought to resonance again. In the primary circuit if V_1 and V_2 were the potentials across the condenser C, the current through it was $C\frac{d}{dt}(V_2-V_1)$ and the current through the equivalent parallel resistance r was $\frac{V_2-V_1}{r}$

Whole current i $= \left(\frac{1}{r} + C\frac{d}{dt}\right)(V_2 - V_1)$

But $V_2 - V_1 = \left(R + L\frac{d}{dt}\right)i$ Rbeing the resistance of the remainder of the primary circuit.

Therefore $\left(R + L\frac{d}{dt}\right)i + \left(\frac{1}{r} + C\frac{d}{dt}\right)^{-1} i = 0$

Assuming $i = e^{zt}$

$$CLz^2 + \left(CR + \frac{L}{R}\right)z + \frac{R}{r} + 1 = 0$$

where $z = -\frac{1}{2}\left(\frac{R}{L} + \frac{1}{Cr}\right) \pm \sqrt{-\frac{1}{CL} + \frac{1}{4}\left(\frac{1}{Cr} - \frac{R}{L}\right)^2}$

Putting $z = p \pm jq$, if z_1 and z_2 were two values of z,

$$p^2 + q^2 = z_1 z_2 = \frac{1}{CL}\left(\frac{R}{r} + 1\right)$$

In the secondary circuit the current J was given by

$$\left(L_1\frac{d^2}{dt^2} + R_1\frac{d}{dt} + \frac{1}{C_1}\right)J + M\frac{d^2 i}{dt^2} = 0$$

where L_1 R_1 and C_1 were the inductance, resistance and capacitance respectively of the circuit. Substituting for $\frac{d^2 i}{dt^2}$,

$$J = \frac{Mz^2 i}{\left(L_1 z^2 + R_1 z + \frac{1}{C_1}\right)}$$

Putting $z = p + jq$ the modulus of the denominator Δ was given by

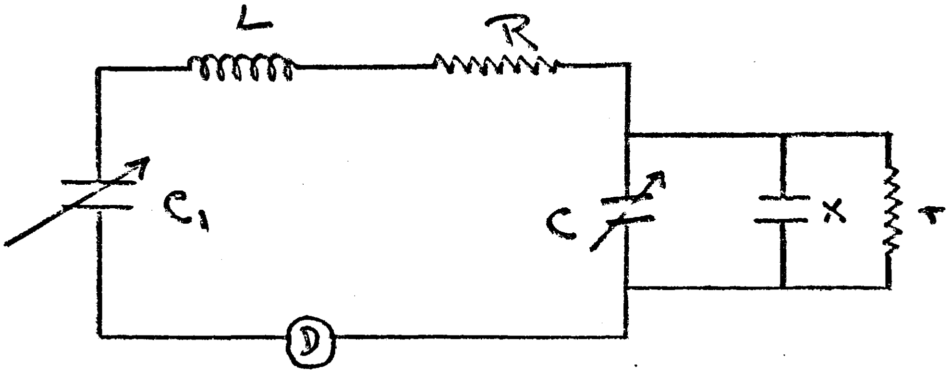
$$\Delta^2 = \left[L_1(p^2 - q^2) + R_1 p + \frac{1}{C_1}\right]^2 + q^2(2pL_1 + R_1)^2$$

and the current J was a maximum when Δ was a minimum. L_1 C_1 and R_1 were all functions of the ~~inductance~~ length used, x, but it was shown that, since R_1 was small its variation was negligible. Putting $L_1 = \lambda_1 x$ and $C_1 = kx$ where λ_1 and k were constants, Δ was differentiated with respect to x and shown to be a minimum when

$$\left[\lambda_1 x(p^2 + q^2) + \frac{1}{2}R_1 p\right]^2 = \left(\frac{1}{kx} + \frac{1}{2}R_1 p\right)^2$$

$$\text{i.e.}\quad L_1 C_1 = (p^2 + q^2)^{-1} = \frac{CL}{\frac{R}{r} + 1}$$

Since R was very small and r large, $\frac{R}{r}$ was neglected with respect to unity, and the condition for resonance



R.T.Lattey's Method.

was given by

$$CL = C_1L_1$$

The ratio of the test condenser to the standard was given directly by the ratio of the two corresponding values of LC at resonance in the secondary, and, the capacity of the air-filled test condenser being calculated, the dielectric constant of the liquid from measurements on the liquid-filled condenser was known. For water it was found to decrease from 90.36 at 0°C to 37.97 at 83°C, and for alcohol it decreased from 24.5 at 12°6 to 16 at 64.2°C.

R.T.Lattey (Phil. Mag. 1921 P.829) measured the dielectric constants of electrolytic solutions by a current resonance method. An oscillator was used which gave a fairly pure sine wave of frequency 300k.c.s. to 3m.c.s. The secondary circuit was as shown, the coupling inductance L being made of rectangular brass strips of very low resistance, R. C_1 and C were variable air condensers and X was the test condenser with effective parallel resistance r. D was a coil of wire leading to a thermal converter and a galvanometer. The circuit was tuned to resonance by suitably adjusting C_1 and varying C. It was then re-tuned to resonance with X disconnected. The impedance of the circuit was given by

$$Z^2 = \left(R + \frac{r}{1+p^2r^2(C+x)^2}\right)^2 + \left(pL - \frac{1}{pC_1} - \frac{1}{pC+px+p^{-1}r^{-2}(C+x)^{-1}}\right)^2$$

with X connected, where p was the angular frequency of the oscillations. With X disconnected,

$$Z^2 = R^2 + \left(pL - \frac{1}{pC_1} - \frac{1}{pC}\right)^2$$

If i and I were the resonance currents corresponding to values C_2 and C_0 for C in these two cases,

$$\frac{C_0}{C_2 + x} = 1 - \frac{\frac{I^2}{i^2} - 1}{K^2} = \frac{1}{K^2} \cdot \frac{r}{R}\left(\frac{I}{i} - 1\right) = \frac{\frac{r}{R}}{\frac{r}{R} + \frac{I}{i} + 1}$$

where

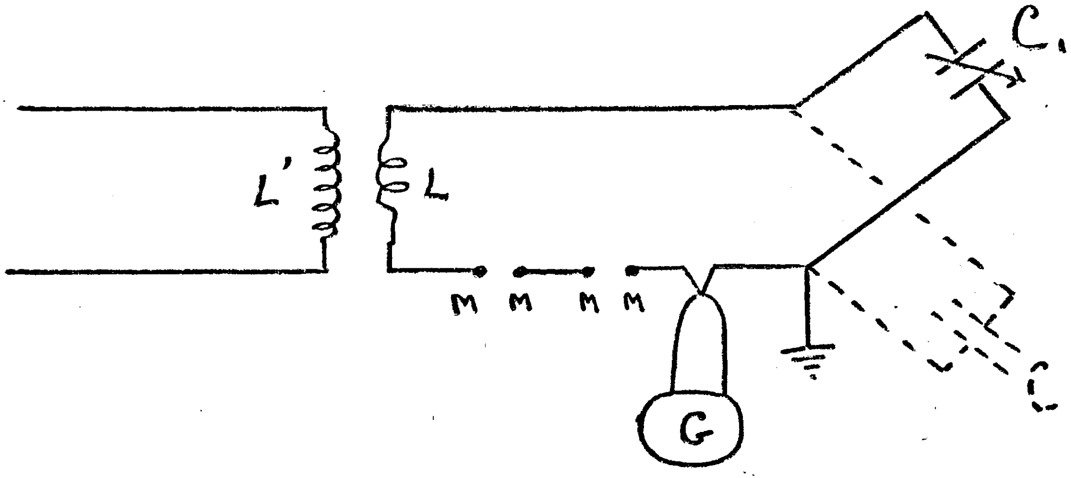
$$K^2 = \frac{L - \frac{1}{p^2 C_1}}{C_0 R^2}$$

$$\therefore \frac{I}{i} = \left[K^2 + \left(1 + \frac{r}{2R}\right)^2\right]^{\frac{1}{2}} - \frac{r}{2R}$$

and

$$C_2 + x - C_0 = \frac{C_0 R\left[1 - \frac{r}{2R} + \left(K^2 - \left\{1 + \frac{r}{2R}\right\}^2\right)^{\frac{1}{2}}\right]}{r}$$

For a perfectly non-conducting dielectric, $r = \infty$ and $C_2 + x - C_0 = 0$ The right hand side of the last equation was therefore a correction term for the conductivity, and was reduced to a minimum by making R and K as small as possible. The correction term then became $\frac{2C_0 R}{r}$ and the current i was large enabling accurate adjustment of C to be made. For very small values of R and large values of r i.e. poorly conducting dielectrics $\frac{2C_0 R}{r}$ could be neglected. Values of K were determined, and since these were small for the particular dielectrics and apparatus used $C_2 + x = C_0$ was assumed. The lead capacity had a relatively large effect on the measured capacity of the test condenser and a very lengthy correction had to be applied. The capacity of the condenser with salt, glycerol and sugar solutions as dielectric was determined and the dielectric constants calculated. It was found, contrary to previous experiments that electrolytes in solution appeared to lower the dielectric constant of water, but the assumptions and approximations made in the theory gave considerable room for error.

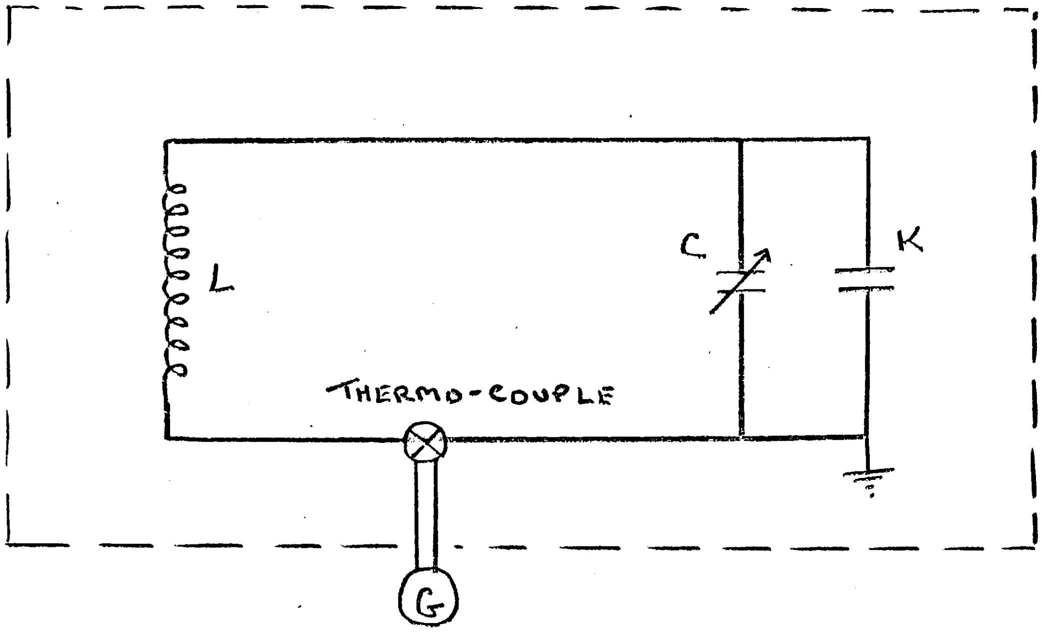


A.B.Bryan's Method.

A.B.Bryan (Phys. Rev. 1923 P.399) investigated dielectric losses at radio-frequencies in liquid dielectrics by a current resonance method. The oscillator was variable between 200k.c.s. and 1.2m.c.s.and the secondary circuit was coupled to it as shown by means of the inductance L. Heavy moveable copper links were provided by means of which the variable air condenser C_1 or the test condenser C could be connected in the circuit. The current was measured by a thermo-couple T in connection with a micro-ammeter G. The mercury cups M were provided so that special high frequency resistances, consisting of short strips of manganin wire which varied very little in resistance with frequency, could be inserted in the circuit. The test condenser C was two concentric silver cylinders in a stoppered glass cell which could be immersed in an oil bath for temperature variation measurements. C_1 was a precision condenser with a micrometer screw control and a maximum capacity of 1,500p.f. The whole apparatus was supported on ebonite posts several inches above the bench to decrease leakage losses and stray capacitances. The coupling to the oscillator could be varied. With the test condenser in the circuit the oscillator was tuned to resonance with the measuring circuit. Resistances were then inserted until the resonance current was the same as before, the resistance added being equivalent to the series resistance of condenser C , C_1 having negligible conductance. Thus the capacity of C was measured with air ~~an~~ and liquids as dielectric at different temperatures and frequencies and the dielectric constants of the liquids and power factors of the condensers formed were determined, the frequency being measured by a wavemeter. Measurements were made with nitrobenzene specially purified, and ~~with~~ dried, and also with distilled water. Absolute values of the dielectric constant were not considered reliable owing to difficulties in calibrating the test condenser with its stray capacities and lead inductances, but graphs were plotted for nitrobenzene showing that at 30°C there was no variation of dielectric constant over the frequency range 300k.c.s. to 1.5m.c.s. but that, at 625k.c.s. there was a decrease of dielectric constant as the temperature increased from 10°C to 140°C.

H. Race (Phys. Rev. 1931 P.430) used a similar method to A.B.Bryan to examine variations with temperature and frequency of dielectric losses in viscous insulating oil over a rather wider frequency range, 40k.c.s. to 3m.c.s. He also made measurements at lower frequencies with a modified power factor bridge. The secondary circuit was loosely coupled to a variable oscillator and consisted of a coupling coil in series with several link resistors of short straight non-magnetic resistance wire sealed in evacuated glass tubes, a vacuum thermo-couple of constant heater res-

RESONANCE CIRCUIT.



J.W.Smith's Method.

istance which was connected to a high sensitivity galvanometer by high frequency chokes and ac by-passing condensers, and a high frequency switch by means of which the test condenser or standard variable air condenser could be joined in the circuit. The test condenser consisted of two concentric cylinders of a chromium- nickel steel alloy insulated from each other by fused quartz which was shown to have negligible leakage and loss. The tests were carried out in a specially shielded room. The experimental procedure and theory were as for Bryan's method previously described.

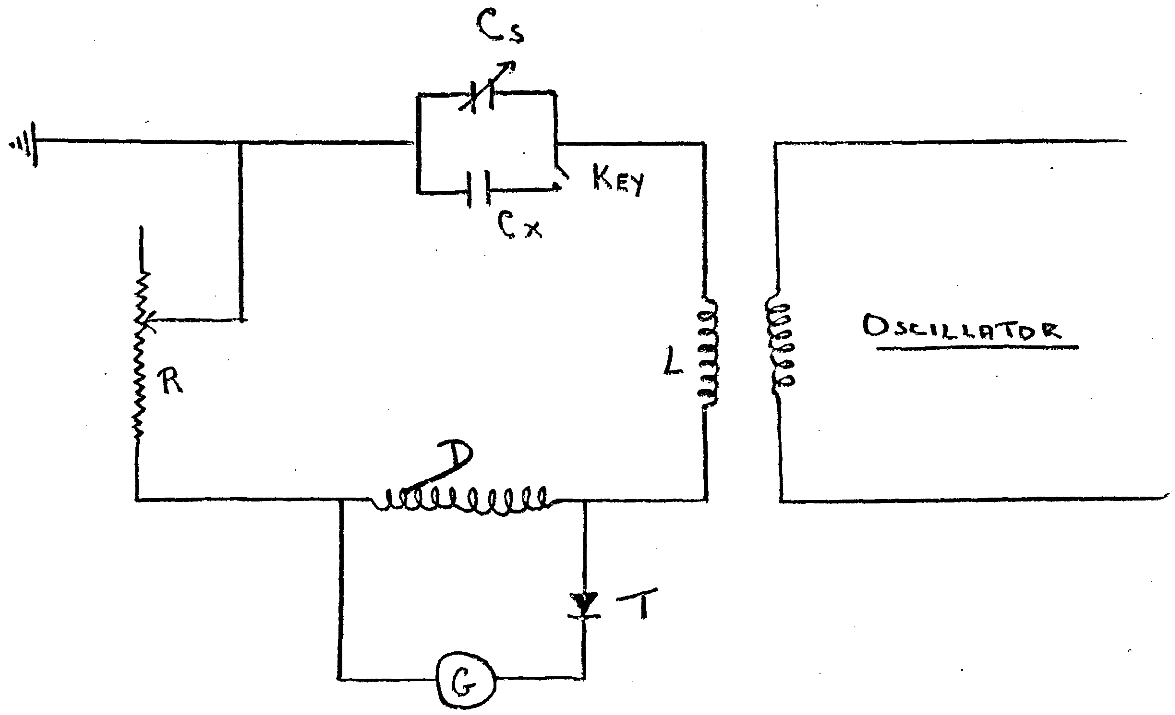
J.W.Smith, (Proc. Roy. Soc. 1932 P.251) used a simple current resonance method of determining the dielectric constants of various solutions. A powerful quartz crystal controlled oscillator was specially designed to be independent of slight impedance changes in its circuit and oscillated at approximately 1.23m.c.s. The resonance circuit was placed a considerable distance away from it and was enclosed in an electrostatic screen, which reduced the pick-up very little. The coupling coil was of bare copper wire specially wound to be of constant inductance, and this was connected in series with a vacuum thermo-junction J of low resistance, 1.26ohms, which did not damp the secondary oscillations. K, the test condenser, and C, a standard variable air condenser, were joined in the circuit as shown by thick short leads dipping in mercury cups. The test condenser was of the type used by Sayce and Briscoe in their resonance method described later, and had a very low leakage resistance, being made of heavily silvered coaxial glass cylinders immersed in an earthed water thermostat. The apparatus was not used for absolute measurements; By tuning C, current resonance curves were plotted, first with K removed, then with K filled with dry air and then with K containing different solvents, which were assumed non-conducting. Assuming the static values of the dielectric constants to hold for the solvents, the value of that part of K which was variable by changing the dielectric could be found. C_0 was the value of C with K removed, the leads being in position, and C_1 and C_2 were the values of C when K was filled with air and water respectively. Then

$$C_0 = \frac{C_1 \epsilon - C_2}{\epsilon - 1}$$

Current resonance was found using various solutions, the effect of conductivity being ignored although this was not really justifiable, and curves were plotted to determine the resonance values of C since the current peaks were not symmetrical for conducting solutions. For a resonance value C_x the dielectric constant was given by

$$\epsilon_x = \frac{C_0 - C_x}{C_0 - C_1}$$

A simple voltage resonance method was used by Kitchin and Muller (Phys. Rev. 1928 P.979) in an examination of anomalous dispersion, absorption and Kerr

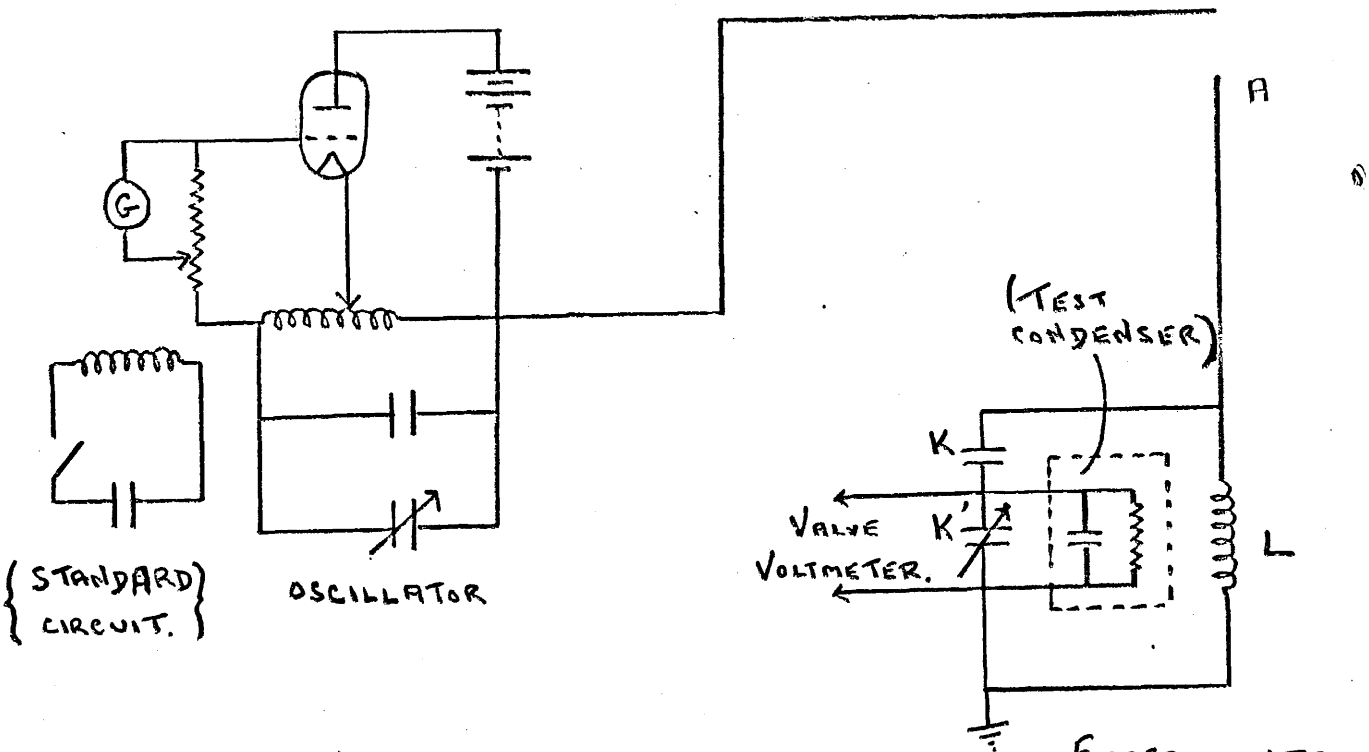


Method of Kitchin and Muller.

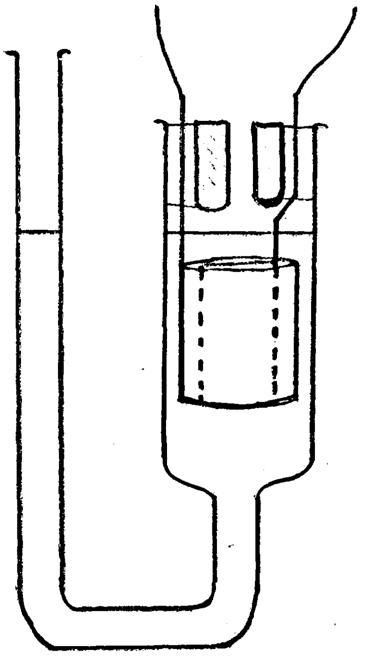
effect in viscous dielectrics. The secondary circuit was coupled to a valve oscillator of frequency 0.4 to 10.9m.c.s. by means of the inductance L. C_s was a variable standard condenser connected in parallel with the test condenser C_x which consisted of two coaxial cylinders whose capacity was considered invariant with temperature. R was a variable resistance and D a drop coil, the effective voltage across this being noted by a galvanometer in series with a crystal rectifier. C_s was tuned to resonance with C_x in the circuit and also with C_x removed by opening the key. Readings were taken at constant frequency using castor oil and rosin with the temperature increasing from -20°C to 100°C and from 20°C to 160°C. In each case R was varied to bring the resonance galvanometer deflection to same value, thus eliminating the effect of conductance due to the dielectric. The value of the dielectric constant was given by the ratio of the change in C_s on inserting C_x in the circuit when it had dielectric and then air between the plates. At each frequency an absorption band was detected within the temperature range examined, the temperature corresponding to the maximum dielectric constant varying with frequency.

A.Astin devised a method of measuring dielectric constants of conducting liquids by voltage resonance and used it for water, methyl alcohol, glycerine and sugar solutions. (Phys. Rev. 1929 P.300). It was desirable

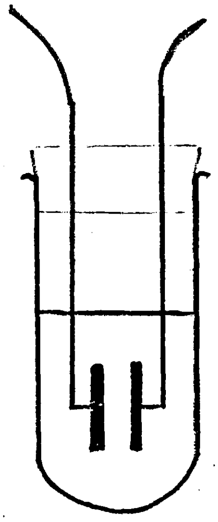
A.Astin's Method.



Experimental circuit.



Cylindrical condenser.



