ABSTRACT

The definitions and types of measurement of hardness are reviewed with special reference to the static indentation test. The present state of the theory of static indentation hardness testing of metals is described, together with previous measurements of the distortions of metal surfaces caused by indenting. Multiple beam interferometry is employed to measure the surface distortions. The apparatus and techniques employed, which include an application of multilayer dielectric films to the interferometric study of metal surfaces, are described.

OPTICAL STUDIES OF DISTORTIONS
OF METAL SURFACES PRODUCED BY INDENTING

BY

JOHN A. BELK

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[2] Volume of the flow pattern. This is found to be smaller than the volume of the indentation for all the indentations studied. Possible explanations of this discrepancy are discussed.


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The definitions and types of measurement of hardness are reviewed with special reference to the static indentation test. The present state of the theory of static indentation hardness testing of metals is described, together with previous measurements of the distortions of metal surfaces caused by indenting. Multiple beam interferometry is employed to measure the surface distortions. The apparatus and techniques employed, which include the application of multilayer dielectric films to the interferometric study of metal surfaces, are described.

The specific aspects of the surface distortions studied are:-

1. Flow pattern shape, with special reference to the transition from ridging to sinking impressions, and variations due to the geometric shape of the indentation.

2. Volume of the flow pattern. This is found to be smaller than the volume of the indentation for all the indentations studied. Possible explanations of this discrepancy are discussed.

3. Shallowing or elastic recovery of ball indentations. The shallowing of ball indentations in a wide range of metals is studied, and its variation
(4) The shape of recovered ball indentations. Shallow ball indentations are found to be not strictly spherical, in contrast to deeper ball indentations. Measures of their deviation from sphericity are proposed and an explanation of these deviations is discussed.

(5) Critical specimen size. The mode of deformation occurring when a hardness test indentation is made near to the edge of the specimen is studied. The results are compared with the mode of deformation for wedge indentations in ideal and real metals.

On the basis of the results a fuller description of the processes involved in ball hardness tests is possible. The surface distortions from the onset of plasticity to the fully plastic stage are described in detail, and support Tabor's theory of ball indentation. The treatment of the hardness test as predominantly a problem in plasticity is justified.
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CHAPTER 1.

HARDNESS.

1.1 DEFINITIONS AND MEASUREMENTS.

Experience has led man to classify the materials with which he comes into contact as solids, liquids and gases. The transition between solids and liquids is not sharply defined but nevertheless the classification is a fundamentally useful one. The natural scientist seeks to tabulate the properties of these three types of material in order to enable him to gain some fundamental knowledge about them, and to make use of them in the most satisfactory manner. To this end he has defined concepts, such as density, and more specific concepts such as rigidity for solids, surface tension for liquids and viscosity for gases. Under the heading of specific concepts for solids come elasticity, specific heat, coefficient of expansion, which are of interest as they furnish information about the solid state, and yield strength, brittleness and hardness which are of interest in the utilisation of the materials. The hardness of a material achieved importance as a measure of the suitability of the material for a particular application. When it became necessary to measure hardness the complexity of the concept was revealed. The overwhelming difficulty encountered in any discussion or definition of hardness is that it is not a fundamental property of the material but a complex aggregate
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of its mechanical properties. A further difficulty is that owing to the wide range of applications for which hardness data are required a number of different hardness tests have arisen, none of which measures exactly the same thing.

In normal conversation the terms hard and soft may be ascribed to materials for a wide variety of reasons. For instance a material might be termed hard because it was resistant to wear, or could not be scratched easily, or could not be permanently deformed easily, or would scratch most other materials. These various aspects of the quality which is loosely called hardness have all given rise to methods of measurement, which led L.B. Tuckerman (1925) to regard hardness as "a hazily conceived conglomeration or aggregate of properties of a material more or less related to each other" whose attributes were "resistance to abrasives, resistance to scratching, resistance to plastic deformation, high modulus of elasticity, high yield point, high strength, absence of elastic damping, brittleness, lack of ductility and malleability, high melting temperatures, magnetic retentivity, etc."

It is also important however, not to lose sight of the value of hardness measurements to the engineer, metallurgist, physicist and mineralogist, whether it be from a fundamental or an empirical viewpoint.

All definitions of hardness imply a resistance to deformation; resistance to scratching, permanent deformation
3.

and abrasion, are employed as measures of hardness. It will be instructive to survey the main types of hardness measurement in some detail, under the headings of scratch, indentation, dynamic, abrasive and magnetic hardness.

**Scratch Hardness.**

The measurement of hardness as the ability of one material to scratch another, or be scratched by another, was probably the earliest type of test employed. It was developed by mineralogists and the first name associated with it is that of Reaumur. In 1722 he produced a metal bar whose hardness increased from one end to the other. The hardness was determined by the position on the bar which the metal under test would just scratch.

Mohs (1822) put the test on a semi-quantitative basis by introducing ten minerals as standards. These were numbered from one (talc) to ten (diamond) and were chosen so that each one would scratch all those below it in the scale. This hardness scale has been and is widely used by mineralogists but is not suitable for metals as the intervals are not well spaced in the higher ranges of hardness. The test is also unsatisfactory in that it depends critically and unpredictably on the shape of the point and its inclination to the surface under test.

A modern development of scratch hardness is the micro-character. A sharp diamond stylus, under a definite load, is
drawn across the surface under test, and the width of the resulting scratch determines the hardness. This method is of use in measuring the change of hardness across, for instance, a grain boundary, but as the apparatus is difficult to operate and the scratching process depends in a complicated way on the elastic, plastic and frictional properties of the surface, the test has achieved no widespread use.

**Static Indentation Hardness.**

The first static indentation hardness test was performed by Reaumur in 1722. He applied pressure to two triangular prisms whose axes were at right angles to each other, and defined the indentation hardness as the extent of their mutual deformation. Foeppel and Schwerd (1897) and Haigh (1920) followed this type of test.

Since 1900 the static indentation hardness has been measured by pressing, under known load, a hard indenter into the flat surface of the test specimen, and measuring the size of the impression produced. Spherical, conical, or pyramidal indenters are used and are made either of hardened steel or diamond. The hardness is defined as the ratio of the