Parallel Plate Condenser

that the series resistance of the condenser containing the liquids should be as low as possible, and it became very low at high frequencies. The effect of frequency on lead inductances was examined and it was found experimentally that no correction was necessary for frequencies below 3m.c.s. It was found that resonance detection by means of a thermo-couple was unsatisfactory as the detecting circuit affected the secondary. Observations on the grid current in the primary circuit were sensitive only when the two circuits were tightly coupled, and this caused frequency variations on tuning the secondary. For resonance detection a valve voltmeter was therefore used, this having only a capacitive effect on the secondary. The experimental arrangement was as shown. The oscillator was tuned to a frequency of 2.7m. c.s. and the frequency output was kept constant by closing the standard circuit and adjusting the variable oscillator condenser for minimum deflection of the grid galvanometer. The secondary circuit was very loosely coupled to the oscillator by means of the antenna A, and an electromotive force, E_o, was assumed to be induced into the circuit by a pure inductance, resonance in the circuit under these conditions being independent of the dielectric conductance in the test condenser. Determinations were made with varying degrees of coupling. For very loose coupling the values obtained were consistent, but for tight coupling there was disagreement. If R was the series resistance of the circuit C the capacitance of K' with the test condenser in parallel, x the parallel resistance of the test condenser and E_0 the induced electromotive force producing a current I, the impedance of the circuit, Z, was given by

$$Z = j\omega L + R - \frac{1}{\omega K} + \frac{1}{j\omega C + \frac{1}{x}}$$

and

$$\frac{|E_0|^2}{|I|^2} = |Z|^2 = \left(R + \frac{x}{1+x^2\omega^2C^2}\right)^2 + \left(\omega L - \frac{1}{\omega K} - \frac{x^2\omega C}{1-x^2\omega^2C^2}\right)^2$$

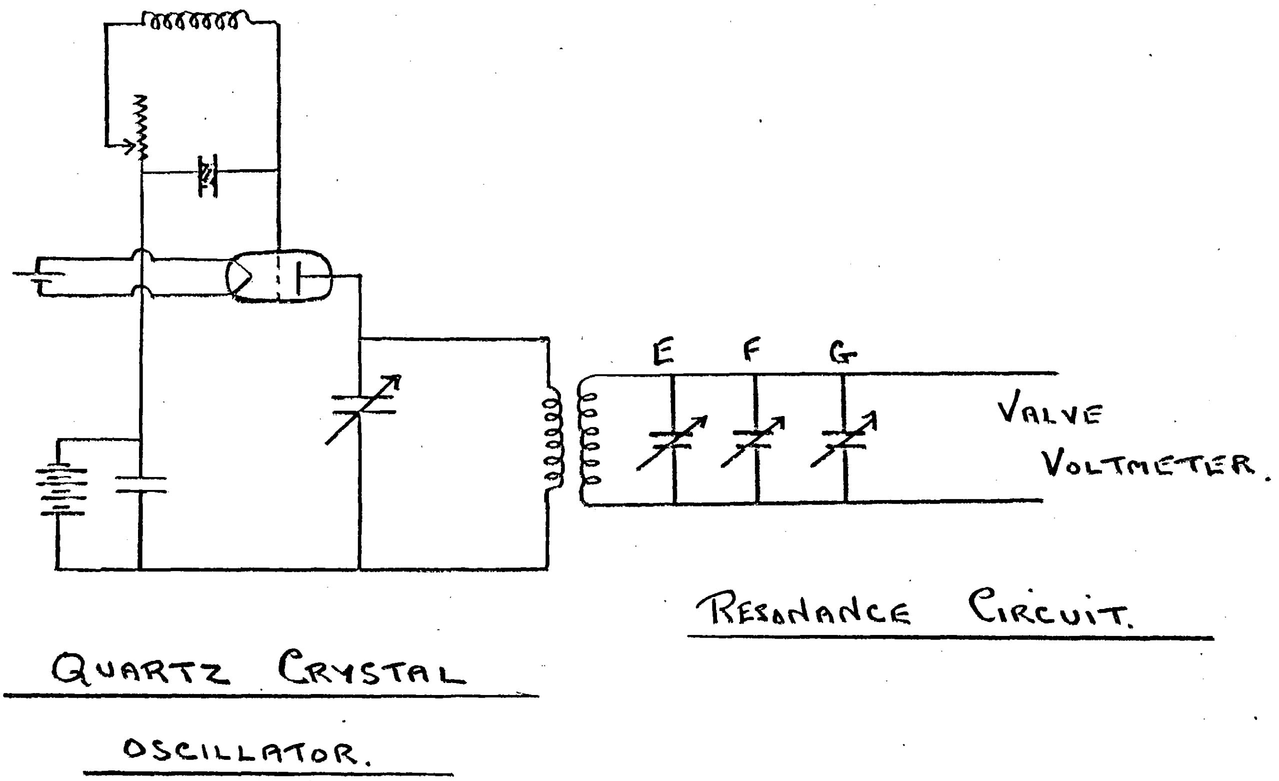
If E_c was the potential difference across C,

$$\frac{|E_c|^2}{|I|^2} = \frac{x^2}{1+x^2\omega^2C^2}$$

Eliminating I from these two equations, the maximum value of E_c was found to correspond to a value of C given by

$$C = \frac{K(\omega^2LK-1)}{R^2K^2\omega^2 + (\omega^2LK-1)^2}$$

which was independent of x, and showed that C was a function of terms which could be kept constant. K was a fixed mica condenser of 700p.f. and K' a precision air condenser variable by means of a long arm to avoid body capacitances. Two test condensers were used as shown both having platinum electrodes, and good agreement was obtained between the results for both of them, thus verifying that resonance was independent of the parallel resistance since this was quite different in the two cases. K' was tuned to resonance with the test condenser filled first with benzene and then with other liquids. The dielectric constant of benzene was determined by an alternative method for non-conducting liquids at 7.7k.cs and the value was assumed as a standard under experimental conditions. If k_1 and k_2 were the dielectric constants



Method of Johnstone and Williams.

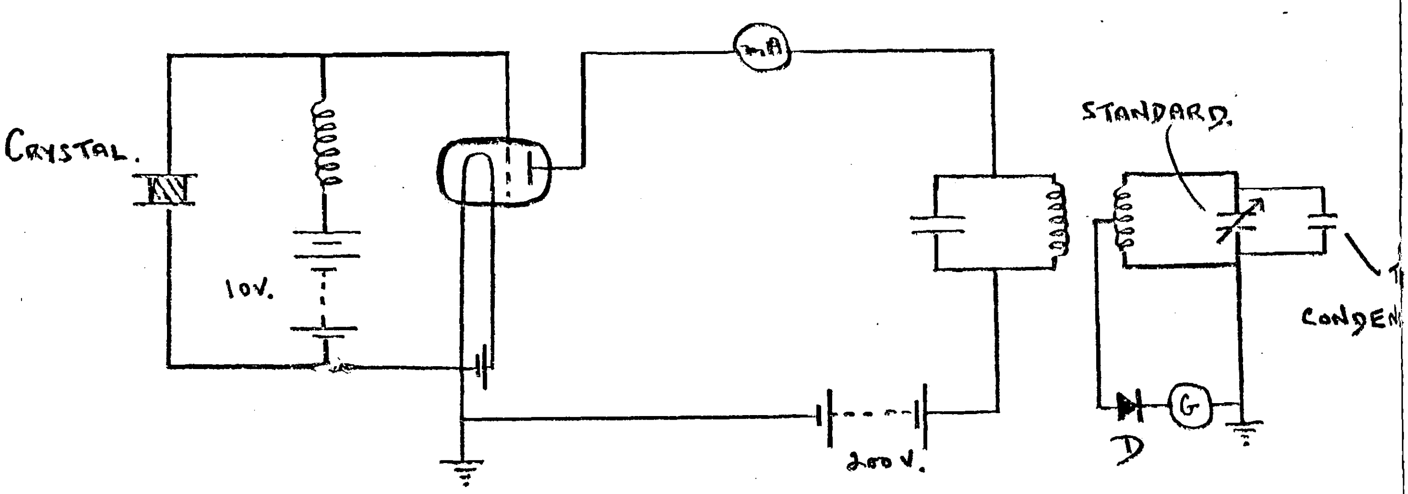
of benzene and another liquid, and c_1 and c_2 the corresponding values of K, then

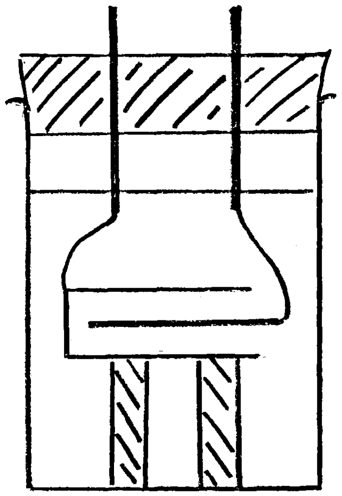
$$\frac{k_1 - 1}{k_2 - 1} = \frac{c_1}{c_2}$$

The errors in the determinations were estimated to be about 0.5% for water and for sugar and dilute potassium chloride solutions increasing to 1.5% for strong solutions. The method could have been simplified and extended by using a crystal-controlled generator of stronger output.

J.H.L.Johnstone and J.W.Williams(Phys. Rev. 1929 p.1483) examined the variation in dielectric constant with frequency of different solutions at varying concentrations in order to test the validity of Debye's theory. The oscillator used was crystal-controlled, six frequencies ranging from 2 to 10m.c.s. being available. The resonance circuit was connected as shown, E and G being variable standard air condensers and F a variable experimental condenser of the type previously described as used by Krchma and Williams. With this condenser filled with benzene, E and G were tuned to voltage resonance for two positions A and B of the plates F. This was repeated with the condenser empty and with it filled with mineral oil at different frequencies, and no variations with frequency of the dielectric constants were found. It was repeated using solutions of nitrobenzene in oil and the dielectric constant was found to decrease with increasing frequ-

J.A.Wehrle's Method.





TEST CONDENSER

ency. No lead corrections were made.

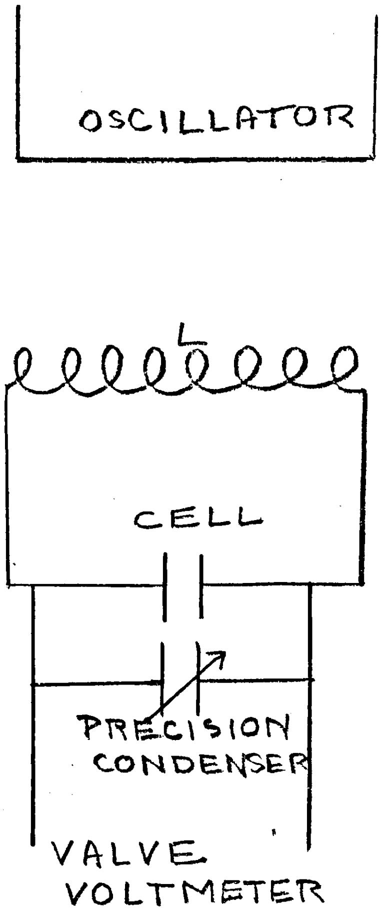
J.A.Wehrle (Phys. Rev. 1931 P.1135) for determinations on molecular association used the simple resonance circuit shown. This was coupled by the inductance L to the quartz crystal oscillator supplying a frequency of 571k.c.s. Resonance in the secondary circuit was determined by the maximum deflection of the galvanometer G in the galena crystal rectifier circuit tapped across a few turns of the absorbing inductance. The test condenser was a parallel plate condenser supported inside a stoppered vessel by insulating posts as shown. The circuit was brought to resonance by varying the standard condenser first with the test condenser removed but with its leads in position, then with the empty test condenser in the circuit, and finally with the required liquid mixtures in the condenser. If C_s and C_a were the capacities of the test condenser containing air and solution, A being the area and d the distance between the plates, n in number, the dielectric constant was given by

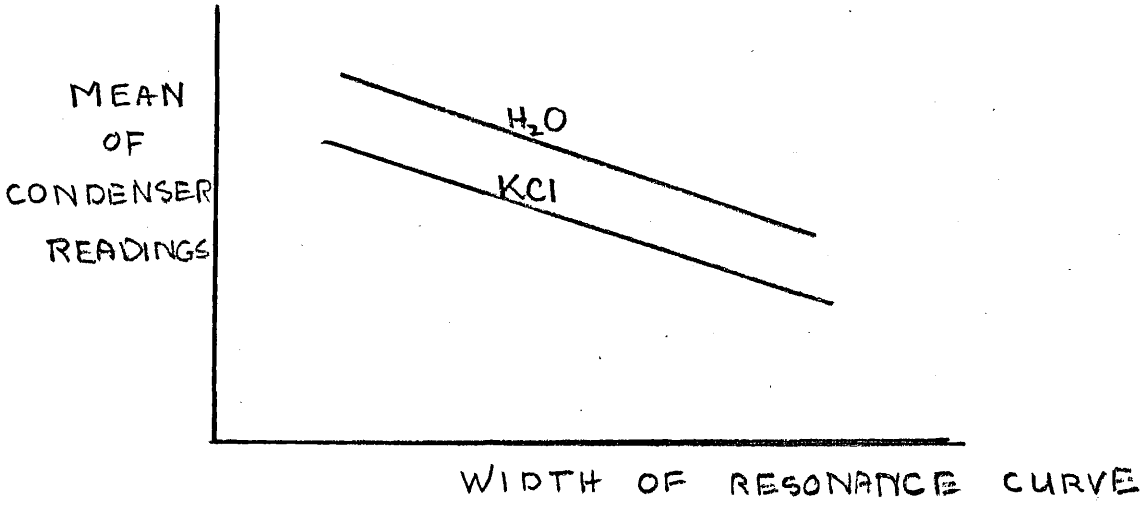
$$\epsilon_s = \frac{C_s}{C_a} = \frac{\epsilon_s A n}{4\pi d} \cdot \frac{4\pi d}{\epsilon_a A n}$$

This value was uncorrected for lead inductances etc. Determinations were made in this way for solutions of nitrobenzene in carbon disulphide, and these were plotted against values obtained by Williams which were assumed true, giving a calibration curve for the condenser. The method was used to examine solutions of two polar substances in non-polar liquids, e.g.ether and nitrobenzene in benzene, and chlorobenzene and nitrobenzene in benzene. It was simple to apply and of fair accuracy but was unsuitable for measuring dielectric constants greater than 10 owing to the decrease in galvanometer deflection obtained and to the setting up of displacement currents in the dielectric which gave rise to flattened resonance peaks.

W.Anderson (Phil. Mag. 1932 P.986) used a similar method to Johnstone and Williams for determining dielectric constants of solid dielectrics at 150 and 1500k.c.s. The secondary circuit consisted of two variable condensers and the test condenser in parallel with a coupling coil and with a valve voltmeter joined across the test cell. The capacity of the test cell was determined from the difference in resonance settings of the variable condensers with the cell in the circuit and removed from it by opening a key. The method was extended for power factor measurements by noting the change in the variable condenser required to obtain half the resonance voltage each side of the resonance peak. The test condenser consisted of circular brass plates six inches in diameter, and measurements of the capacity were made with air between the plates and with samples of glass, porcelain, ebonite, paxolin and bakelite between them. No variations of the dielectric constants with frequency were observed. The oscillator was calibrated for frequ-

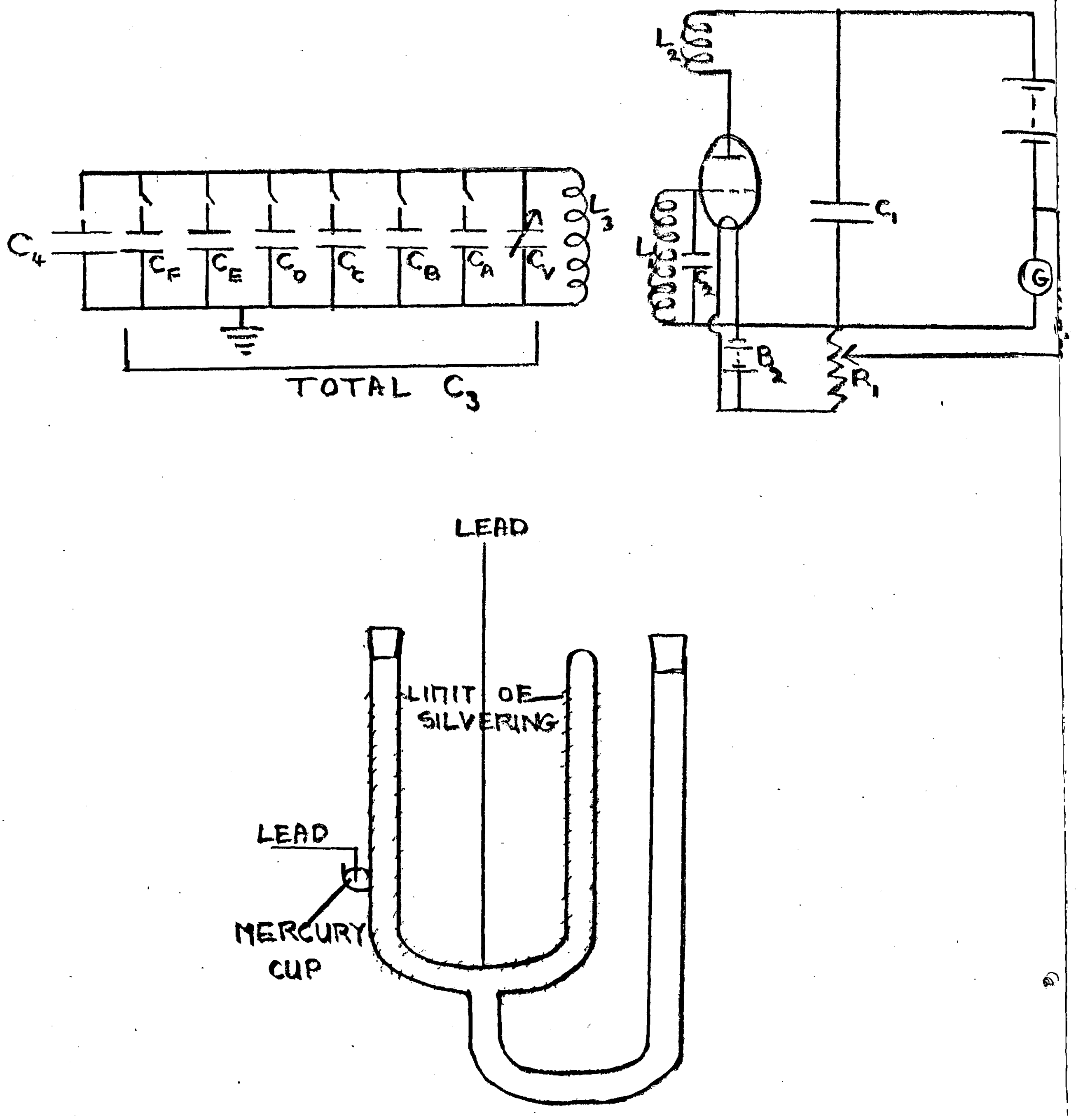
Method of Malone, Ferguson and Case.





ency output by means of a heterodyne wavemeter. Values of dielectric constant obtained were estimated to be accurate to about 1%.

Malone, Ferguson and Case devised a voltage resonance method (Journ. of Chem. Phys. 1933 P.836) and applied it to measure the dielectric constants of aqueous solutions of potassium chloride. The oscillator used gave a stable output of 3 to 15m.c.s. and the secondary circuit, which was assumed to have negligible resistance, was coupled to it by means of an inductance, assumed pure, at a distance of 90cms. The test condenser consisted of a small, platinum, parallel plate condenser held rigidly in a small glass bottle by means of its short leads, and its capacity was determined by tuning the precision condenser to resonance. This value was obtained very accurately by observing the capacity readings for the same voltage reading each side of the resonance peak, which, in practice, was not quite symmetrical. Readings were then plotted, as shown, of the mean of these two condenser readings against the difference between them, and a straight line was obtained, which was extrapolated to give the resonance peak value. This procedure was followed with pure water, used as a standard, in the condenser, and then with solutions of potassium chloride of varying concentrations. These results did not agree with those obtained by Astin, for, at a constant frequency of 10m.c.s. the dielectric constants of the



Method of Sayce and Briscoe.

solutions were found to increase with concentration. The assumption that the coil used was a pure inductance was of very doubtful validity, and whenthe experiments were repeated with the coil immersed in benzene the value of the dielectric constant remained the same as that of pure water at all the concentrations used.

L.A.Sayce and H.V.A.Briscoe (Journ.Chem.Soc. 1925 P.315) measured the dielectric constant of benzene at 25.5°C and 65k.c.s. detecting resonance by means of the reaction on the oscillator. The anode current in the oscillator circuit was a function of the current in the grid circuit containing L_1 and C_2. The secondary circuit, which was loosely coupled to L_1 by the coil L_3 consisted of the test condenser C_4, a number of fixed condensers C_A---C_F of 10, 20, 40, 80, 160, and 320p.f., and a very accurate variable condenser of maximum capacity 10p.f., C_v, all connectedin parrallel with L_3. When the natural frequency of this circuit approached the frequency of the primary circuit there was a sudden increase in the amount of energy withdrawn from that circuit, and this fact was used in determining the resonance point. The fixed condensers were all calibrated against the precision condenser C_v in steps. The frequency of the oscillator was adjusted to cause resonance in the secondary with C_4 out of circuit and with a large value of C_3, C_{30}, and this was measured by means of a wavemeter. C_3 was then adjusted to resonance with C_4 in the circuit, first when filled with air and then when filled with benzene. Then

$$\frac{C_{3\,\text{benzene}} - C_{30}}{C_{3\,\text{air}} - C_{30}} = \text{dielectric constant of benzene,}$$

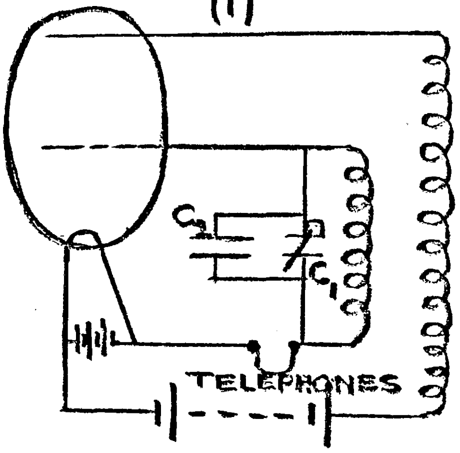
which was given as 2.2389±0.0009.

J.Wyman (Phys. Rev. 1930 P.623) also used the reaction on the anode current of an oscillator to detect resonance. The method was devised to measure the dielectric constants of pure biological compounds and had to be appropriate for liquids of low, but measurable conductivity. The oscillator used was variable to an upper limit of 120m.c.s. variations being made by interchanging inductances and by using a rotating condenser driven by a reducing gear. The frequency was determined by coupling with a quartz crystal oscillator so that audible beats were produced with one of its harmonics. The effective resonating circuits used consisted of a number of specially designed resonators, which were suspended in turn by a fine thread near the oscillator, first in air and then in the medium under test, and the oscillator frequency was varied to give resonance. Each resonator was treated as a series resistance, R, inductance L, and capacitance, C, having an impedance

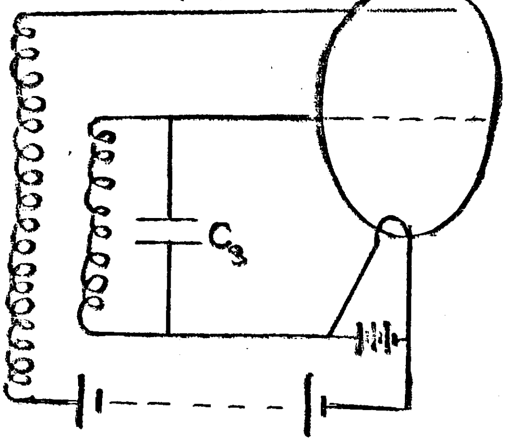
$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

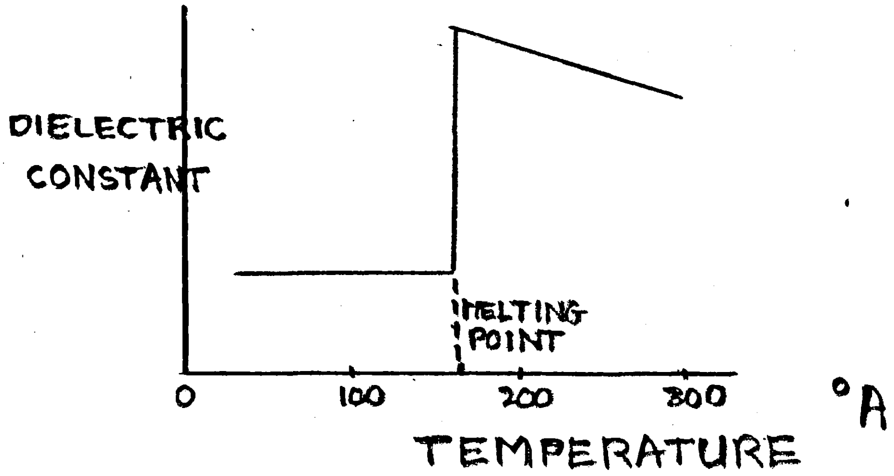
which was a minimum at the resonance frequency given by $\omega = \sqrt{\frac{1}{LC}}$. On immersing it in a liquid, L remained constant but C changed from C_0 to C_1 where $C_1 = \epsilon C_0$, ϵ being the dielectric constant of the liquid. Hence $\epsilon = \frac{\omega_0^2}{\omega_1^2}$. This assumption was true only for liquids of negligible conductivity. A correction term was worked out for conducting liquids, but the results obtained were not in agreement with those obtained by other methods and it was supposed that the behaviour under these conditions was more complex than had been assumed. Two types of resonators were used; one type being made of a number of concentric cylinders joined by a metallic conductor and the other type being a metallic spiral. Their natural frequencies varied from 10 to 1000m.c.s. and results obtained with different resonators, except for the one with the lowest natural frequency, were in agreement. This was due to the fact that the conductivities of the liquids used became negligible at the higher frequencies, but at 10m.c.s. they were large enough to produce error. Owing to the frequency limitations of the oscillator, relative values between the liquids used had to be determined using different resonators in order that every resonant frequency could be kept above 10m.c.s.Thismethod introduced additional error into the final absolute value of dielectric constant obtained, but could have been avoided if an oscillator with a higher frequency range had been available. The assumption that the maximum change of anode current occurred at the resonant frequency was tested by adapting the resonator to include a thermo-junction to give a maximum deflection on a series galvanometer at resonance. Resonant frequencies determined by both methods were in good agreement but only under conditions of negligible conductivity. Measurements were made for

(1)



(2)





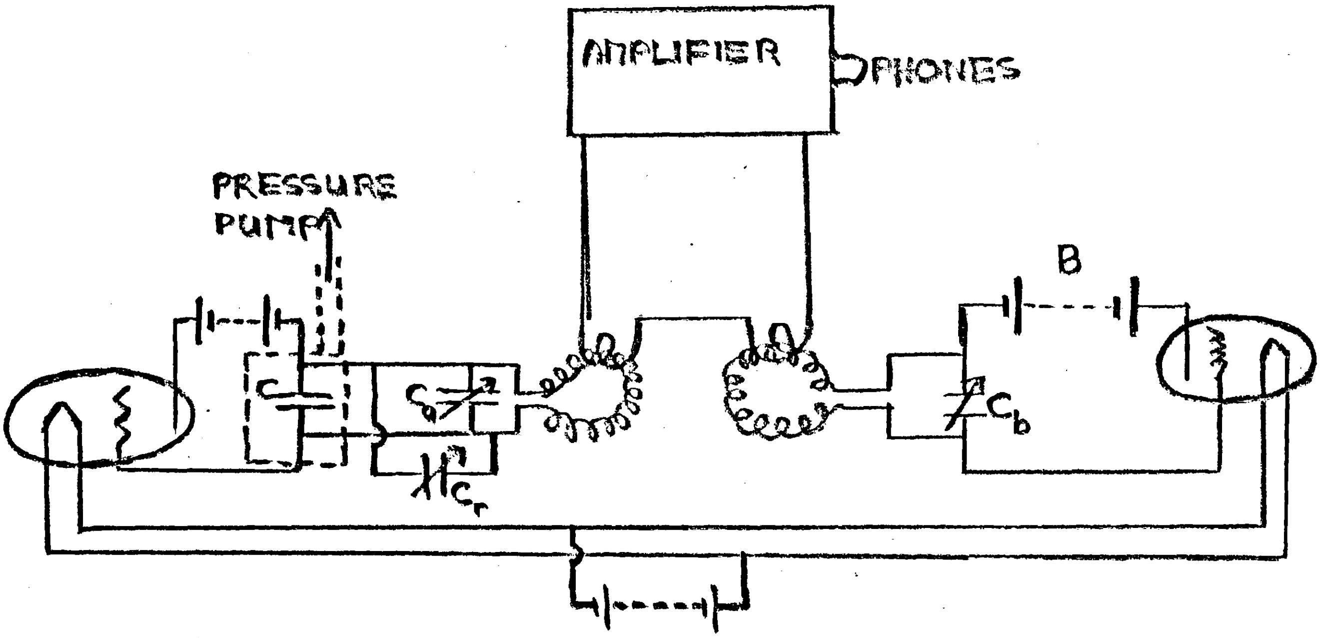
TYPICAL GRAPH

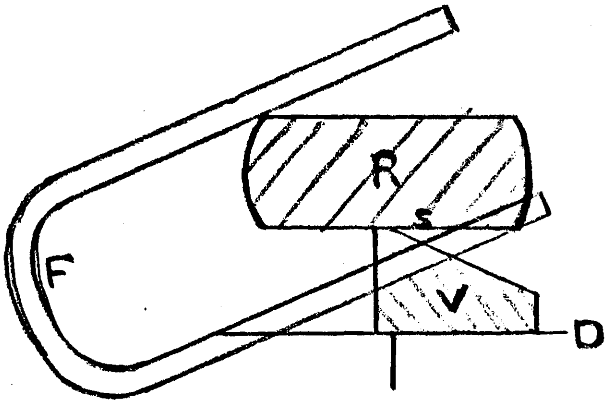
L.C.Jackson's Method.

water, which gave a sharp resonance peak, and values were estimated to have an errer of less than 0.1%. They were in agreement with the values obtained by Drake, Pierce and Dow using an improved line method, and in reasonable agreement with other observers. A linear decrease of dielectric constant with temperature was observed. Providing resonance frequencies of the order 1000m.c.s. were used, it was found that measurements could be made on liquids of conductivity twenty times that of water using the reaction on the oscillator for resonance detection. For conductivities greater than this up to one hundred times that of pure water measurements could be made using the thermal detector.

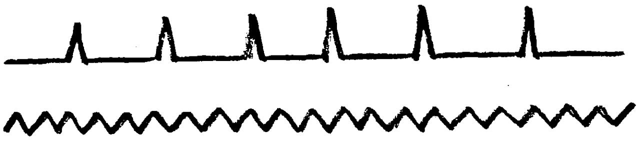
L.C.Jacksen (Phil. Mag. 1922 P.481) measured the dielectric constants of a number of formates and acetates at very low temperatures and 470k.c.s. by a heterodyne beat method. (2) was a standard oscillator which remained at a fairly constant frequency. (1) was an oscillator containing a variable standard condenser, C_1, in parallel with the test condenser C_2. Coupling between the oscillators produced a heterodyne beat note detectable in the telephones which could be adjusted to zero by tuning C_1 since the time of oscillation of the circuit depended directly on the inductance and capacitance in the grid circuit. The apparatus was simple and not designed for a high degree of

E.C.Fritt's Method.





RECEIVING SYSTEM



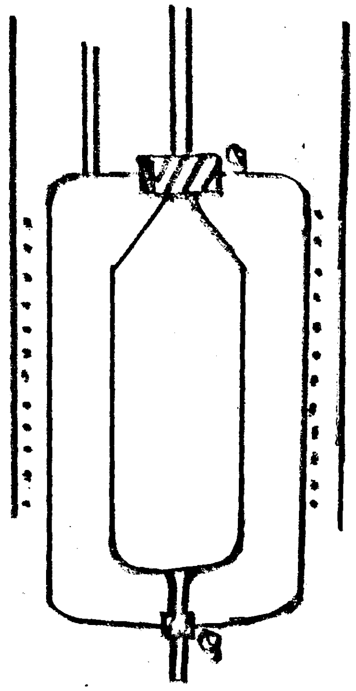
EXPOSURE

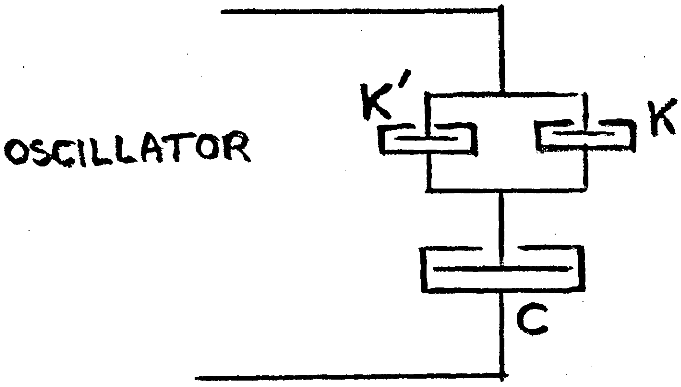
a wave-meter. The results showed that a sudden drop in the value of dielectric constant took place on solidification, as suggested by Debye, and also showed a progressive change of the constant along each series with methyl, ~~ethyl~~ ethyl, n-propyl and n-butyl radicles.

E.C.Fritts (Phys. Rev. 1924 P.345) measured the dielectric constants of five gases by a heterodyne beat method at frequencies ranging from 250k.c.s. to 800k.c.s. The oscillator B was maintained at a constant frequency by fixing the value of C_b, while the frequency of the oscillator A was varied. Oscillations from both A and B were picked up by the coupling coils and the beat note was amplified and received in the headphones. C was the test condenser, so designed as to have a minimum quantity of permanent insulating dielectric. The final form used consisted of six coaxial cylinders, alternate ones being rigidly supported at both ends in opposite metal end pieces arranged in four separate quadrants. The condenser was placed in a bell-jar with a thermo-junction at the base and a gas outlet. C_a and C_b were similar high grade variable condensers and C_r was a smaller variable condenser. In this method C_a was tuned so that A oscillated at a frequency 100c.p.s. fewer than B. To the diaphragm of the telephone, D, was attached a light vane V, and a standard tuning fork, F, was so arranged that it formed a triangular slit, S, with V and with one side of a razor blade R. Light was concentrated on the slit S and, passing through it, fell on to a strip of photographic film rotated at constant speed by a spring motor. Thus as F and V vibrated, opposite ends of the slit were modified and the resulting exposure was similar to that shown. A record was made of the beat frequency, N, with C evacuated and joined in the circuit. A gas at a given pressure was admitted and a second record of the new beat frequency was made, n_2, The condenser C_r was then adjusted a small kmown amount dC_r and another record of the beat frequency, n_3, was made. C_r was accurately calibrated and read by means of a telescope. Since $\frac{dn}{dC} = \frac{n}{2C}$ and n and C were approximately constant, $dn \propto dC$. The change in capacity of C due to the admission of a gas of dielectric constant ϵ was $(\epsilon - 1)C$.

Therefore $$\frac{(\epsilon-1)C}{dC_r} = \frac{n_2 - n_1}{n_2 + n_3}$$

The value of C was determined by bringing the circuits A and B to the same frequency by varying C_a, having C first disconnected and then connected in the circuit. Difficulties were experienced due to changes in the filament temperatures of the oscillators. The size of the batteries seemed to affect the results and some trouble was caused by insufficient shielding. Also the capacity effect of the leads was not fully corrected for. Experiments carried out with two different condensers indicated that the effect of the solid dielectric spacing could be neglected. The values of dielectric





C.T.Zahn's Method.

constant found for carbon dioxide and hydrogen were in agreement with values found by previous observers, whereas those found for nitrogen, oxygen and air were rather lower. On the whole, the high accuracy attained in the method of measuring beats was notborne out by the rest of the apparatus.

When C.T.Zahn, (Phys. Rev. 1924 P.400)devised a method of measuring the dielectric constants of gaseous hydrogen halides in order to calculate the electric moments of their molecules, relative values previously obtained were in fairly good agreement, but absolute values differed by as much as 10%, suggesting inaccuracies in the calibration of the apparatus used. The method he used was similar to that of Fritts, two similar oscillators A and B being used, the frequency of A being kept constant during each set of readings while the frequency of B was varied by means of a test condenser C in the anode circuit. K' was a fixed condenser and K a smaller variable precision condenser, these being connected in parallel with each other and in series with C. The oscillators were completely shielded to prevent any action of B on the output ofA during tuning, and were coupled electrostatically to an amplifier fitted with head-phones. A operated at a frequency of about 1m.c.s. and B was always tuned so that a beat note of 1000c.p.s. was produced in the head-phones, this being ensured by adjusting the beat note to give zero beats with a tuning fork operated by an auxiliary valve circuit at 1000c.p.s. This method of detection, although less accurate than that of Ffitts was less complex and gave sufficient accuracy for the measurements involved. The cylindrical condenser used was made of a platinum and palladium alloy of gold, and had small quartz disc insulators Q. It was resistant to chemical action at the high temperatures used, these being produced by a heating coil and measured by means of a mica insulated platinum resistance thermometer The whole apparatus was enclosed in a Pyrex tube and mounted in a shield of nickel-plated brass.. With C evacuated, K was adjusted for zero beats. The gas was introduced and K readjusted for zero beats, so that the change in capacity ΔC caused by the introduction of the gas could be calculated. If ΔK was the corresponding change in K,

$$\frac{1}{K+K'} + \frac{1}{C} = \frac{1}{K+K'+\Delta K} + \frac{1}{C+\Delta C}$$

i.e.

$$\Delta C = -\left[\frac{C^2}{(K+K'+C)\Delta K+(K+K')^2}\right]\Delta K$$

ΔC was small since the dielectric constants of the gases concerned were small, so that the value approximated to

$$\Delta C = -\left[\frac{C}{K+\frac{1}{2}\Delta K+K'}\right]^2\Delta K$$

The capacity C consisted of a capacity C_0 which changed with the gas in the condenser and a non-variable capacity due to the quarz insulators, the lead capacities, the capacity formed with the thermometer wire etc. The The total value of C was determined by replacing C by a precision condenser and tuning it for zero beats, and by short circuiting each of the lead capacities in turn these could be measured and subtracted from the total capacity C, together with a calculated correction for the insulators. These corrections were found to be about 3% of C. By means of a diffusion pump the condenser could be evacuated to a pressure of 10mm. of mercury which was necessary to rid the system of water vapour. The condenser was also baked and closed with a phosphorous pentoxide tube and liquid air trap, and only very carefully purified gases were used since the method was sensitive enough to detect very small traces of impurity. The variation of dielectric constants with temperature was examined, difficulties being experienced in obtaining uniform temperatures owing to the very good heat insulation. The dielectric constants were found to increase linearly with temperature, and, in later experiments on water vapour using a pressure condenser, the variation with pressure at different temperatures up to 165°C was examined. These experiments were of less accuracy than the earlier ones, but it was shown that the dielectric constant increased linearly with pressure at the higher temperatures but not below 100°C, due to surface condensation.

For measurements of the dielectric constants of binary mixtures of very small conductance, J.W.Williams and I.J.Krchma, (Journ. Am. Chem. Soc. 1926 P.1888) used a heterodyne beat method. Asimple wavemeter circuit containing an inductance and capacitance and a two stage amplifier was maintained in oscillation at a constant frequency of about 1m.c.s., care being taken to prevent variations in the batteries and impedances. Telephone receivers in the anode circuit of the second amplifier valve were used to detect heterodyne beats between this oscillator and another, consisting of a simple Hartley circuit containing two variable standard air condensers in parallel of total capacity C_1, in parallel with the test condenser C_x, which has already been described in Williams and Krchma's bridge method. By tuning C_1 for resonance, with C_x in two different settings, first with air and then with various liquids as dielectric, the dielectric constants of the liquids were calculated, thus eliminating the necessity for lead corrections. Values obtained for benzene and toluene were 2.282±0.0020 and 2.378±0.0030 respectively.

H.E.Watson (Proc. Roy. Soc. 1928 P.43) made an apparatus for determining the absolute dielectric constants of ammonia, phosphine and arsine. The change in capacity of the test condenser when filled with gas was compensated by means of a variable condenser in series so as to maintain the frequency of the oscillating system constant, the exact setting being obtained by the production of beats with a second oscillating system of fixed frequency. The valves used were very much underrun to prevent fluctuations in output, and the output could be tested by observations on meters arranged in the circuit. All components were separately shielded and carefully spaced. The whole apparatus was shielded, and the variable condenser was adjustable by means of a system of pulleys and observable through a telescope fitted in the shield. The oscillator could be operated over a range from 300k.c.s. to 2m.c.s. The test condenser used was similar to that constructed by Zahn, but having a glass bulb fitted inside to reduce the volume of dielectric needed. It was kept at constant temperature by immersion in freezing mixtures or in an oil bath, heating coils not being used in order to avoid stray capacity effects. All condensers used had very low power factors. Long leads were so arranged that stray capacities were small, and their inductances were allowed for. A specially standardised condenser of silvered quartz tube was used to calibrate the variable condenser. Allowances were made for the effects of temperature variations on the condensers. Corrections were also made for the capacity effects of the leads and mutual capacities of the test and variable condensers. The circuit for beats production was placed 2.5metres from the test circuit and was well shielded. It consisted of an oscillating detector and note amplifier leading to an audio-oscillator of controllable intensity, which modulated the output at 600c;p.s. This was further amplified and led to a loudspeaker, and the beat note between the high frequency sources could be adjusted to the exact frequency of the audio-frequency note by observing beats between the two. Readings were taken with the test condenser alternately evacuated and filled with gas, and an average value was taken for each gas. Difficulties were experienced in obtaining a uniform temperature throughout the gas condenser, especially when it was evacuated. Determinations were made with phosphine, ammonia and arsine, and also with air, carbon dioxide and benzene in the condenser for comparison with values obtained by other observers. The values obtained for carbon dioxide and air tended to be higher than those previously obtained, and the benzene value was considerably higher, so that the absolute values obtained were of doubtful accuracy.

An attempt to improve on the above method was made by Watson, Rao, and Ramaswamy (Proc. Roy. Soc. 1931 P.569). The audio-oscillator was replaced by a 750c.p.s. tuning fork, valve driven, and a crystal controlled oscillator of 958k.c.s. was used as a standard frequency source. The effect of external conditions on the variable frequency oscillator was carefully investigated and counteracted, The gas condenser was made so that the constant temperature bath could pass through the centre, making it much easier to obtain uniform temperatures. In spite of the use of very thick metal cylinders for the test condenser it was difficult to avoid small errors due to deformation by the gas pressures. One serious error in the previous determination was due to the inductance of the fixed condensers, and attempts to correct this were not very successful. Corrections for condenser distortion by the gases under pressure were made by using benzene as a standard. The rare gases and hydrogen were examined their dielectric constants being measured after each stage of purification, constancy of readings being taken as a criterion of purity. From the values obtained it was found that none of the gases had an electric moment.

J.D.Stranathan (Phys. Rev.1928 P.653) used a heterodyne beat method to measure the dielectric constants of dilute solutions of polar liquids in non-polar solvents, and to relate the values to the Debye Theory. The standard oscillator, crystal-controlled at 476k.c.s., and the variable oscillator containing the test and variable precision condensers in parallel, were carefully shielded from each other and from the amplifier, by means of which a beat note was heard that was adjusted to 1000c.p.s. by comparison with a standard tuning fork. The frequency output of the variable oscillator could be checked by the substitution of a precision condenser for the variable capacity components. No variations were experienced due to synchronisation of the oscillators, owing to the shielding arrangements. The test condenser was a three plate cylindrical brass condenser, the plates of which were separated by three Pyrex spacers. This fitted into a thermos flask, and the temperature was read by means of a mercury thermometer. Lead capacities were eliminated by making measurements with dummy leads, and the effect of the glass spacers was allowed for by measurements made on a condenser with twice the number of spacers. The capacity of the empty condenser was found to be a linear function of the temperature and this effect was allowed for Corrections were necessary for the difference in evaporation rates of the components of the mixtures used, especially at high temperatures; e.g. with methyl alcohol in benzene the alcohol evaporated considerably. Care was necessary to prevent the formation of bubbles during readings taken at high temperatures.

C.H.Schwingel and J.W.Williams, (Phys. Rev. 1930 P.854) examined the variation with temperature of the dielectric constants of carbon disulphide and nitrous oxide. The variable oscillator and crystal controlled standard, oscillating at 498.6k.c.s., were separately shielded and coupled by a single wire, the amplified beat note being reduced to a zero of 20 beats in 10 seconds. This low frequency beat note may have caused some pulling of the variable oscillator. The arrangement of the test and variable condensers and the procedure and calculation involved were similar to those of Zahn. The condenser consisted of 23 parallel plates of heavily gold-plated brass with a spacing of three mms. and insulated by two strips, of Mycalex, the effects of which were ignored owing to their size and position. The capacity of this condenser was checked before and after every series of readings, and allowances were made for lead capacities. Five to eight readings were taken at each temperature, the condenser being alternately filled and evacuated, and a test made with ethyl ether was found to give the accepted value for its electric moment. This method was later used by E.W. Greene and J.W.Williams (Phys. Rev. 1932 P.119)to examine the variation of the dielectric constants of the ethylene halideswith temperature at different pressures. The condenser used consisted of four concentric cylinders of polished Monel metalwith four mica spacers, supported in a large Pyrex tube which fixed into a vertical iron tube heated bya nichrome wire coil. Pressures were measured by a Daniel's gauge to 0.2mm. of mercury, and temperatures were registered by two chromel-alumel thermocouples to an accuracy of 0.05°C.

For the determination of the polarisation and electric moment of tung oil A.A.Bless, (Phys, Rev.1931 P.1149) measured the dielectric constant using a crystal oscillator at a frequency of 100 and 300k.c.s.

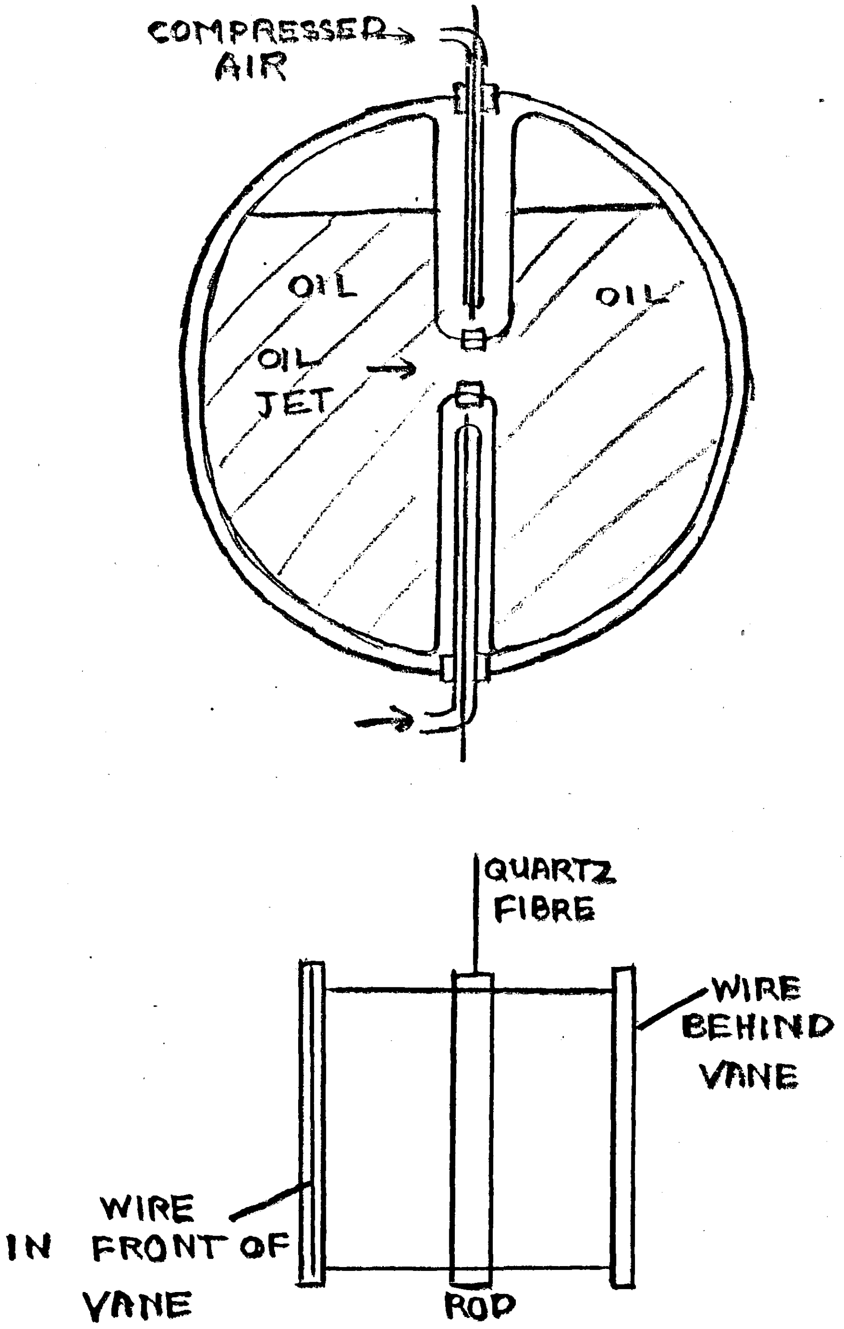
and 1m.c.s. and a Hartley oscillator containing a variable and test condenser, these being adjusted to zero beats. The test condenser needed only a small quantity of dielectric yet had a fairly large capacity, consisting of two concentric nickel cylinders fitted into a glass tube of annular cross-section provided with a tube for a thermometer.

W.R.Pyle, (Phys. Rev. 1931 P.1057) used a standard oscillator crystal controlled at 85.5k.c.s. coupled by a single wire to a Hartley oscillator, the beat note being amplified and adjusted to zero frequency for readings. The variable components were controlled by pulleys outside metal shields. The test condenser consisted of two coaxial brass cylinders cemented with sodium silicate to a ground glass base, and the end effects of this were neglected; The test oscillator was often checked for frequency drift. Stray capacities were avoided by careful grounding and shielding, and the lead capacity was found by making measurements with dummy leads. At the frequencies used lead inductances were considered negligible. The calculated test condenser capacity was estimated to be 2.5% in error due to variations in the diameters of the cylinders, and to the fact that they were non-coaxial. For liquids of high dielectric constant the test condenser was filled only to a suitable known height owing to the limited range of the precision condenser, and this may have introduced error, besides complicating the procedure and calculations. The method was only possible for liquids of very small conductivity, and was found unsuitable for measuring the dielectric constants of acetone and aniline, these being found by a bridge method. Good agreement was found with previous observers for ethyl ether, nitrobenzene, o-xylene, m-xylene and p-xylene, but the apparatus was not suitable for use over a large temperature range.

J.D.Stranathan, (Phys. Rev.1935 P.538) developed a very stable heterodyne beat apparatus for measuring the dielectric constant of water vapour. Two oscillators were tuned to a beat note of 1000c.p.s. as in his previous methed described earlier, the standard oscillator being crystal controlled at 545k.c.s. All the valves were run from a common a.c. power supply, and great care was taken with shielding to prevent synchronising of the oscillators. The temperature was controlled to within 0.1°C so that, after warming up, there was no frequency drift. Experiments were carried out with different condensers, the spacers being arranged in the weakest parts of the field, and the effect of using spacers of different materials was examined. For the variable condenser, a small standard variable condenser was used in parallel with the unknown condenser, and an accurate method of calibration was described, giving values to a few thousandths of a picro-farad. This gave better results than a large variable standard in series with a small fixed precision condenser owing to shielding difficulties and inter-condenser capacity effects. The dielectric constant of water vapour was found to vary linearly with pressure over a larger range than was hitherto thought until saturation was closely approached, variations then being due to adsorption.

A.C.Tredidga (Phys. Rev. 1940 P.294) measured the variation with pressure of the dielectric constant of water vapour at different temperatures using a crystal-controlled oscillator at a frequency of 42m.c.s. as a standard. The heterodyne beat note formed with the variable oscillator was amplified and impressed on the plates of an oscilloscope so that the condition for zero beats could be determined with precision. The test condenser was made of conical-shaped parallel plates of Invar steel, held apart by three equidistant quartz pins It was mechanically strong yet had only small edge effects and was invariant with temperature. Owing to difficulties in making absolute measurements at the high frequencies used, relative measurements were made with air as the calibrating gas. Capacity measurements were made with the condenser alternately evacuated and filled with gas, readings being taken every eight minutes until the temperature was constant. A linear relationship with pressure for the dielectric constant was obtained at temperatures of 71.9°C, 99.8°C and 147°C.

L. G.Hector and D.L.Woernley (Phys. Rev. 1946 P.101) measured the dielectric constants of eight gases by a heterodyne beat method. Previous results, while giving constant results for each apparatus, disagreed among themselves, and possible sources of error were investigated. A standard high frequency was obtained by demodulation of a radio broadcast signal. The screen grid of a pentode was used as anode in the test oscillator circuit, and the output signal was taken from the anode. A buffer amplifier mixed the signals from the standard and test oscillators so that coupling was prevented between the two sources, and this amplifier was tuned to the second harmonic of the oscillator to reduce coupling further and to double the beat note. The test condenser consisted of multiple plates mounted on three isolantite pillars and shielded by a thick, brass cylinder in a heavy, steel pressure chamber. Data taken with two condensers of different sizes allowed the non-variable capacity to be calculated. The oscillator was tuned to the same frequency as the standard signal with the empty test condenser in the circuit, and the change in frequency on admission of a gas was measured on a string oscillograph, a standard audio-frequency being simultaneously recorded on a second string. The values of dielectric constant obtained were lower than the average of previous values for air, helium, neon andargon, but in good agreement with those for oxygen, hydrogen, carbon dioxide and nitrogen.



J.D.Tear's Method.

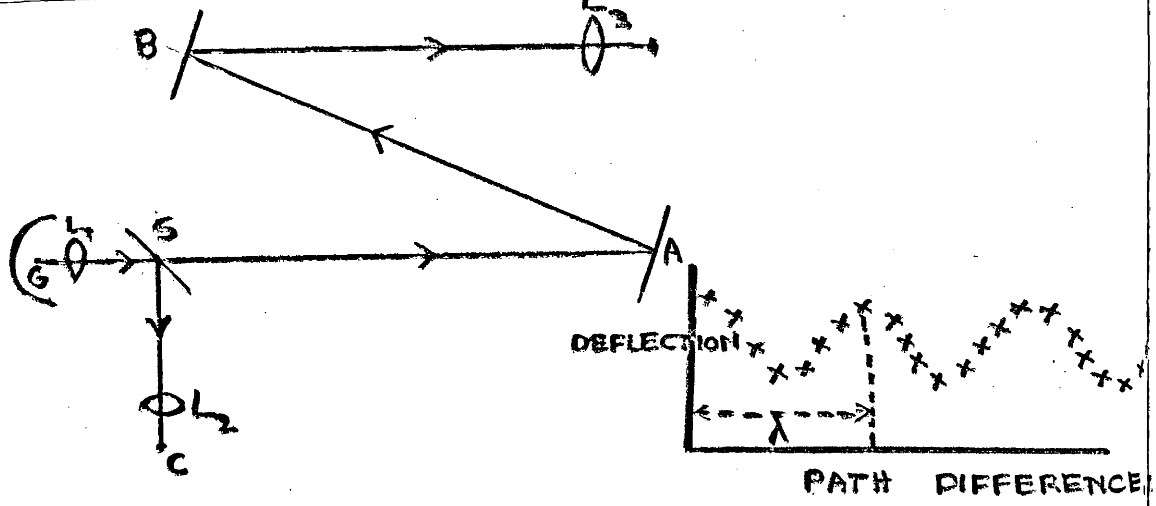
Free Wave Methods.

Free wave, line, guide and resonator methods of determining dielectric constants involve the measurement of wavelength in air or in a vacuum and in the material under test. From the ratio of these wavelengths the refractive index can be found, which is, for poorly conducting dielectrics, the square root of the dielectric constant. The earliest used, and basicly simplest of these methods are the free wave methods, and generally they are less accurate than the others. In actual determinations, Hertzian oscillators were mainly used, which were unreliable and caused inaccuracies in wavelength measurements. Quite apart from the oscillators used, difficulties due to diffraction usually arose.

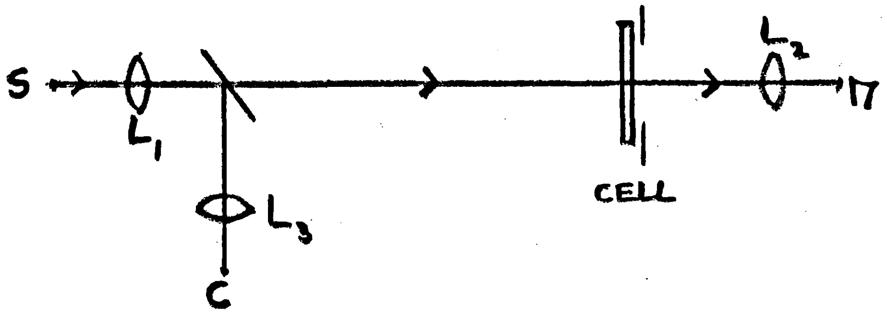
J.D.Tear (Phys. Rev. 1923 P.611) used an optical method for determining the refractive index and absorption of certain liquids for electric waves of 0.42 to 2.7cms. A specially designed Hertzian oscillator was used consisting of a tungsten doublet with an oil jet directed at the spark gap as shown. The radiatien from this was made parallel by a double convex lens, and it was received on a number of resonating wires attached to radiometer vanes so that the incident energy of the resonant frequency caused heat to be generated and the vanes to deflect. The very light moveable vane were suspended from a quartz fibre enclosed in a glass vessel which was evacuated and which was fitted with an ebonite window for transmission of the incident

J.D.Tear's Method.

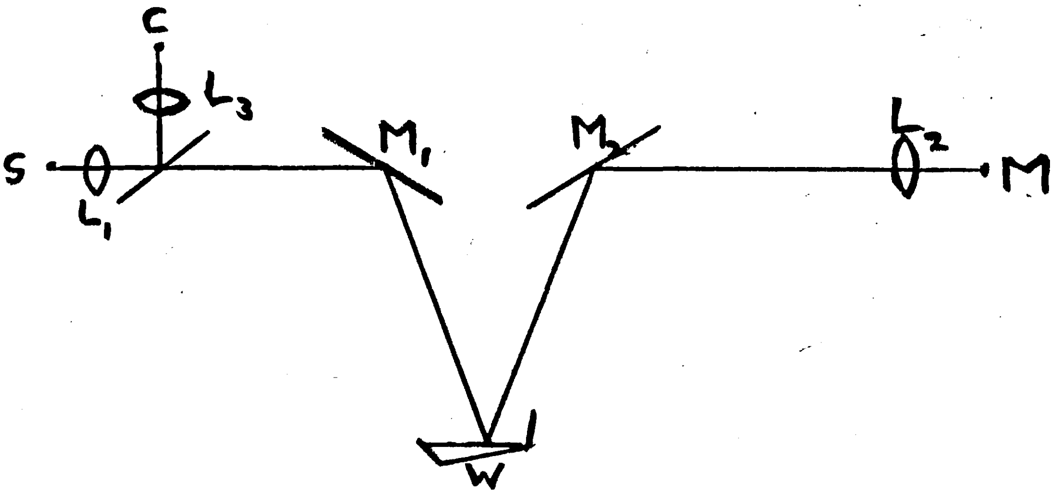
Wave-length measurements



Transmission Measurements,



Reflection Measurements.



energy and a glass window to enable observations of the deflection to be made with a telescope. The oscillator and receiver were tuned to the same frequency and arranged so that the electric vector of the radiation incident on the receiver was parallel to the resonating wires. The radiation wavelength in air was measured by means of interference fringes produced by a Boltzman interferometer. The oscillator was placed at the focus of a concave mirror G, and the radiation was reflected from a plane brass mirror A to the interferometer brass plates at B and received at the focus M of the lens L_3. The check receiver C was used in all measurements, since the Hertzian oscillator could not be considered a constant source of radiation and had to be checked continually. Measurements of the absorption of liquids were made by comparing the deflections of the receiver when exposed directly to the radiations and when a liquid cell of thickness d was interposed between them. Glass cells, owing to their high reflection coefficient, were found to be unsuitable as they produced considerable interference, and this difficulty was lessened by using ebonite cells to contain the liquid under test. From measurements on layers of different thicknesses d, th value of the extinction coefficient, α, in the formula

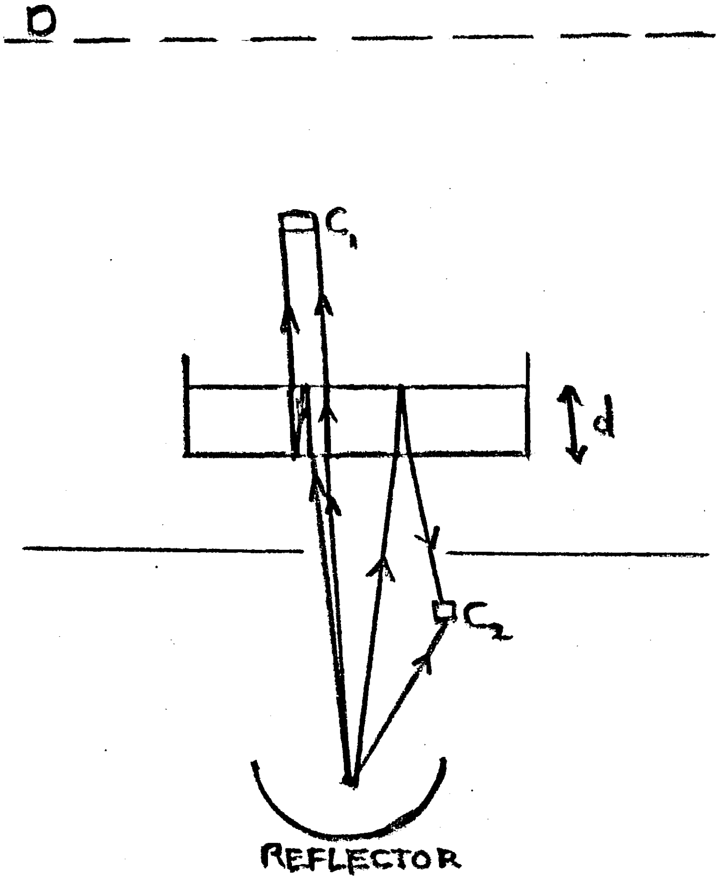
$$I = I_0 e^{-\frac{4\pi\alpha}{\lambda} . d}$$

could be found. The logarithm of the transmission, log $\frac{I}{I_0}$

was plotted against d, the slope of the straight line obtained being $-\frac{4\pi\alpha}{\lambda}$. Measurements of the reflection coefficient were then made by the arrangement shown, M_1 and M_2 being silvered plate glass and W the liquid contained in a rubber tray, tilted to avoid back surface reflections. The radiation received at M was compared with that received when a mercury surface was substituted for the liquid, mercury being perfectly reflecting to centimetre waves to within ½%. The refractive index was then calculated from the formula

$$n = \frac{b}{\cos\varphi} + \sqrt{\frac{(b^2-1)}{\cos\varphi} - \alpha^2} = \sqrt{\epsilon}$$

the electric vector of the radiation being in the plane of incidence with α the angle of incidence, λ the wavelength in air, $b = \frac{1+R}{1-R}$, R being the reflection coefficient. Measurements of refractive index were also made by examining the interference fringes produced in thin films of liquid, these being extrapolated to ~~infinitely thickness~~ infinitely small thickness, but corrections should have been made for the absorption of the films and the results were not very accurate. Also the method was quite unsuitable for liquids such as kerosene, which had a low reflecting power, or water which had a high absorption for the wavelengths used. Two absorption bands were found for water in the regions examined, and glycerin, ethyl alcohol and methyl alcohol all showed signs of anomalous dispersion, although some of these effects may have been due to the



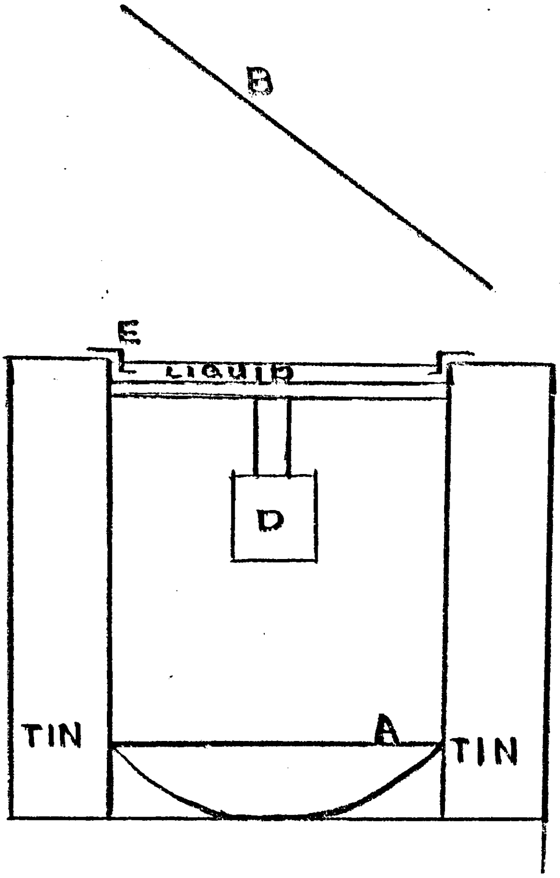
T.T.Goldsmith's Method.

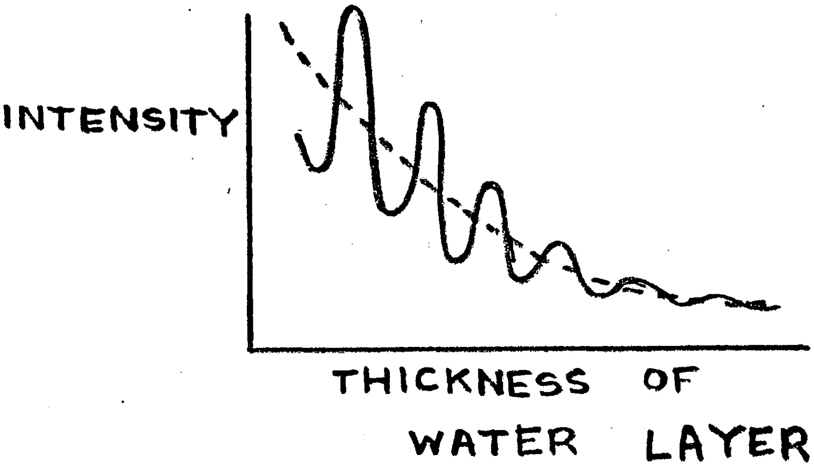
presence of water as impurity.

T.T.Gold smith (Phys. Rev.1937 P.245) measured the refractive index of water for electromagnetic waves 8 to 24cms. in length, using magnetron and positive grid oscillators as sources, which were of very much greater stability and gave a purer wave form than the Hertzian oscillators previously used. Polarised radiation from a transmitting amtenna T was allowed to fall on the crystal detectors C_1 and C_2, fllowing the paths indicated. Each crystal received two beams of energy, one of which had been retarded by traversing the water thickness d twice more than the other. By varying the depth of the water, the amount of radiation received at each crystal passed through maxima and minima due to interference, the change in depth Δd of the water corresponding to two successive maximum or minimum values being equal to half the wavelength of the radiation in water, $\frac{\lambda'}{2}$. Similarly, by placing the moveable reflector D above C_1 and producing interference by varying the path difference in airof the beams received at C_1 directly from T and after reflection from D, the wavelength of the radiation in air was found, and the dielectric constant and refractive index were given by

$$\sqrt{\epsilon} = n = \frac{\lambda}{\lambda'}$$

The curves obtained from the crystal C_2 were more successful than those from C_1 since the interference





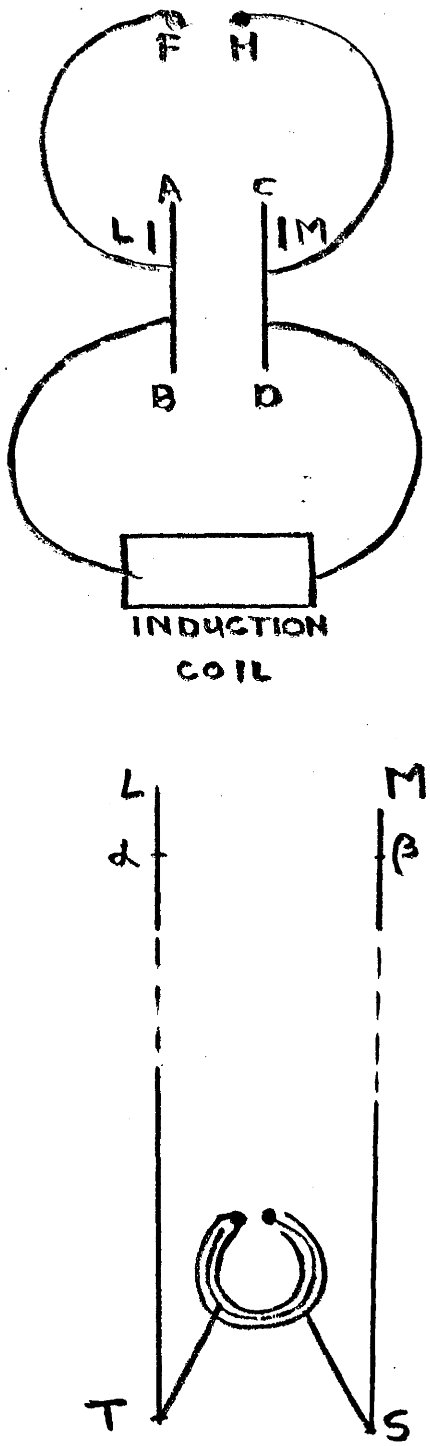
H.W.Knerr's Method.

curves from the latter were superposed upon the absorption curves for the increasing water thickness, making the peaks difficult to locate. The valve oscillator was found to vary as the water thickness increased, so that there was not complete agreement between results obtained using this and the magnetron oscillator. A linear decrease in refractive index with decreasing wavelength was found, thus indicating anomalous dispersion in the region investigated. The absolute values obtained were rather high compared with those of other observers, as no correction for absorption was made.

H.W.Knerr (Phys. Rev. 1937 P.1054) examined the spectrum of water for wavelengths of 5 to 20cms. For wavelengths up to 12cms. he used a free wave method, while for longer wavelengths a method using guiding lines was found considerably more successful. Previous disagreement on the presence of absorption bands was considered to be due to the damped sources of oscillation used, improvements being found when continuous wave sources were used. In this case a magnetron generator was used consisting of split anode magnetron tubes producing a beam of waves which was concentrated by a parabolic brass mirror. For wavelength measurements up to 12cms. this was directed to an echelette grating at an angle of incidence of 6½° and the reflected beam was received by a second parabolic mirror having the detector at its focus. This consisted of a tungsten wire and iron pyrite crystal, which had previously been calibratedfor receiving plane polarised radiation by means of Hertzian grids, and was found to respond linearly to the incident energy. The absorption cell was constructed as shown, radiant energy being reflected from the aluminium mirror B through the liquid layer to the mirror A, from which it was focussed on to the detector D. The tin guard strip T provided the slit aperture inside the liquid to minimise diffraction difficulties, but these were still considerable. As described in the previous method, maximum and minimum deflections were observed by varying the thickness of the liquid layer, these being superposed on an exponential absorption curve as shown. The path difference đ and phase difference for beams suffering up to 4 internal reflections were calculated and the resultant obtained, and by rather complex analysis the consequent displacement of the interference maxima and minima could be allowed for and the wavelength in the liquid found from the corrected distance between successive maxima. The refractive index was found, as before from the ratio of the wavelength in air and in the liquid, and hence the dielectric constant was calculated.

Line Methods.

Transmission lines have been used for the measurement of dielectric constants with considerable accuracy for waves of 10cms. to 1m. in length. For waves shorter than 10cms. wave guides have proved of greater use owing to high losses in lines as the wavelength decreases. For waves longer than 1m. the necessary apparatus for wavelength measurement becomes unwieldy. Many of the earliest line methods used consisted in setting up stationary waves along lines in air and then immersed in the test dielectric, the wavelength in each case being derived from the distances between the nodes and antinodes. Other methods used have involved the measurement of the capacity of a condenser joined across the ends of the parallel wires used, first with the condenser filled with air and then with the dielectric under test. With parallel wires the field is not confined strictly to a limited space around the wires, so that readings are sensitive to external variations. Also, inthe method requiring the immersal of the wires on the test dielectric, the quantity of dielectric needed was very large, andthere was always a tendency for part of the field around the wires to remain outside the dielectric, which introduced considerable error. Later coaxial lines were used, the outer tube acting as a shield so that the field was entirely concentrated in the annular space between the



J.J.Thomson's Method.

two conductors, thus overcoming the earlier difficulties.

J.J.Thomson, (Proc. Roy. Soc. 1889 P.292) measured the dielectric constants of a number of solids at a frequency of 25m.c.s. Oscillations were excited in the condenser ABCD by the induction coil as shown, the wavelength being $2\pi\sqrt{LC}$ where C was the capacity and L the inductance of the condenser unit. The condenser consisted of circular zinc plates 30cms. in diameter, and F and H were highly polished balls. L and M were small zinc plates placed close to the condenser plates and having two insulated wires about 20m. long attached to them, so that oscillations were induced along the wires. The wavelength of the oscillations was measured by means of a resonator consisting of a circular wire coated with rubber and tin foil and broken at one point, the ends being terminated by two metal balls whose distance apart could be regulated. The inner and outer conductors of the resonator were joined to two metallic contacts which slid along the transmission lines as shown. Sparks passed between the balls of the resonator when the contacts were at different potentials As shown in the diagram, one contact was kept at T and the other moved from S along the wire until, at β, no sparks were observed. β was then at the same potential as T. The contact at T was then moved along the wire until, in the position α , no sparks passed across the resonator air gap. α, β, T and S were then known to be at the same potential, and, since T and S were at the far end of the wires the potential there was a maximum and the distances $T\alpha$ and $S\beta$ were one wavelength. The wavelength was then changed by inserting a slab of dielectric between the plates of the condenser ABCD, and it was measured again by the same process. Then if λ_1, λ_2, C_1 and C_2 were the respective wavelengths and corresponding capacities producing them,

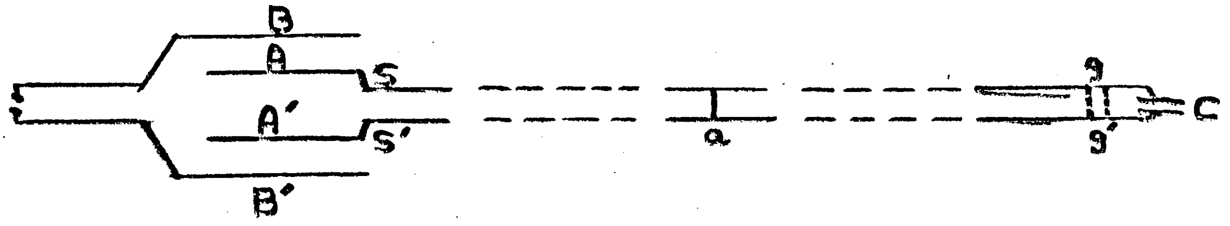
$$\frac{\lambda_1^2}{\lambda_2^2} = \frac{C_1}{C_2} = \epsilon$$

, the dielectric constant of the material under test. Allowance had to be made for the capacity which was not due to the parallel plates and was not altered by the insertion of the dielectric slab, and this was done by calculating the plate capacity C , and measuring the total capacity in air by a tuning fork method, the difference giving the constant part of the capacity, C'.

Then
$$\frac{\lambda_{\text{Dielectric}}}{\lambda_{\text{Air}}} = \sqrt{\frac{C\epsilon + C'}{C + C'}}$$

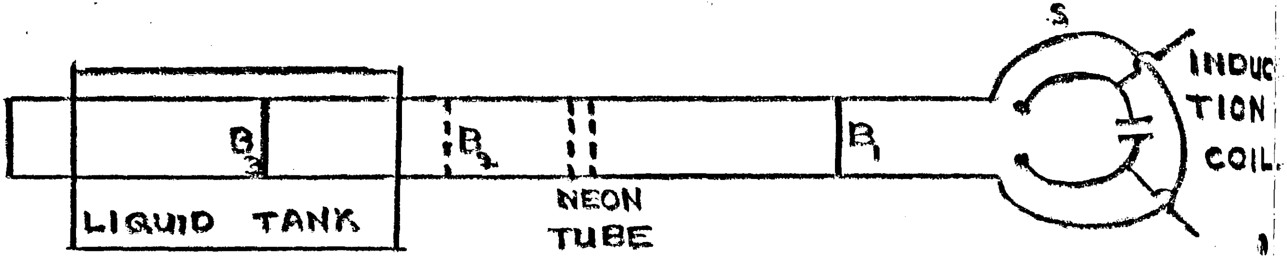
Measurements were carried out on glass, ebonite and sulphur, the value for glass being 2,7, thus approaching the value obtained from the refractive index using visible light and being much smaller than the static value, which showed that absorption takes place at wavelengths longer than 10m. The values for sulphur and ebonite were in agreement with the static values, indicating an absence of absorption bands at longer wavelengths.

E.Lecher, (Phil. Mag. 1891 P.172) measured the dielectric



Method of E. Lecher.

constants of ebonite, glass and petroleum by three different methods at frequencies of 2, $2x10^3$ and $3.3x10^7$c.p.s. For the highest frequency a Hertzian oscillator was used to send waves from AA' along two wires joined at the far ends by a condenser C, the wires being 31cms. apart and 1,122cms. long. The condenser had circular plates of radius 9.68cms. which could be moved parallel to each other by means of a micrometer screw, the distance between them being accurately known. On bridging the lines in certain positions with a metal slider 'a', the vacuum tube gg' could be made to glow, the positions of 'a' depending on the value of C. The value of C was changed by varying the distance apart of its plates, and the corresponding distances of 'a' from s were measured for the maximum glow of gg'. This was repeated with a slab of dielectric between the plates of C. Each determination of the position of 'a' was repeated many times, the greatest deviations for each determination of a legth 'as' being 5 to 6cms. With air as dielectric a curve was plotted of distance between the condenser plates against the corresponding length 'as', so that with any dielectric used, the equivalent air capacity giving the same length 'as' could be found, and this was equal to the capacity of the dielectric-filled condenser. If t_1 was the separation of the plates with air as dielectric, t_2 the thickness of the dielectric slab and e the separation of the plates with this in-



P. Drude's Method.

serted,

$$C = \frac{A}{4\pi\left(\frac{e}{\epsilon} + t_2 - e\right)} = \frac{A}{4\pi t_1}$$

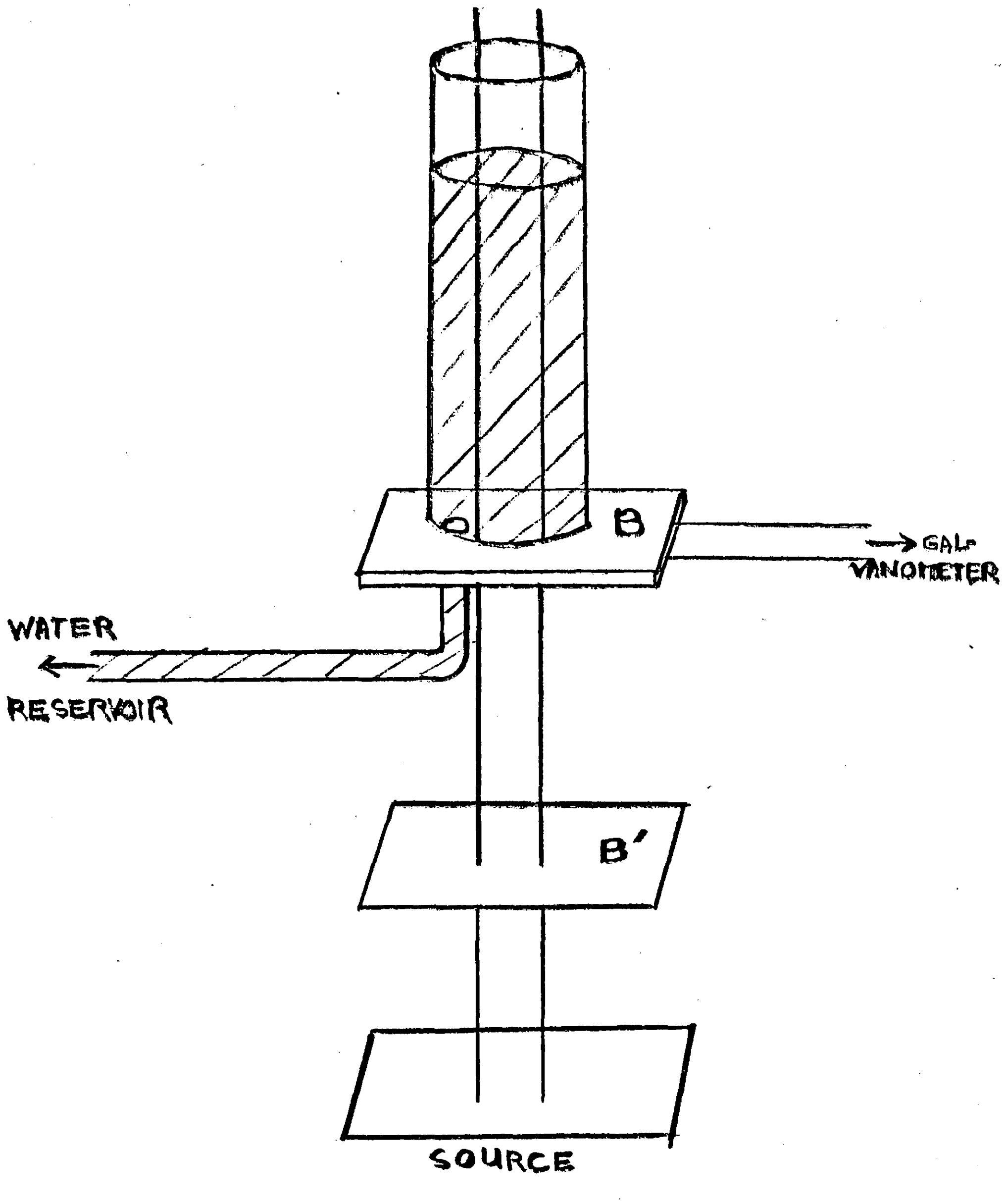
A being the area of each plate, so that

$$\epsilon = \frac{e}{e - (t_2 - t_1)}$$

The average thickness of the ebonite and glass specimens used was accurately measured by a micrometer. The petroleum was enclosed in a glass cell with wooden sides and allowance was made for the glass. Since the glass was thin difficulties were found due to the liquid pressures distorting the cell. Other errors were due to the presence of water in the oil. The value of the dielectric constant obtained for ebonite was high compared with Thomson's value, while for glass it was 7.31, considerably greater than Thomson's value. Attempts were made to measure the dielectric constant of water, but owing to the high conductivity an infinite value was obtained.

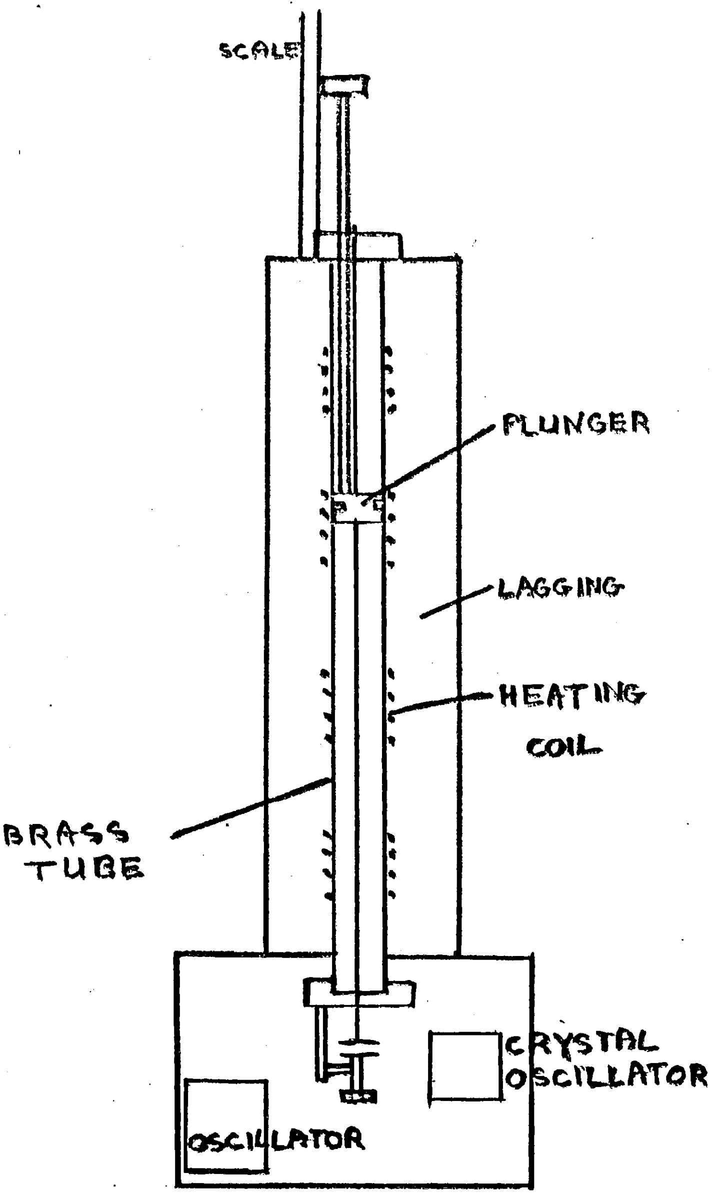
P.Drude (Wied. Ann. 1895 P.633) measured the dielectric constants of a number of liquids by a method which did not require a condenser, at wavelengths varying from 6 to 384cm. in air. A source of damped oscillations from an induction coil was used to induce secondary oscillations in the wire S, and these travelled along the parallel wires shown. With the liquid tank removed and with bridges B_1 and B_2 suitably placed, standing waves were set up along the line and the distance between consecutive nodes and antinodes was measured by means of the neon tube, this distance being half the wavel length in air. By immersing part of the line in liquid leaving B_1 in its former position but adjusting B_3 to various positions inside the liquid so that stationary waves were once more produced, the wavelength in the liquid could be found, and the ratio of the two wavelengths gave the refractive index of the liquid. For water, at a frequency of about 830m.c.s., the dielectric constant was given as 76±6. The same value was obtained for copper sulphate solution, although the waves were rapidly damped and readings difficult to obtain for all but dilute solutions. Values were also obtained for ethyl alcohol, glycerin and petroleum, the value of 2 for the latter being smaller than Lecher's value observed at a lower frequency.

G.C.Southworth (Phys. Rev. 1924 P.631) examined the dielectric properties of water using a line method with a source of continuous waves. The oscillator used had a large rectangle of wire of variable size joining the anode and grid, which controlled the output frequency. Many preliminary experiments were carried out on the production of stationary waves along wires, and it was shown that, for wires having a resistance less than that of 18 gauge copper wire, the effect of the resistance on the wavelength could be ignored. Owing to imperfect reflection, it was found that the distance of the first node from the end of the line was not equal to a quarter or half wavelength for open or



G.C.Southworth's Method.

short-circuited ehds respectively. The apparatus finally used consisted of vertical wires passing through air and through a water column as shown. B was a stationary bridge consisting of a halved brass plate joined by means of a constantan-steel thermo-element, which was connected to a galvanometer. B' was a moveable brass sheet attached to the wires by means of spring clips which ensured good contact. With the upper tube empty, the position of B' was adjusted to the nearest point to B which gave resonance, and the distance between the two bridges was then equal to half the wave-length in air. As water was allowed to enter the tube, the galvanometer gave alternately maximum and minimum deflections with increasing depth of water, the sharp minima obtained corresponding to antinodes at the water surface. The distance between consecutive antinodes was thus half the wave-length in water and the refractive index was obtained from the ratio of the wave-lengths ~~in water and~~ in air.and in water. The wavelength in a ir was checked to ensure that the oscillator had not varied. Measurements were made with different wire spacings, different diameters of the glass tube and different degrees of coupling to the oscillator. The first two variations were found to have little effect on the results, but a coupling area of 0.3x1.0cm.caused a progressive error of about 3% for wavelengths of 200to 300cm. in air. For final results this coupling area was reduced to

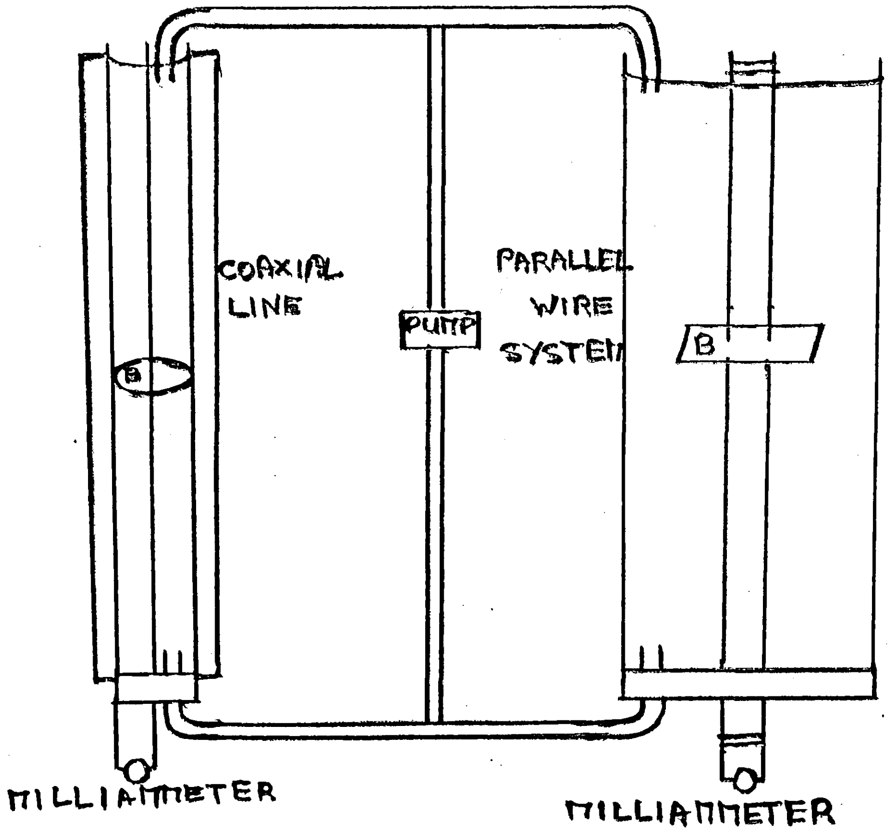


Method of Drake, Pierce and Dow.

0.2x0.3cm. Over the wave-length range examined, from 124 to 276cm., no appreciable dispersion was found, the mean value of the dielectric constant being 79.1±0.5 For copper sulphate solutions the value was approximately the same.

F.H.Drake, G.W.Pierce and M.T.Dow (Phys. Rev. 1930 P.613) measured the dielectric constant of water and aqueous solutions of potassium chloride for air wave-lengths ranging from 3.918 to 25.47m. using a co-axial line. The frequency of the oscillations used was determined by to tuning to zero beats with a crystal oscillator and the air wave-length was hence determined. The line used was a brass tube with a concentric copper wire as shown, the moveable brass plunger being fitted with spring contacts to the wire and to the outer tube. A sensitive galvanometer recorded changes in the anode current of the oscillator, the maximum change occuring when the plunger was situated at a node. The tube was filled with water which could be maintained electrically at different temperatures, and by adjusting the plunger the node positions could be determined to 0.1mm The distance between consecutive node positions was half the wave-length in water, and the ratio of the wave--length in air to that in water gave the refractive index of the water. Inorder to determine the dielectric constant, a correction was applied for the conductivity of the water, and the dielectric constant ϵ was given

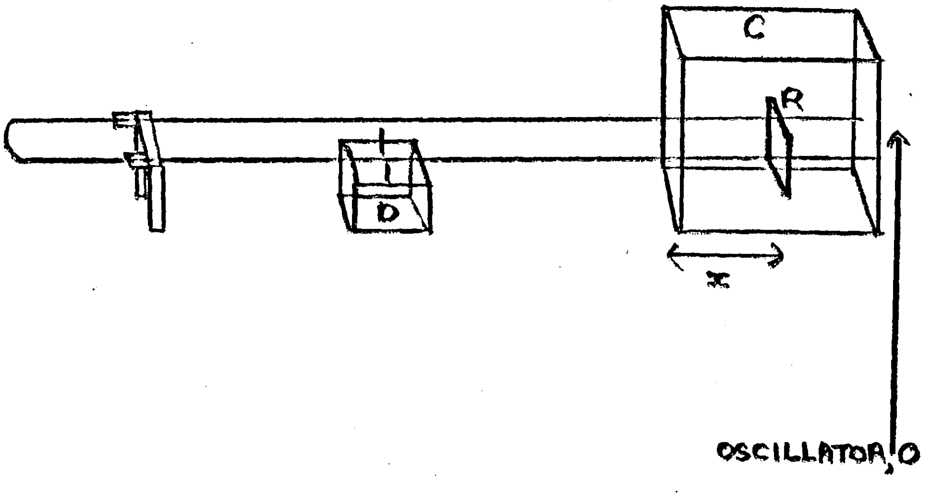
$$n = \epsilon^{\frac{1}{2}} \left[\frac{(1+h)^{\frac{1}{2}} + 1}{2} \right]^{\frac{1}{2}}$$

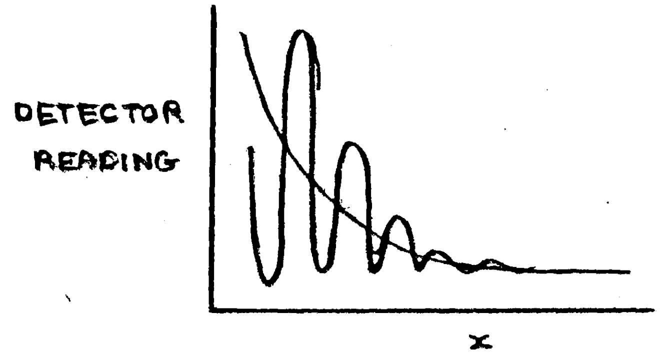


Method of Malone, Case and Ferguson.

where $h = \frac{2\gamma T}{\epsilon}$, ϵ and the specific conductivity of water, σ, being expressed in electrostatic units. T was the time period of the oscillations. At 25°C, the dielectric constant was found to have a mean value of 78.57±0.05, there being no dispersion observed over this wave-length range. A decrease from 87 to 67 was observed as the temperature increased from 3°C to 60°C, the rate of decrease being small between 4°C and 10°C. The dielectric constants of $\frac{1}{170}$N potassium chloride and $\frac{1}{70}$N potassium chloride were measured by the same method and were found to vary less than 1% from the value for distilled water. This variation may have been due to increased errors introduced by the higher attenuation

Malone, Case and Ferguson, (Journ. of Chem. Phys. 1933 P.842) used a similar method for measuring the dielectric constants of liquids, also having a source of undamped waves. The experiments were carried out using Lecher wires and a co-axial tube, three different methods of node detection being employed. The two line systems were mounted vertically as shown, B in each case being a moveable shorting bridge with spring contacts. The Lecher wires were seven feet long and ½in. apart, and were mounted in a tube 5in. in diameter, the whole system being heat insulated. When B was adjusted for maximum current in the milliammeter shown, it was at a node in the stationary wave system set up. The distance between consecutive nodes was measured with the tube filled with air and with various liquids. A similar procedure was adopted for the co-axial line, which consisted of an axially situated phosphor-bronze wire in a brass tube, the wire and the inside of the tube being silver-plated. The valve oscillator used was placed a distance of several feet from the line systems. Preliminary measurements were carried out on water and alcohol and the results agreed to within 0.75% with those obtained by Wyman. It was fonnd that the Lecher line measurements gave lower values than the co-axial line since part of the field extended outside the glass tube containing the liquid. This error was increased by placing the wires further apart. measurements were then made with potassium chloride and copper sulphate solutions, no corrections being made for the conductivity, so that the dielectric constant was calculated from the formula $\epsilon = n^2$, and a decrease in the constant was found with increasing concentration. The experiments were repeated with a valve voltmeter connected across the line input terminals and the milliammeter removed. Using this method of detecting the nodes an increase in dielectric constants with increase in concentration of electrolytes was found. Although the values obtained for water and alcohol were consistent, the valve voltmeter was found to be very sensitive to changes in its surroundings, and the precision was not so high as with the milliammeter detector. A

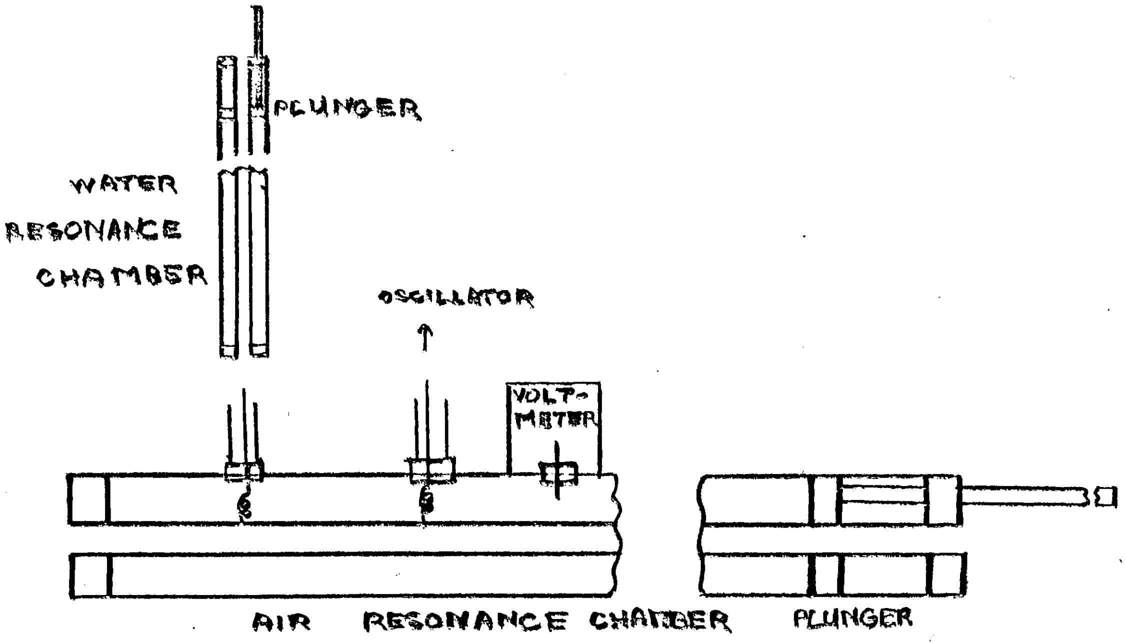




H.W.Knerr's Method.

third method of detecting nodes was used in which the maximum reaction on a valve voltmeter connected in the circuit of the oscillator was observed. This detector was also sensitive to changes in its surroundings and gave values for the dielectric constants which increased slightly with increasing concentration. It was concluded that the anomalous results obtained were due to the absence of correction for conductivity and not, on the whole, to faulty action of the detecting devices.

In addition to the free wave method previously described, H.W.Knerr (Phys. Rev. 1937 P'1054) used a line method in his investigations of the spectrum of water for wave-lengths of 5 to 20cms. A cell of soft wood, C, lined with rubberized cloth, had two copper wires 1.9mm. in diameter and spaced 1cm. apart sealed into the walls as shown. The length of the wires outside the cell could be varied from 73 to 83cm. by adjusting the trombone arrangement at the end. A magnetron oscillator O was placed near the cell, and was joined to the reflector R which bridged the wires inside the cell. The detector used consisted of an iron pyrite-tungsten crystal connected in series with a milliammeter and supported in a wooden mounting, the pick-up wires being placed with their tips a few millimetres from the wires. The copper wires used had a very low resistance, and this was assumed to have a negligible effect on the waves set up along them. There was no evidence that the presence of the detector disturbed these waves.The cell C was filled with water and the reflector R was placed well back in the cell so that the waves reflected from R were highly damped by their passage through the water and were not detectable outside the cell. The length of the line was then adjusted to produce standing waves and the detector was adjusted to a position of maximum reaction i.e. an antinode. When the reflector R was moved nearer to the front face of the cell, the transmitted wave was reflected from the end of the line as before, and, travelling back into the cell, was reflected at R, but this time the oscillation was not completely damped and, appearing along the line outside the cell, set up another system of standing waves with its component reflected from the far end. For certain positions of the reflector, the nodes and antinodes of this system coincided with those of the first system, so that, as the distance of R from the front cell wall, x, was varied by means of a micrometer screw, the detector reading passed through maximum and minimum values. As the distance x became very small, it was possible for waves reflected three or four times at R to be detected outside the cell, so that, when the detector reading was plotted against x a series of maxima and minima were obtained superposed on an exponential curve, the



Method of Conner and Smyth.

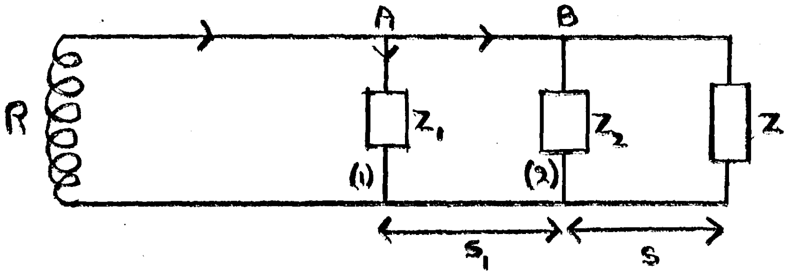
distance between two peaks being ½ of the wavelength in water. The wave-length in air was twice the distance between two antinodes produced when no reflected component from R appeared along the line. The refractive index, given by the ratio of the wave-length in air to that in water was found to be 8.80 to within0.6% at 22°C, and as the conductivity of the water used was very low, the dielectric constant was the square of this value This method was considered far more accurate than the free wave method, and all the results were reproducible.

W.P.Conner and C.P.Smyth measured the dielectric constants of some simple amino-acid polypeptides using a coaxial tube method for frequencies of 300 to 750m.c.s. The apparatus consisted of a stand-pipe oscillator coupled to the inner tube of a copper coaxial tube air resonance chamber by means of a short coaxial line. Standing waves were set up in this chamber by adjusting the shorting plunger, the antinode positions being determined by a simple probe-type voltmeter coupled capacitively to the inner tube of the resonance chamber, the outer tube being slotted so that its position could be adjusted and the wave-length, twice the distance between adjacent antinodes, could be calculated. The resonance positions were sharp , and distances could be measured to 0.01cm. The oscillator coupling was reduced until the grid current changed by less than 0.5milliamps. at resonance. The water resonance chamber was similar to the air resonance chamberbut was smaller, and was coupled to the inner tube of the latter as shown in order to lessen its effect on the oscillator. After determining the wave-length in air, the plunger of the water chamber was adjusted to give a minimum reading of the voltmeter. Then maximum energy was being withdrawn from the air chamber and the water chamber was at resonance. The distance between two adjacent positions of the water plunger was half the wave-length in water, and these positions could be determined to within 0.015cm. Only about four positions could be found owing to the high attenuation of the water, and these could be most accurately determined by plotting voltmeter readings against the plunger settings. The most reliable results were obtained from the symmetrical curves which were given when the air chamber had first been set in resonance with the water chamber completely out of resonance, the only assymmetry then being due to damping. The conductivity of the water used was small and its effect on the dielectric constant was considered negligible, so that the latter was calculated from

$$\epsilon = \frac{(\lambda_{Air})^2}{(\lambda_{Water})^2}$$

This method was very tedious to useas it involved suitable adjustment of three plungers so as not to produce unstable or distorted frequencies from the oscillator.

G.Williams (Proc. Phys. Soc.1944. P.63) adaptedthe double bridge method of measuring impedances devised by Flint



G.William's Method.

and Williams for measuring the dielectric constant of transformer oil at 150m.c.s. Oscillations were induced in the line system at R. Z_1 and Z_2 were the unknown impedances of bridges (1)and (2) carrying current measuring devices and Z was the test condenser. The ratio of the currents I_1 and I_2 was given by

$$\rho^2 = \left|\frac{I_1}{I_2}\right|^2 = K_1 + \frac{K_2 + K_3 \sin 2(b+\beta s)}{\sinh^2 a + \sin^2(b+\beta s}$$

where β was the propagation constant $\frac{2\pi}{\lambda}$, s the distance between bridge (2) and the end of the line, and a and b were given by

$$\frac{Z_0 - Z}{Z_0 + Z} = e^{-2(a+ib)}$$

K_1 and K_3 were constants for a fixed distance s_1 while K_2 was a constant involving the impedance Z. The resistive and reactive components of Z could then be found from

$$Z = Z_0 \tanh(a+ib)$$

where Z_0 was the line characteristic impedance, The separation of the bridges was first adjusted to make K_3 ~~z~~ zero. The condenser was removed and the ends of the line were short-circuited so that a and b became zero. Then

$$\rho^2 = K_1 + K_2 \operatorname{cosec}^2 \beta s + 2K_3 \cot \beta s$$

and a graph of ρ^2 against s gave an unsymmetrical curve until s_1 was adjusted to make K_3 zero. Then the minimum values of ρ^2 occurred whene $s = \frac{\lambda}{4}$. By plotting a graph ,of ρ^2 against $\operatorname{cosec}^2 \frac{2\pi s}{\lambda}$, K_1 was found from the intercept on the axis of ρ^2 . Z_1 and Z_2 were chosen to make K_1 as small as possible. With the condenser C across the ends of the line, maximum values of ρ^2 occurred when $\sin^2(b+\beta s)=0$ i.e. when $b = \frac{(n+1)\pi}{2} - \frac{2\pi s}{\lambda}$ where n was an integer, and hence, s and λ being known, b could be found. In order to find 'a', $\frac{1}{\rho^2-K_1}$ was plotted against $\sin^2(b+\beta s)$ and a straight line was obtained, the negative intercept on the $\sin^2(b+\beta s)$ axis giving $\sinh^2 a$. Alternatively 'a' could be found from

$$\frac{\rho^2_{min.} - K_1}{\rho^2_{max} - K_1} = \tanh^2 a$$

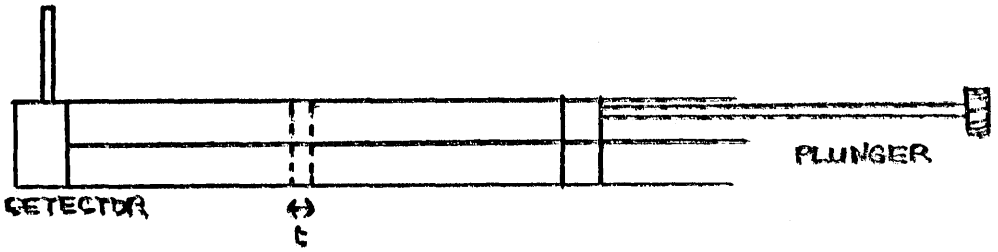
With the condenser C first in air and then in transformer oil, ρ^2_{max} was very large and $\rho^2_{min.}$ almost zero owing to the small losses, so that 'a' was very small and could be neglected. The dielectric constant was then given by

$$\epsilon = \frac{C_{oil}}{C_{air}} = \frac{\tan b_{oil}}{\tan b_{air}}$$

Williams later outlined a coaxial line method of measuring dielectric constants, (Phil. Mag. 1944 P.283) in which the line was used vertically with its lower end short-circuited and containing the test liquid in place of the condenser used in the Lecher line method. The impedance of the liquid filled line was then given by

$$Z = Z_0' \tan \frac{2\pi}{\lambda_1} s_1$$

Z_0' being the line characteristic impedance, λ_1 the wave-length of the oscillations in the liquid, and s_1 the depth of the liquid in the line. The reactance would then become infinite and ρ^2 a maximum for values of s_1 equal to $\frac{(n+1)\lambda_1}{4}$, from which λ_1 could be ob-



C.R.Englund's Method.

tained. For liquids having considerable loss the dielectric constant could not be obtained from the relation $\epsilon = \left(\frac{\lambda}{\lambda_1}\right)^2$, and a method of correcting for this was given.

C.R.Englund, (Bell System Tech. Journ. 1944 P.114) measured the dielectric constants and power factors of a number of solid dielectrics. The theoretical case of an open-ended wire was first considered in which resonance would be obtained for a line equal in length to $\frac{1}{4}\lambda$, λ being the air wave-length. With a dielectric plug in the end of the line, the length of the line would have to be changed in such a way that the line reactance remained the same. In practice a half wave-length line was used short-circuited at both ends, and the resonant length was found with the line filled with air and with a dielectric plug of thickness t at its centre. The resonant length for the air-filled line was $\frac{\lambda}{2}$ from which λ was obtained, and the difference in the resonant lengths was Δl. The reactance of the dielectric plug was then given by $\sqrt{\epsilon}\tan\left(\frac{\pi\sqrt{\epsilon}t}{\lambda}\right)$ and that of the line it displaced was given by $\tan\frac{(\Delta l+t)\pi}{\lambda}$, and equating these and rearranging,

$$\frac{\pi t}{\lambda}\tan\frac{\pi(\Delta l+t)}{\lambda} = \frac{\pi\sqrt{\epsilon}t}{\lambda}\tan\frac{\pi\sqrt{\epsilon}t}{\lambda}$$

Putting $y = \frac{\pi t}{\lambda}\tan\frac{\pi(\Delta l+t)}{\lambda}$

and $X = \frac{\pi\sqrt{\epsilon}t}{\lambda}$

we have $y = X\tan X$

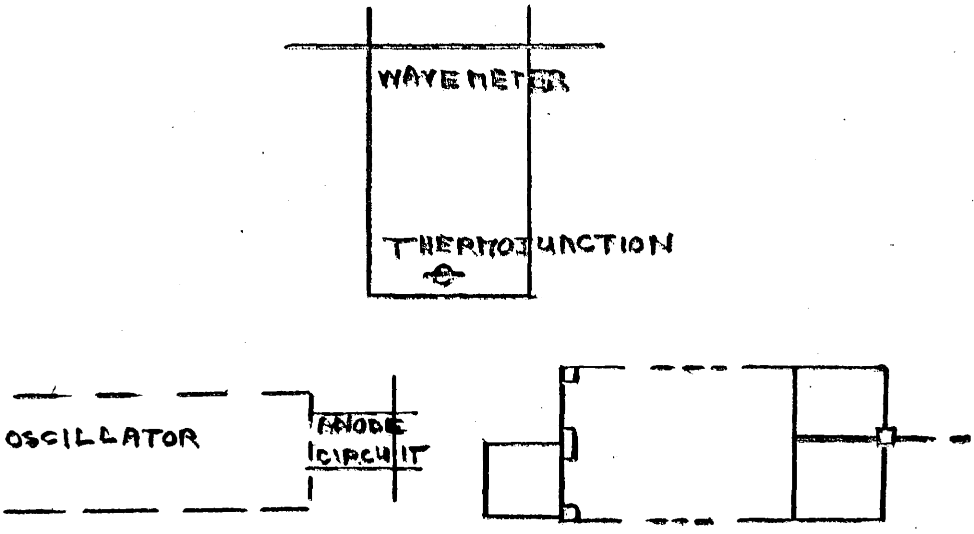
y was determined by direct measurement and this gave

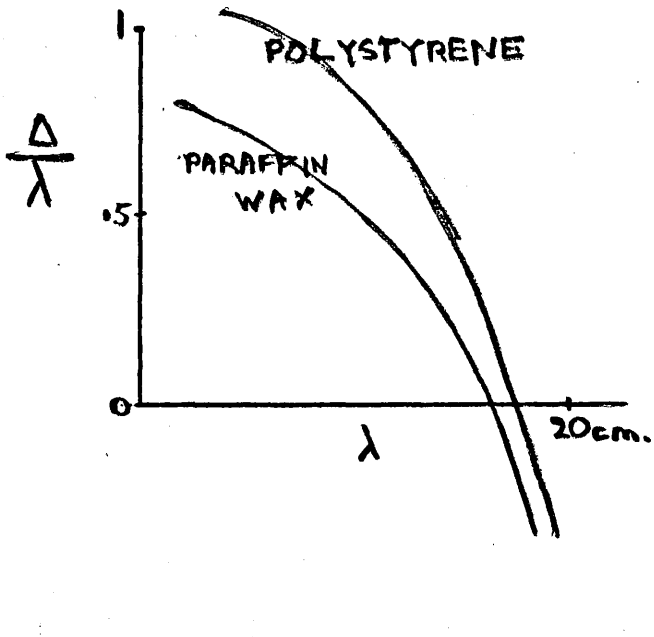
XtanX, from which X could be determined by means of td tables and hence ϵ found. The oscillator used produced air wave-lengths of 22.5cm. and 10cm. and the current input was through a coaxial plug tapped across a half-wave-length line well off tune, which could be tuned to control the amplitude. Two types of crystal detector were used, each mounted in a head which could be fitted on to the end of the resonating line. The crystals had been calibrated at 60c.p.s., but any slight variation in calibration in the megacycle range, while causing error in the loss determinations, could not affect the values obtained for the dielectric constants.

Wave Guide and Resonator Methods.

These methods involve the measurement of the lengths of waves of known frequency in guides or resonators filled, or partially filled, with the test dielectric. By suitable choice of the dimensions of the apparatus, undesired modes of propagation can be eliminated and the chosen mode transmitted with negligible attenuation apart from loss in the dielectric, and for this reason guides and resonators are now used almost exclusively for measuring dielectric constants for frequencies greater than 3000m.c.s.

H.R.L.Lamont (Phil. Mag. 1940 P.1) described methods of using wave guides for the measurement of micro-wave dielectric constants. Amagnetron oscillator was used which produced waves variable from 10 to 15cms. in air, the anode circuit being a pair of parallel wires fitted with a sliding bridge. A wavemeter, coupled loosely and at right angles to the anode circuit, consisted of a pair of parallel wires closed at one end and fitted with a moveable reflecting plate, so that the wavelength of induced oscillations could be accurately found by measuring the distance between two adjacent positions of the bridge for which the vacuum thermojunction shown gave minimum deflections of a microammeter. These readings were consistent to 0.1%. Also loosely coupled to the oscillator was the test guide, a copper tube of 7in. diameter and 5ft. in length, which was closed by a move-





H.R.L.Lamont's Method.

able piston, attached by spring contacts to the tube. A longitudinal slot was cut in the tube so that a micro-ammeter probe could be inserted and the field investigated along the tube, this being found not to disturb the field appreciably. Resonance in the tube was observed by noting maximum reaction on the oscillator. The wave-length in the guide at resonance when it was empty or filled with liquid could then be found either by plotting the field along the tube or by adjusting the piston to different positions of resonance. The distance between two such adjacent positions, or the distance between two field maxima or minima was half the required wave-length. The air wave-length in the tube, λ_t was then compared with the free wave-length, λ , given by the wavemeter through the formula

$$\frac{1}{\lambda_t} = \sqrt{\frac{1}{\lambda^2} - \left(\frac{k}{2\pi}\right)^2}$$

k being the tube constant for the E_0 wave propagated, $\frac{2.405}{\text{radius of tube}}$, and agreement was found to within 0.25%. The dielectric constant of the test liquid was then found from

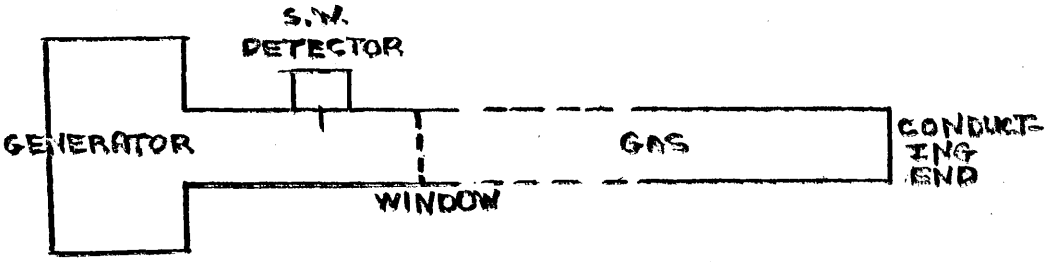
$$\frac{1}{\lambda_{tl}} = \sqrt{\frac{\epsilon}{\lambda^2} - \left(\frac{k}{2\pi}\right)^2}$$

where ϵ was the dielectric constant and λ_{tl} was the ~~w~~ wave-length measured in the liquid-filled guide. At the higher frequencies the first H mode was propagated in addition to the E mode. The main disadvantage of this method was the large quantity of liquid needed, 20gallons Also it could not be used for solid dielectrics. Of the two methods of wave-length measurement, that of plotting the field along the tube was found to be the more accurate, but it was tedious. For measurements on solid dielectrics, a short cylinder of the test solid was made so that it could slide smoothly inside the tube. By considering the boundary conditions at the two plane surfaces of the dielectric, it was shown that the slab inserted effectively shortened the resonant length of the tubeby an amount δ which was a function of the thickness of the slab, of its position in the tube and of its dielectric constant. By differentiating, expressions were obtained for the maximum and minimum shifts necessary to restore resonance, δ_1 and δ_2, and the dielectric constant was obtained from the combination of these expressions,

$$\epsilon = \frac{2}{\beta s_1} \tan^{-1} \left\{ \tan \frac{\beta(s_1+\delta_1)}{2} \tan \frac{\beta(s_1+\delta_2)}{2} \right\}^{\frac{1}{2}} \left[\frac{\tan \frac{\beta(s_1+\delta_1)}{2}}{\tan \frac{\beta(s_1+\delta_2)}{2}} \right]^{\frac{1}{2}}$$

s_1 being the thickness of the slab and β the propagation constant in the air-filled tube, $\frac{2\pi}{\lambda_t}$, The dielectric slab was moved along the tube in steps of one centimetre, and corresponding shifts from the resonance position for the air-filled tube were determined, these ~~position~~ shifts being plotted against the distance of the slab from the fixed end of the guide. From this graph the maximum and minimum shifts were found, and these were then found more accurately by taking repeated measurementsover theapproximate values indicated. The dielectric constant was then found by substituting in the formula quoted above..A simpler method of calculating the dielectric constant was also carried out, which required only the difference in the maximum and minimum shifts, Δ, for a given air wave-length, λ. It was shown that for the particular value of λ, $\lambda = \frac{2\pi}{k}\sqrt{\frac{\epsilon}{\epsilon+1}}$ Δ was zero, and plotting $\frac{\Delta}{\lambda}$ against λ for different wave-lengths a curve was obtained which intersected the λ axis at the required wave-length. Substituting this value in the above equation, ϵ was found. This method was adapted for liquids using a celluloid container for the liquid under test, a correction being made for this. Certain errors were caused in all the methods described due to irregularities in the tube, these being greatest in. the first method owing to the pressure of the liquid inside the tube. Values obtained by this method showed a maximum variation of 1.4%, The method employing a solid slab gave results which had a minimum variation of 1%, but the simpler graphical method of calculation involved a possible error of 2%. The results obtained using this method for liquid paraffin agreed satisfactorily with those from the other method, but for liquids of high dielectric constant the errors introduced were magnified by the calculation and the method was not satisfactory.

W.D.Hershberger, (Journ. of App. Phys. 1946 P495) measured the absorption and dielectric constants of gaseous ammonia and a number of organic gases for 1.25cm.

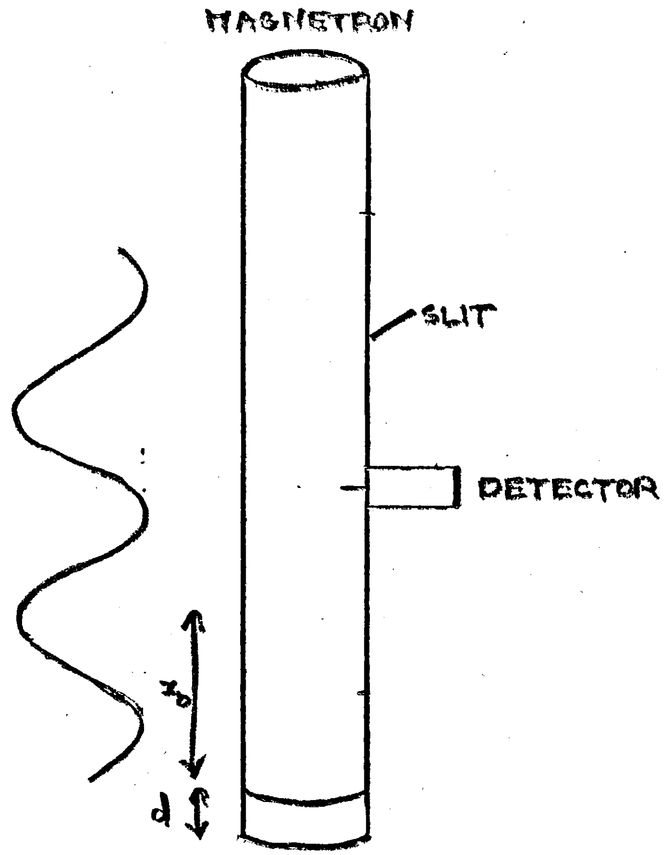


W.D.Hershberger's Method.

waves. A rectangular wave guide was connected to the generator as shown and used in the H_{01} mode. For dielectric constant measurements the far end of the guide was closed by a metal diaphragm, and a gas tight window was placed across the tube at A. The portion of the tube between A and the generator was slotted so that a standing wave detector could be used to investigate the field. Standing waves were set up along them guide with the air-tight portion of the tube evacuated, and the standing wave detector was located at a minimum. The gas under test was then allowed to enter the guide, thus shortening the wave-length, and the standing wave detector was adjusted to remain still at the same minimum as before. Then, by applying the wave guide equations, and assuming that the dielectric constant $\epsilon = 1 + \delta$, where δ was small, it was shown that

$$\delta = \frac{2\Delta L}{L}\left[1 - \left(\frac{\lambda_{vs}}{2b}\right)^2\right]$$

L being the length of the gas-filled tube, ΔL the change in the position of the minimum, λ_{vs} the free space wave-length in a vacuum and b the width of the guide. The reflections from the window were compensated byreflections set up by a tuning element. The accuracy of the method obviously would increase theoretically with the length of guide used; but the length could not be increased indefinitely due to losses in the copper guide, which would have caused the standing wave pattern to be poorly defined. There was, in addition,



Method of Roberts and von Hippel.

some absorption due to mis-matching.

S.Roberts and A.vonHippel, (JOURN. of App. Phys. 1946 P.610)measured the dielectric constants and loss at centimetre wavelengths of solid and liquid dielectrics. A magnetron oscillator, at one end of a wave guide radiated waves vertically down the guide to the far end, which contained a quantity of dielectric to a depth d. By means of a slot cut along the length of the guide, standing waves set up by reflection at the bottom of the guide could be investigated with a standing wave detector..Attenuation in the air-filled portion of the guide was considered negligible, and this was verified by examining the standing wave pattern set up with no liquid in the guide and showing that the minimum deflection on the indicator was almost zero. By the use of the usual guide equations, expressions were obtained for the resultant electric and magnetic vectors of the incident waves and those reflected from the dielectric-filled end of the guide, and from these the wave impedance at the dielectric surface Z_o was found to be

$$Z_o = Z_1 \coth \phi$$

Z_1 being the characteristic guide impedance and ϕ a complex quantity $\rho + j\psi$ such that

$$\frac{A_r}{A_i} = e^{-2\phi}$$

where A_r and A_i were the reflected and incident amplitudes respectively in the air-filled portion of the guide, The standing wave ratio, $S = \coth \rho$, and ψ , the phase retardation due to passage through the dielectric was given by

$$\psi = 2\pi\left[\frac{1}{4} - \frac{x_0}{\lambda_1}\right]$$

λ_1 being the wave-length in the air-filled part of the tube and x_0 the distance of the first minimum from the dielectric surface, where the phase difference of ~~t~~ the incident and reflected waves was π. By substituting for φ, the wave impedance became

$$Z_0 = Z_1 \frac{S - j\tan\frac{2\pi x_0}{\lambda_1}}{1 - jS\tan\frac{2\pi x_0}{\lambda_1}}$$

By considering the wave equations for propagation in the dielectric, the wave impedance Z_0 was found in terms of the dielectric depth, d, and the propagation constant for the dielectric, γ_2, to be

$$Z_0 = \gamma_1 Z_1 \frac{\tanh \gamma_2 d}{\gamma_2}$$

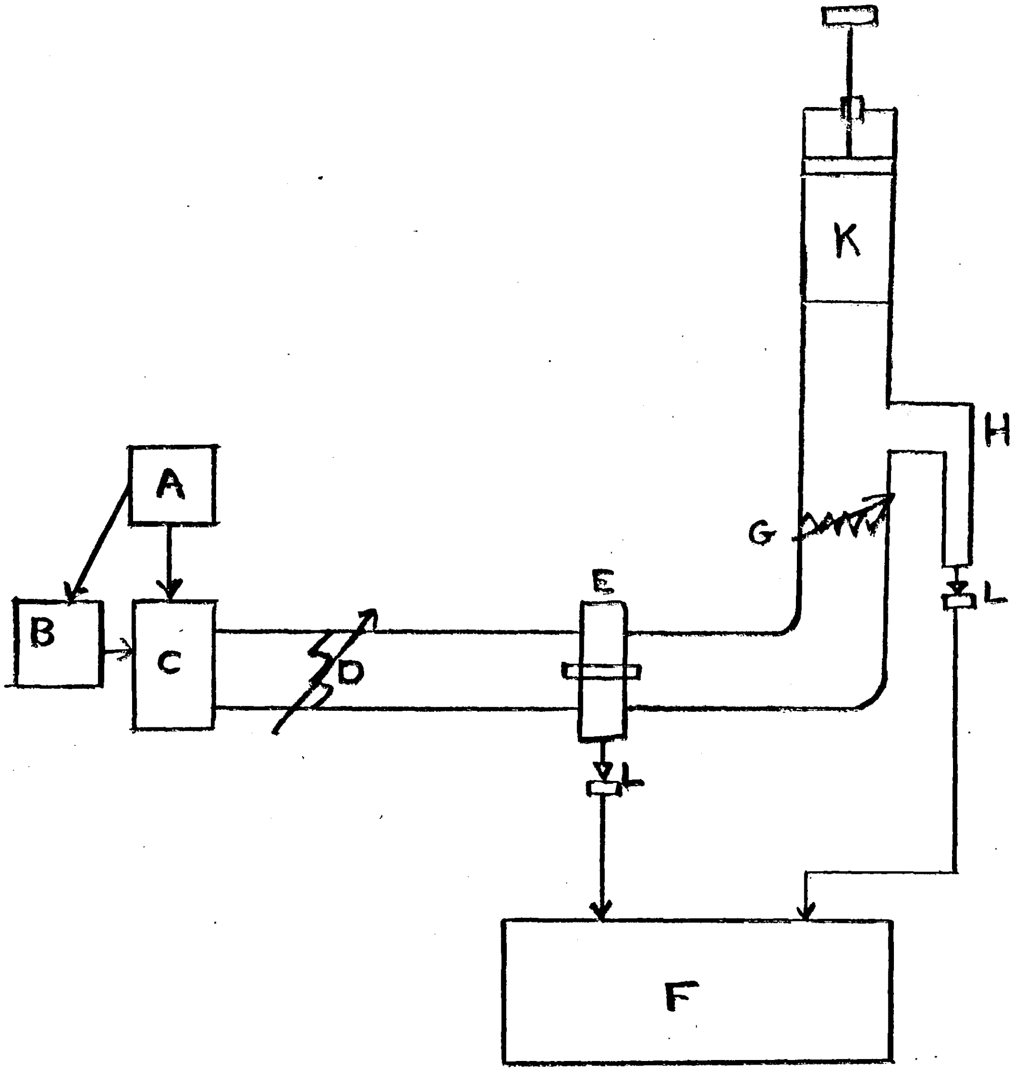
and combining this with the previous equation,

$$\frac{\tanh \gamma_2 d}{\gamma_2 d} = \frac{-j\lambda_1}{2\pi d}\;\frac{S - j\tan\frac{2\pi x_0}{\lambda_1}}{1 - jS\tan\frac{2\pi x_0}{\lambda_1}} = Ce^{j\xi}$$

$Ce^{j\xi}$ being a measurable quantity, γ_2 could be found from charts or by using a series approximation, experiments with one or more thicknesses d of the dielectric being necessary to give a unique value. The complex dielectric constant ϵ^* was then given by

$$\epsilon^* = \frac{\left(\frac{1}{\lambda_c}\right)^2 - \left(\frac{\gamma_2}{2\pi}\right)^2}{\left(\frac{1}{\lambda_c}\right)^2 + \left(\frac{1}{\lambda_1}\right)^2}$$

λ_c being the cut-off wave-length, and the real part of this gave the real dielectric constant while the imaginary part gave the loss component. The original apparatus used consisted of a cylindrical wave guide mounted vertically and fitted with a detachable end-piece for inserting the dielectric. A magnetron oscill-



Method of Heston, Hennelly and Smyth.

ator was mounted at one end of the guide, its output being filtered and stabilised by means of a resonator. The standing wave ratio was measured by a travelling crystal detector driven by a lead screw so that its position could be noted to within 0.001in. The detector was calibrated by measurements using the empty pipe, the field at any point x being given by

$$|E_x| = E_{max} \sin\left[\frac{2\pi(x-x_0)}{\lambda}\right]$$

λ, was determined by measuring the distance between adjacent minima. Measurements were made on solid and liquid dielectrics using 6cm. waves with the intention laterof extending the method for 3cm. and 10cm. waves.

Heston, Hennelly and Smyth (Journ. Am. Chem. Soc. 1948 P.4093) measured the dielectric constants and absorption of organic halides with wavesof 1.25cm. length. The apparatus was arranged diagrammatically as shown, A being a regulated power supply feeding into B, a square wave modulator at 1000c.p.s. C was a microwave oscillator, which was attached directly to the air-filled guide with the decoupling attenuator D to prevent frequency pulling. E was a travelling crystal detector leading to an audio amplifier F which detected the modulation frequency, being linear over the operating range. G was a matching attenuator, matched towards the cell so that it did not reflect energy, and attenuating reflected energy from the right angle bend in the guide. The cell K was a 4in. section of coin silver guide fitted with a brass plunger specially adapted so that its face appeared as a short-circuit to incident waves despite the high contact resistance. A mica window separated the liquid from the rest of the guide, and this was matched out by placing a met~~t~~allic inductive window immediately before the mica window. A proper adjustment of this window and the attenuator G gave a voltage standing wave ratio of 1.04±0.01. The directional coupler H measured the reflected power directly, greatly attenuating incident power from the oscillator but permitting the passage of reflected energy from the cell to the audio amplifier. The method was suited for measurements on liquids with high dielectric loss. It was based on the variation in reflection coefficient of a uniform layer of dielectric as its depth was varied. A resonant cavity wavemeter was used to measure the output wave-length in free space, λ_o, while the wave-length in the air-filled guide, λ_g, was measured by a slotted line identical in dimensions with the cell The cut~~t~~-off wave-length, λ_c, could then be calculated ~~f~~ from the formula

$$\lambda_c = \frac{\lambda_o \lambda_g}{(\lambda_g^2 - \lambda_o^2)^{\frac{1}{2}}}$$

The wave-length in the dielectric, λ_d, was twice the distance between two minima in the cell and was found from the positions of the plunger as minimum readings were recorded. For high loss dielectrics the dielectric constant, ϵ', was then given by

$$\epsilon' = \left(\frac{\lambda_o}{\lambda_d}\right)^2 + \left(\frac{\lambda_o}{\lambda_c}\right)^2 - \left(\frac{\epsilon'' \lambda_d}{2\lambda_o}\right)^2$$

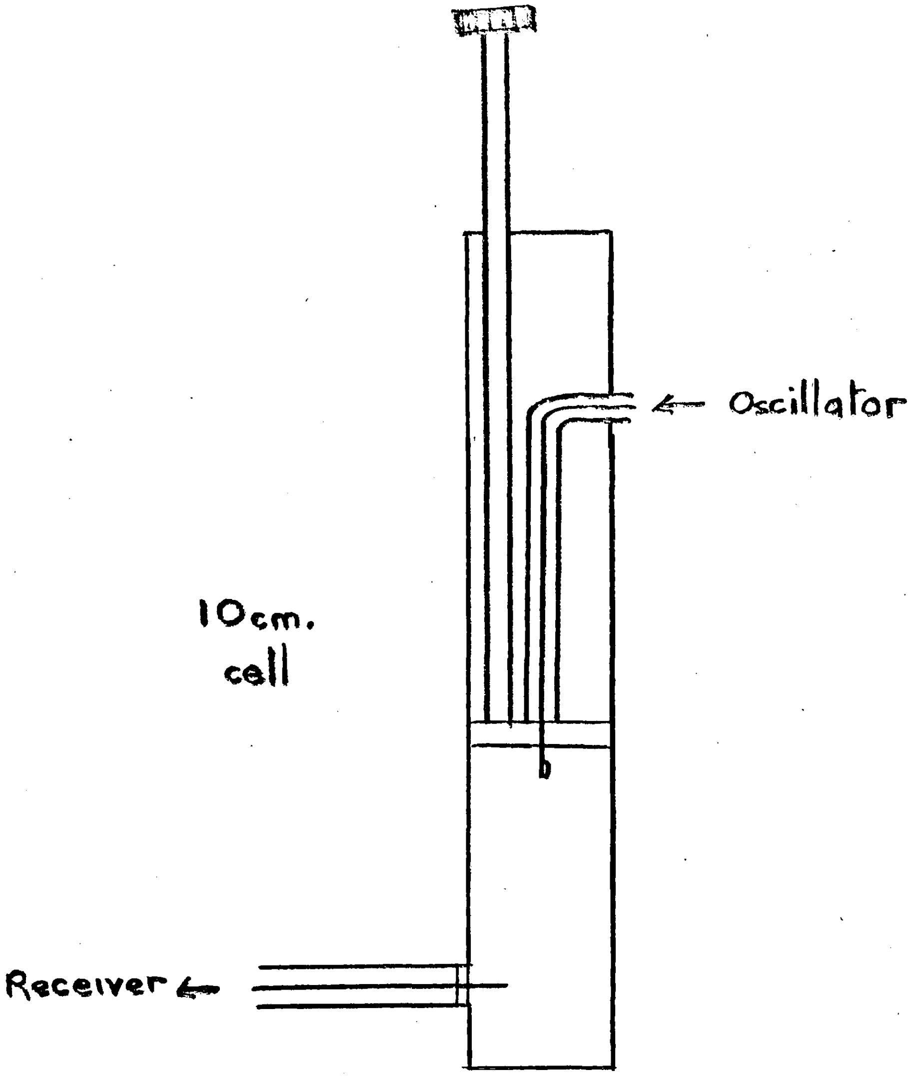
where ϵ'' represented the imaginary dielectric term, this term being neglected for low loss materials, The propagation constant in the liquid $\gamma = \alpha + j\beta$, α being the absorption coefficient and ϵ'' was given by

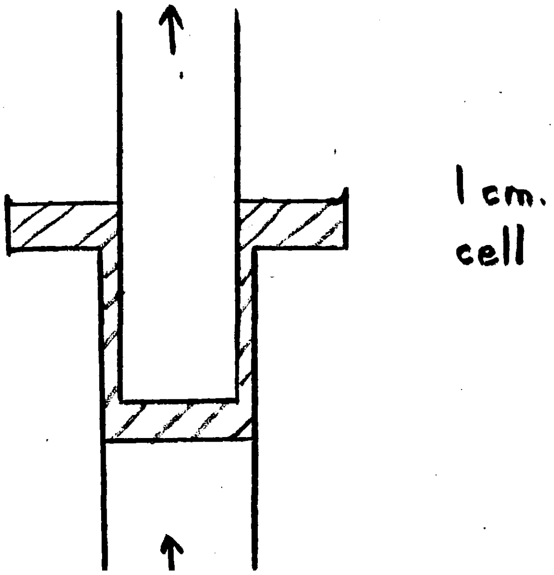
$$\alpha = \frac{\pi \epsilon'' \lambda_d}{\lambda_0^2}$$

With the dielectric cell full of liquid i.e. containing about 5cc., the plunger was placed near the mica window and slowly withdrawn, the maximum and minimum readings on the output meter being recorded with the corresponding positions of the plunger until further withdrawal produced no variation in the meter reading, this reading being equivalent to that due to an infinite layer of dielectric. From the ratio of one of the maximum readings to the infinity reading a value of $\alpha\lambda_d$ could be obtained and ϵ'' calculated, and from this, using the full formula for ϵ' , the real dielectric constant could be obtained. Values obtained using the first and second maxima agreed to within 0.2% and in general the agreement obtained was within the estimated limits of experimental error.

A guide method of measuring the dielectric properties of water was devised by Collie, Ritson and Hasted, (Trans. Far. Soc. 1946 42A P.129) and was later applied to aqueous ionic solutions (Journ. of Chem. Phys. 1948 P.1). 10cm. waves were fed from a coaxial line into a rectangular guide whose dimensions were such that, when filled with water, only the H_{10} mode could be propagated.

Method of Collie, Ritson and Hasted.





The wave was picked up by a probe after passing through a variable depth of water, and was taken by a coaxial line to a superheterodyne receiver. The attenuation of the waves through the water was measured by direct comparison with the attenuation of a ~~atten-~~ calibrated piston attenuator, so that it was not necessary for the receiver to be directly calibrated. The experiment was repeated using a guide of different dimensions. When filled with water, the first guide of cross-sectional dimensions a_1 and b_1 had a propagation constant p_1, where

$$p_1^2 = (n_1 - jK_1)^2 = \epsilon_1 - j\epsilon_2 - \frac{k_1^2 c^2}{\omega^2}$$

ϵ_1 being the real dielectric constant, ϵ_2 the loss factor, ω the frequency in radians per second, c the velocity of waves in free space and k_1 the constant $\frac{\pi}{a_1}$, for H_{10} waves. Similarly with the second guide of propagation constant p_2 and width a_2,

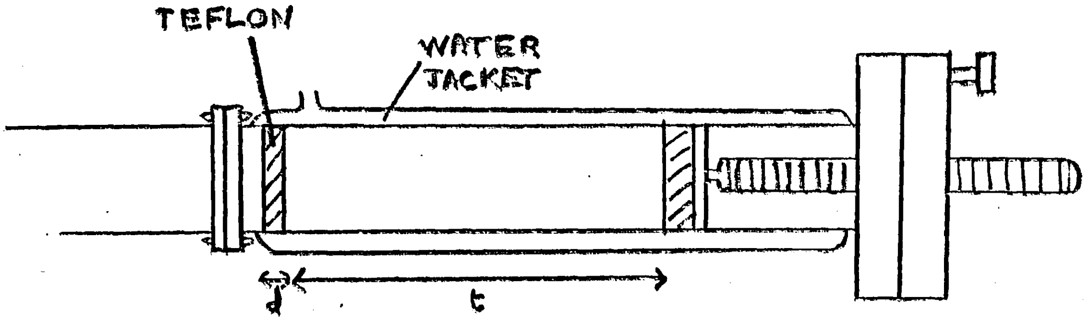
$$p_2^2 = (n_2 - jK_2)^2 = \epsilon_1 - j\epsilon_2 - \frac{k_2^2 c^2}{\omega^2}$$

so that the dielectric constant

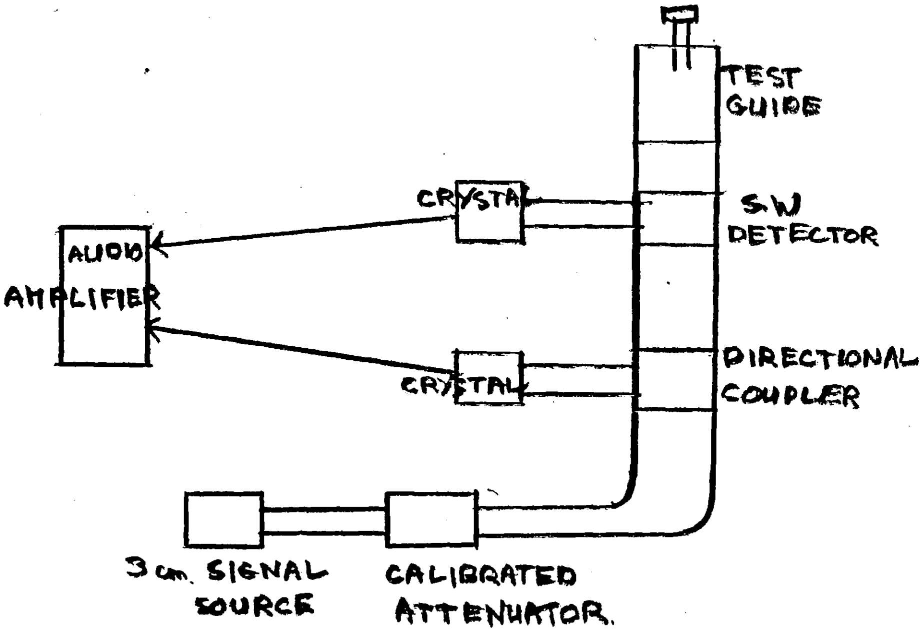
$$\epsilon_1 = \frac{c^2}{\omega^2} \frac{(k_1^2 K_1^2 - k_2^2 K_2^2)}{K_1^2 - K_2^2} - (K_1^2 + K_2^2)$$

K_1 and K_2 being the experimentally measured absorptions, and the guide measurements and frequency being accurately determined. The apparatus was then adapted for measurements at 1.25cm., at which frequency the technical difficulties were very much increased. The oscillator was very much more sensitive to changes in load, voltage and temperature, and care was needed in suitable design

G.E.Crouch's Method.



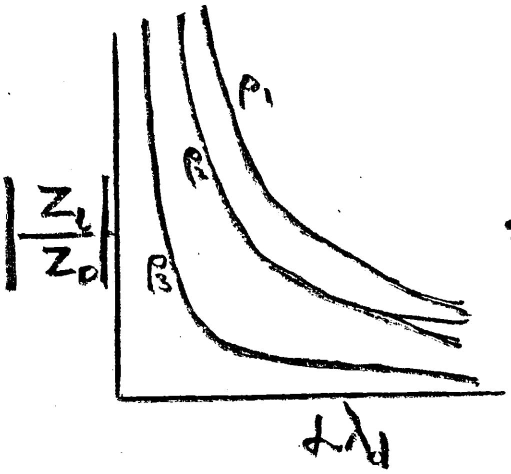
TEST GUIDE

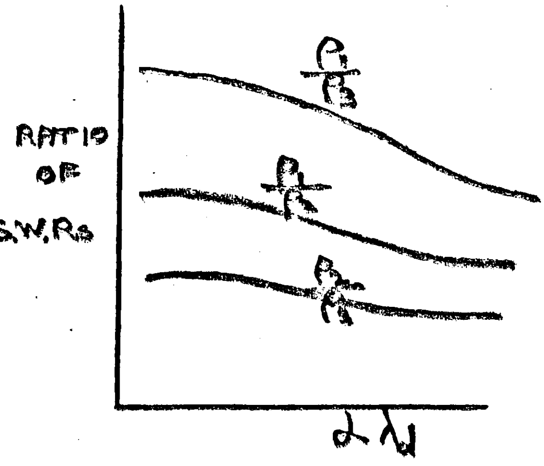


and arrangement of the apparatus to prevent frequency drift. Flexible wave guide was used in place of coaxial line for feeding into the test cell, and this was fitted with marble windows to prevent reflection. The cell was specially constructed of wider guide than the feeding guide so that the waves quickly reached a non-divergent state. The estimated error in the measurement of K was 1% below 30°C. In measurements on ionic solutions the defects of the method were suggested to be mainly technical e.g. the uneveness of the cell walls made absolute measurements uncertain and also mechanical defects of the pick-up probe. In addition there was a tendency for higher modes than the H_{10} to be propagated. Rough agreement was found with results obtained by Wyman and by Drake, Pierce and Dow.

G.E.Crouch (Journ. Chem. Phys. 1948 P.364) used a method similar in principle to that of Heston, Hennelly and Smyth for dielectric measurements on liquids at 3cm. wave-lengths. The cell used contained a teflon window of thickness d equal to a half wave-length and providing a liquid-tight joint. It had a very low loss so that the impedance from the input end was almost the same as that of the liquid-filled guide without the window. The plunger was also of low loss material and so constructed that it provided an open circuit termination for the guide. The microwaves were introduced into the cell as shown in the block diagram, and by slowly withdrawing the plunger through the cell and

G.E.Crouch's Method.





locating its position when two consecutive minimum values were indicated by the directional coupler, the value of half the wavelength λ_d in the dielectric was determined. The standing wave ratio at the minimum values was also determined, the indicator having previously been arranged at a voltage minimum with a short-circuit in place of the cell. For low loss liquids the value of the dielectric constant ϵ' could then immediately be determined from λ_d, the frequency and the guide constant, but for medium loss liquids ϵ'' had first to be evaluated. By the appropriate application of transmission line equations to wave guides it was shown that

$$Z_l = Z\left[\frac{1+e^{-2\alpha l}}{1-e^{-2\alpha l}}\right]$$

Z_l being the input impedance of the liquid filled guide, Z the intrinsic impedance of the liquid-filled guide and α the attenuation constant when l, the length of the liquid column was $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$ etc. From the measured value of $\alpha\lambda_d$, $\left|\frac{Z_l}{Z}\right|$ was plotted against $\alpha\lambda_d$ and curves were obtained of the type shown. The voltage standing ratio was given by

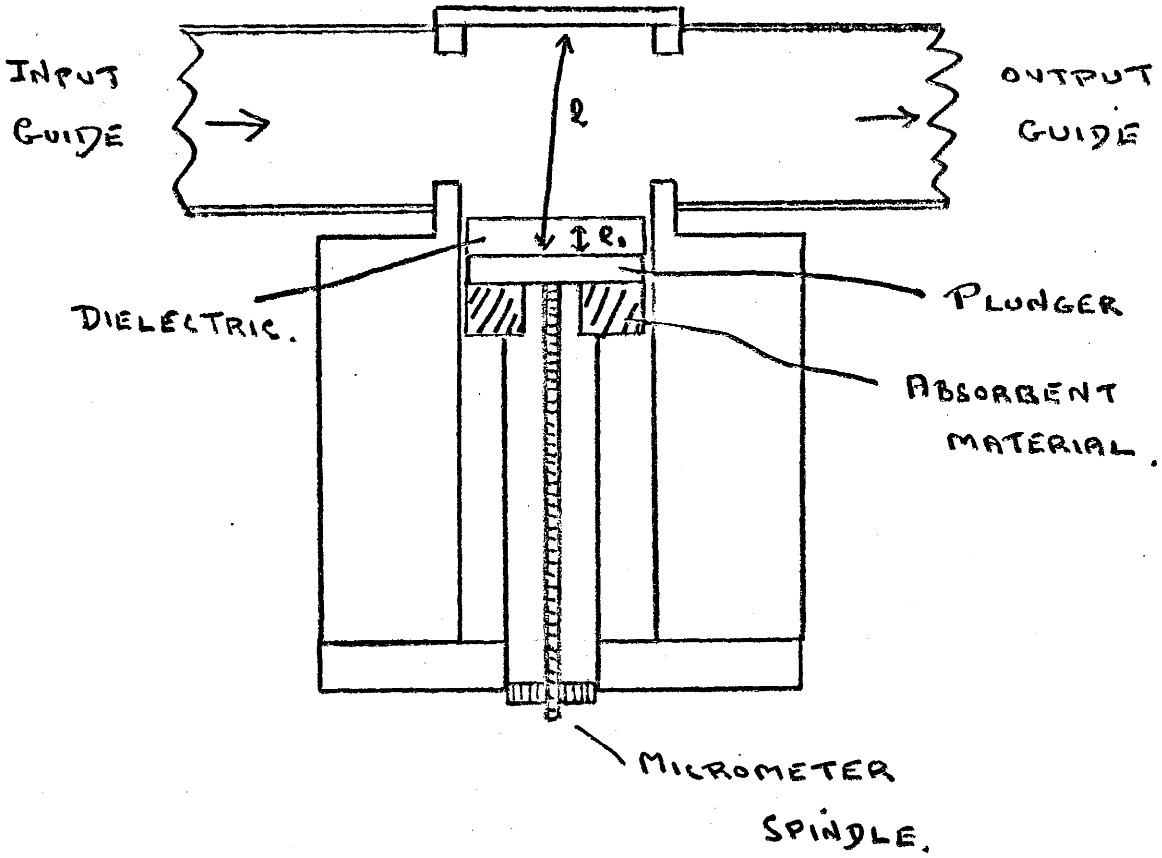
$$\rho = \left|\frac{Z_0}{Z_l}\right|(1 + \text{correction factor})$$

Z_0 being the intrinsic impedance of the air-filled guide. If the voltage standing ratios at succesive minima were denoted by ρ_1 ρ_2 etc. then

$$\frac{\rho_1}{\rho_2} = \left|\frac{\coth\frac{\alpha\lambda_d}{4}}{\coth\frac{3\alpha\lambda_d}{4}}\right|$$

the correction factors being negligible in this expression unless ϵ'' became very large or ϵ' very small. The ratios of standing wave ratios, $\frac{\rho_1}{\rho_2}$, $\frac{\rho_2}{\rho_3}$, $\frac{\rho_1}{\rho_3}$,

R.P.Penrose's Method.



etc. were read off from graph (1)for given attenuations and plotted against attenuation as in graph (2), and from the intersection of these curves at one point on the $\alpha\lambda_d$ axis the required value of $\alpha\lambda_d$ was substituted in the formula

$$\epsilon'' = \frac{1}{\pi}\left(\frac{\lambda_0}{\lambda_d}\right)^2 \alpha\lambda_d$$

and the dielectric constant ϵ' was obtained from

$$\epsilon''\pi^2\frac{\lambda_d^2}{\lambda_0^4} = \left(\frac{2\pi}{\lambda_d}\right)^2 + \left(\frac{2\pi}{\lambda_c}\right)^2 - \epsilon'\left(\frac{2\pi}{\lambda_0}\right)^2$$

R.P.Penrose (Trans. Far. Soc. 1946 42A p.108) measured the dielectric constant and power factor of low loss solids at 25,000m.c.s. using a resonant cavity as a cell. This was caused to resonate in the H_{01} mode by varying the length 1. This mode had the advantage of a low electric field near the boundaryso that the dielectric specimen and the plunger were not required to fit the cavity tightly. Also the resistive losses in the walls were lower than for other modes, making it especially suitable for measurements on low loss materials. Power from a Klystron reflexion oscillator was led into the resonator by an H_{01} rectangular guide containing an attenuating pad to minimise pulling of the oscillator at resonance, and a similar guide conveyed power from the resonator to a crystal dtector. At the frequency employed propagation of higher modes was possible. E modes were precluded, however, by the orientation of the feeding guide and H modes were damped out by high loss material at the back of the plunger, which left the

H_{01} mode undamped. The frequency drift was considerably less than 1m.c.s.and introduced negligible error into the results. The plunger was adjusted to give resonance with the cavity empty and with it containing a slab of known thicknees of the test dielectric, and the corresponding resonant lengths of the cavity could be accurately read off on the screw plunger. By considering the conditions for resonance in the cavity and the ~~bo~~ boundary conditions at the dielectric surface it was found that

$$\left(\frac{\lambda_t}{2\pi l_1}\right)\tan\left[\frac{2\pi(l_2+l_1)}{\lambda_t}\right] = \frac{\lambda_d}{2\pi l_1}\tan\left[\frac{2\pi l_1}{\lambda_d}\right]$$

l_1 being the thickness of the dielectric slab, l_2 the reduction in the resonant length of the resonator when it was introduced, λ_t the tube wavelengthin air, determined from the change in plunger position for two adjacent resonance positions with the cavity filled with air, and λ_d the wavelength in the dielectric. By means of a graph of $\frac{\tan\theta}{\theta}$ against θ, a solution of the above equation for λ_d could quickly be found, and calculated from

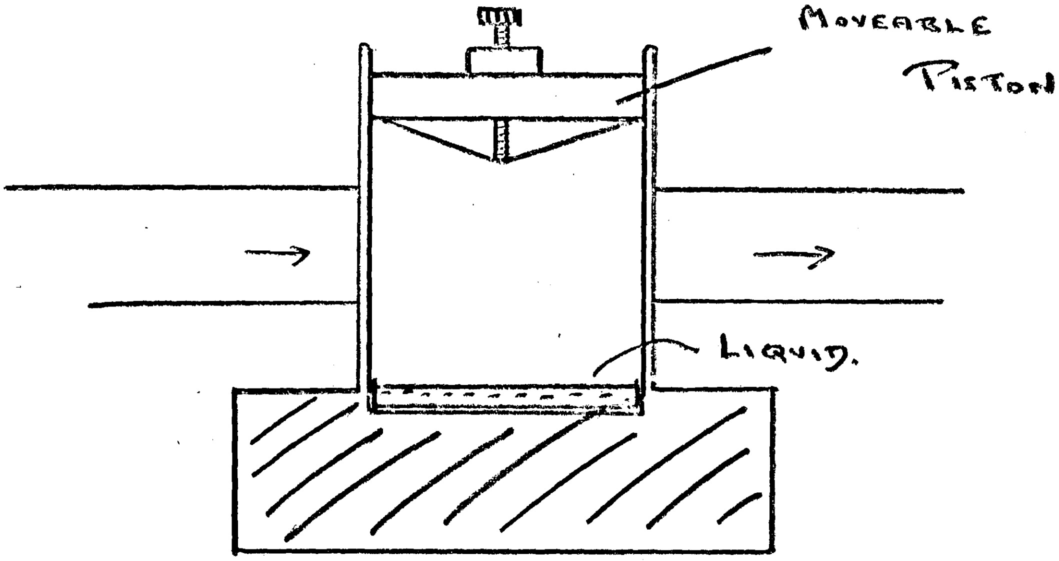
$$\epsilon = \lambda^2\left(\frac{1}{\lambda_d^2} + \frac{1}{\lambda_c^2}\right)$$

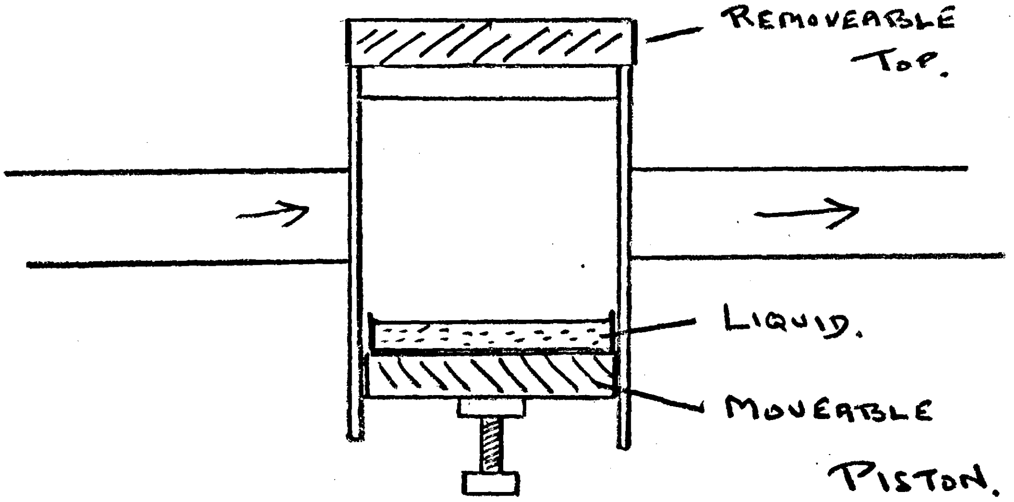
λ being the free space wave-length and λ_c the cut-off wave-length given by

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_t^2} + \frac{1}{\lambda_c^2}$$

Measurements were carried out on 15 solid dielectrics, which were cut in slabs of thickness equal approximately to an integral number of half wave-lengths, so that the surfaces were situated in a weak electric field

Method of Jackson and Powles.



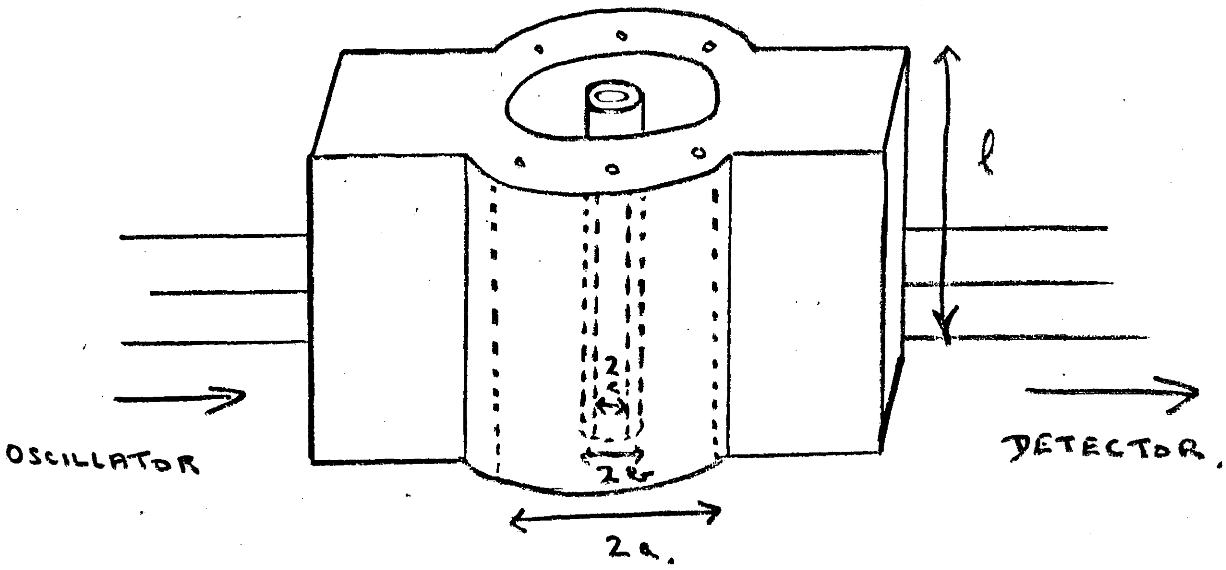


and errors due to inaccuracies of machining were minimised. The experimental~~y~~ values of the dielectric constants observed were found to agree to within 1%.

W.Jackson and J.G.Powles (Trans. Far.Soc. 1946 42A P.101) adapted the above method for use with liquid dielectrics. The cells used at 9.8x10c.p.s. and 2.44x10c.p.s were as shown, the liquid being contained in the first case in a thin-walled recess machined out of the fixed, but removeable, end of the resonator, and in the second case in a thin-walled metal cup supported on the piston. Measurements were made with different rim thicknessesof the container and the results extrapolated to zero rim thickness. The effect of the liquid meniscus was shown to be negligible. At the higher frequency, the results obtained for the dielectric constant were found to be consistently 5% too high and this was shown to be due to the presence of the cup by testing the apparatus with a polythene specimen.

R.Dunsmuir and J.G.Powles (Phil. Mag. 1946 P.747), used a cylindrical resonator excited in the E_{010} mode to measure the dielectric properties of liquids in the 600 to 3200m.c.s. range. For solid dielectrics the same method ,could be usedto give an accuracy of 1%, but for liquids the probable error in the results was increased by the necessity for a container. At the shorter wave-lengths reflection type oscillators were used, while at the longer wave-lengths a coaxial line~~s~~

Method of Dunsmuir and Powles.



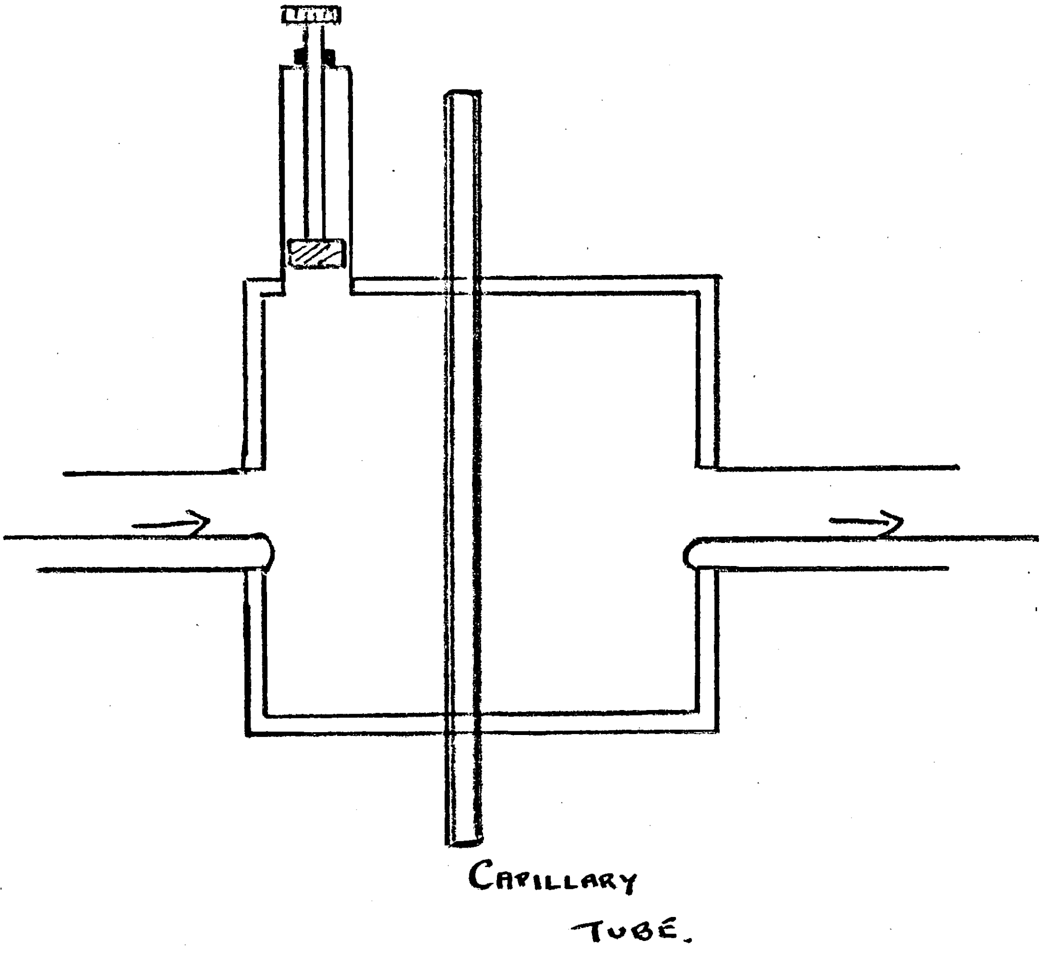
type variable triode oscillator was used, a coaxial line wavemeter being used for calibration. The test bottles used were made to fit coaxially into the resonator, being supported through a hole in the resonator lid, which was found to have no disturbing effect. The material used for the bottle had to be easily workable, ,chemically resistant and of low permittivity and power factor, and quartz was found to be the most satisfactory. A diagrammatic form of the resonator used is shown, a being the resonator radius, l its length, b the outer bottle radius and c the inner bottle radius. The equations for resonance and the boundary conditions at each surface were considered and the amplitude constants eliminated. Since the bottle used had thin walls, (b-c) was small and approximations were made leading to the expression for the dielectric constant of the liquid

$$\epsilon_c = \frac{\frac{b^2}{c^2}\left[1 + \frac{(\beta_a b)^2}{8}\right]\left[M - M_b\left(1 - \frac{(\beta_a b)^2}{8}\right)\right]}{1 + \frac{(\beta_a b)^2}{8}\left[M - M_b\right]}$$

where $$M_b = 2\epsilon_b \frac{(b-c)}{b}$$

and $$M = \left[Y_0(\beta_a a)J_0(\beta_a b) - Y_0(\beta_a b)J_0(\beta_a a)\right]\frac{\pi\beta_a a}{2}$$

The subscripts a, b, and c referred to the air, solid and liquid regions respectively. β was the propagation constant and Y_0 and andJ_0 were Bessel functions whose values could be found so that the function M could be read off from a graphical plot. The dielectric constant of the material of the bottle was required for the

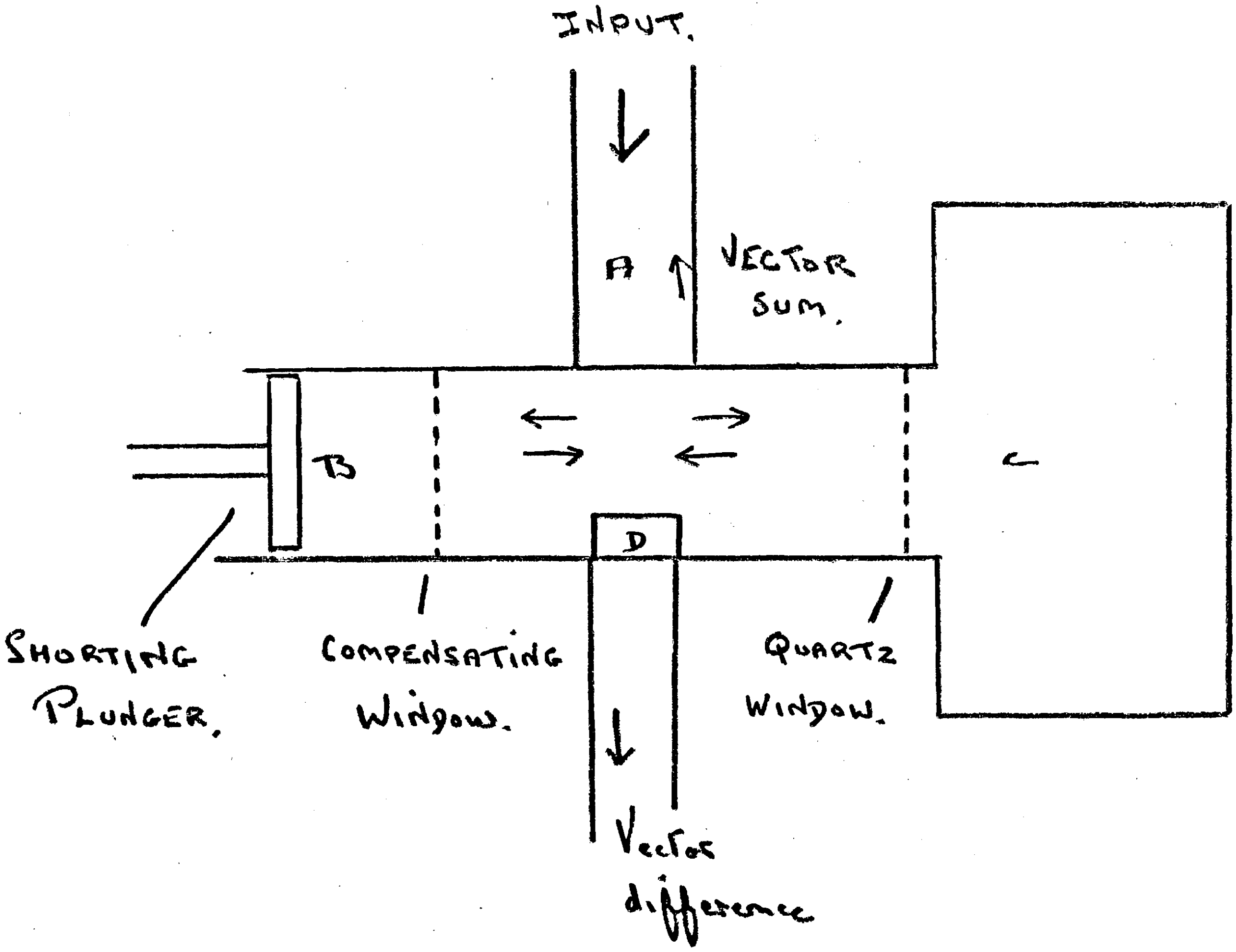


Method of Collie, Ritson and Hasted.

value of ϵ_b , but an approximate value was sufficient. The approximations in the derivation of the above formula introduced an error of less than 1% for values of ϵ_b less than 4 and of $\frac{b-c}{a}$ less than $\frac{1}{40}$. The frequency was adjusted for resonance and the free air wave-length measured, and β_a and β_b were then calculated since $\beta_a = \frac{2\pi}{\lambda}$ and $\beta_b = \sqrt{\epsilon_b}\,\frac{2\pi}{\lambda}$.
Measurements were made using a number of organic liquids, the thickness and size of the bottles being varied to test the theoretical assumptions of the method, which were found to hold within the limits mentioned.

Collie, Ritson and Hasted, (Trans. Far. Soc. 1946 42A. P.129) used a similar resonator method in their investigations of the dielectric properties of water. The resonator was fitted with a capillary tube as shown, and the resonant frequency was found, first with the cavity empty, then containing the empty capillary tube and finally with the capillary tube containing the test liquid final frequency adjustments being made by means of the tuning sleeve. Jackson's analysis previously described was employed, and it was found justifiable within the limits of experimental accuracy to subtract the frequency shift due to the tube from the total frequency shift to obtain the effect due to the water alone. A correction was applied for the imaginary part of the dielectric constant by measuring the Q factor of the resonator and obtaining a value for the absorption, but this was not considered to be of sufficient accuracy, and

C.K.Jen's Method.



the wave guide method previously described was used. The frequency shifts were measurable to an accuracy of about 1%, and the method had the considerable advantage of requiring only very small quantities of liquid. The experimenters considered that, with a suitable method for measuring absorption, the method would have been capable of a good degree of accuracy.

C.K.Jen (Journ. of App. Phys. 1948 P.649) used a resonant cavity method for measuring the dielectric constants of gases. A cylindrical cavity was used, tunable in the range 8,500 to 9?900m.c.s. and was enclosed in a cylindrical brass case. It was sealed from the guide leading into it by a quartz disc coated finely with silver-chromium. The input radiation from the guide A reached the magic tee, D, and divided, half passing to the cavity and half to the guide B, which was terminated by a short-circuiting plunger. In the guide B was a quartz window similar to that at the entrance to the cavity and placed symmetrically with respect to D to annul any reflected waves from the other window. The magic tee was a device which allowed the vector difference of the reflected waves to pass out through the brahch E while the vector sum was returned to A. By analogy with a low frequency resonant circuit, the admittance of the cavity as seen from the guide,

$$Y_c = Y_G \left[\frac{\delta_0}{\delta_1} + j \frac{1}{\delta_1} \frac{f^2 - f_0^2}{f f_0} \right]$$

Y_G being the characteristic admittance of the wave guide,

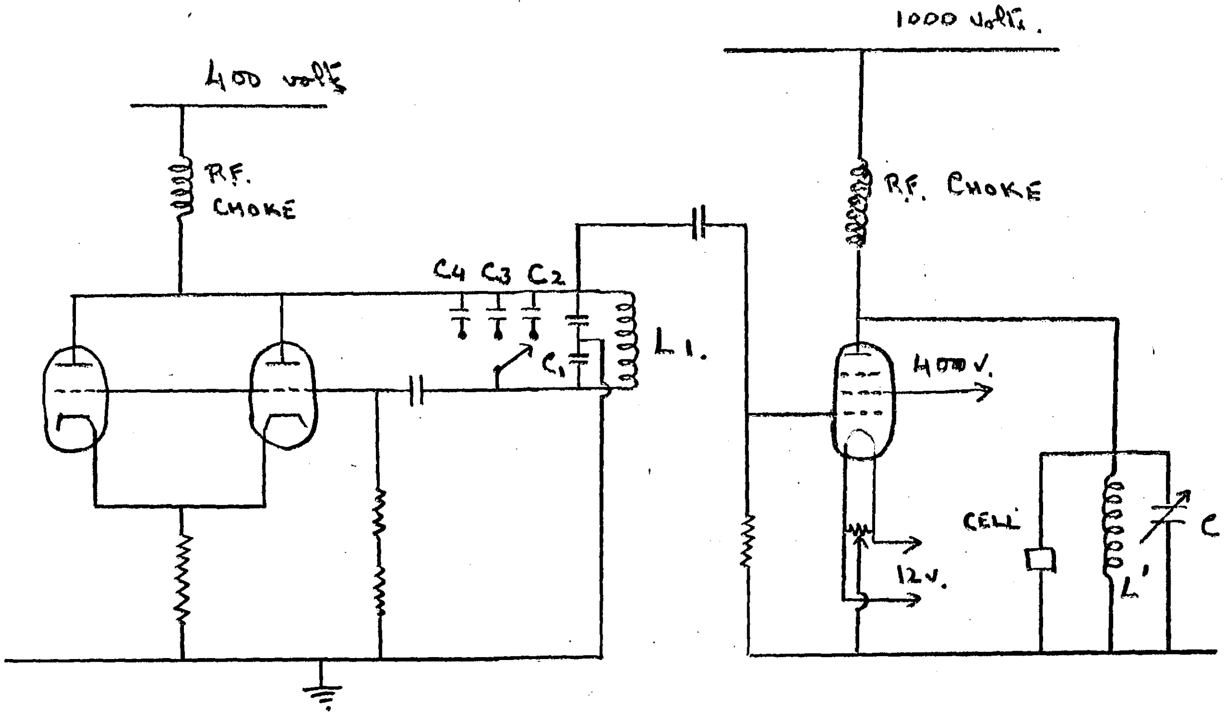
δ, the ratio of the loss per radian to the energy stored, δ_1 the ratio of the energy escaping from the input window to the energy stored, f_0 the resonant frequency of the gas-filled cavity and f the frequency of the incident radiation, For a frequency approaching f_o ,

$$Y_c \simeq Y_G \left[\frac{\delta_0}{\delta_1} + j\frac{2}{\delta_1} \frac{f-f_0}{f_0} \right]$$

For the empty cavity the resonant frequency was f_e, and for the gas-filled cavity it was $\frac{f_e}{\epsilon^{\frac{1}{2}}}$ since the loss part of the complex dielectric constant for the gases considered was negligible with respect to the real part. Consequently, after finding f_e and the resonant ffequency of the gas-filled cavity f_g, the dielectric constant was directly calculated from

$$\epsilon' = 1 - \frac{2}{f_e} \left[f_g - f_e \right]$$

since $\epsilon' \simeq 1$. Measurements of ϵ'' were also carried out since $\epsilon'' = \delta_g - \delta_e$, these being determined from the maximum and minimum vector differences of the reflected waves as measured by the magic tee. The resonant frequencies were detected by means of an amplitude-frequency display on a cathode ray oscillograph, a change in ϵ' of 10^{-5} being detectable. Measurements were carried out using methyl chloride and deuterated ammonia.



Proposed Method for Locating Absorption Bands.

Proposed Method for Locating Absorption Bands in Liquids.

This method is based on the fact that, at absorption frequencies, the effective series resistance of a low loss ~~so~~liquid increases, and thus causes a rise in temperature. Consider a small liquid-filled condenser of capacity C having an equivalent series resistance R and a voltage V across its plates. The power dissipated in the cell,

$$P = \frac{V^2}{\sqrt{R^2+\frac{1}{C^2\omega^2}}} \cdot \frac{R}{\sqrt{R^2+\frac{1}{C^2\omega^2}}}$$

where ω is the frequency in radians per second.

Therefore $$P = \frac{RV^2}{R^2+\frac{1}{C^2\omega^2}} = \frac{RV^2C^2\omega^2}{1+R^2C^2\omega^2}$$

Since C is small, of the order 10^{-19} farads, and R is small, and the average frequency is 1m.c.s. the term $R^2C^2\omega^2$ is small compared with unity and may be neglected. Thus

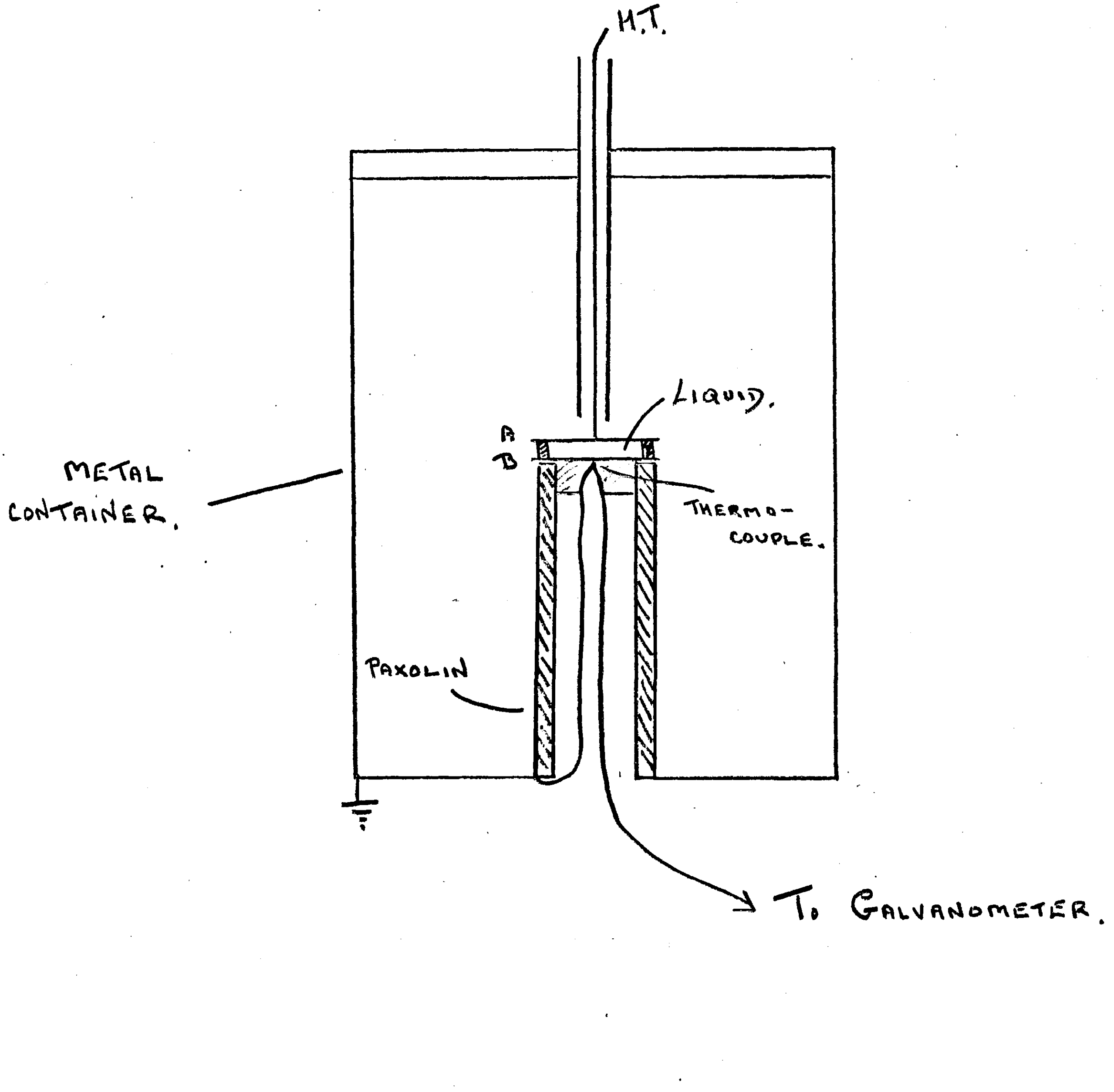
$$P = RV^2C^2\omega^2$$

and this is equal to the heat generated and lost to the surroundings. When the temperature is small most of the heat is used in rising the temperature of the liquid and its container, so that

$$P = RV^2C^2\omega^2 = W\theta$$

W being the water equivalent of the condenser and its contents and θ the temperature rise per second. By suitably arranging the values of V? Cand W for a given frequency range so that θ is small, but just detectable outside absorption regions, a definite rise in θ should be observed when R increases. A diagram of the

Test Cell.


H.T.
LIQUID.
A
B
METAL CONTAINER.
THERMO-COUPLE.
PAXOLIN
To GALVANOMETER.

arrangement of the apparatus is shown. Two CV63 triodes with 400volts on their plates are connected as shown to the tuned circuit L_1C_1, C_1 being a split stator air condenser. By putting C_2, C_3 or C_4 in parallel with C_1 and varying C_1 the frequency output can be varied. L_1 is a coil of about 3½in. diameter and 100turns of copper wire so wound that fewer turns can be tapped off if required, thus giving a further means of varying the frequency. The oscillator output is led via a condenser to the grid of an R.C.A.814 beam tetrode. The anode of this valve is connected via a radio-frequency choke to a 1000volt supply and also to the tuned circuit L',C', L'being a coil 5½in. in diameter of 100turns copper wire variable as in the driving oscillator, and C being a variable condenser immersed in transformer oil to prevent sparking. The driving oscillator is tuned to a frequency measureable by means of a crystal wavemeter, and the amplifier is tuned to resonance at this frequency. The test condenser could be connected across L' as shown with a valve voltmeter to measure V. It is proposed to construct the cell as shown in the diagram. A and B are very thin silver sheets about 1mm. apart which form the plates of the condenser these being held apart by small insulating strips at the outer edges. A thermocouple is joined to B and is used in conjunction with a sensitive galvanometer to note temperature changes. The condenser is supported on a paxolin stand inside a metal constant temperature enclosure. The apparatus is intended for investigating absorption bands in a number of liquids over a frequency range of 100k.c.s. to 10m.c.s. It is thought that it would be useful for approximate preliminary investigations as it requires only small quantities of material.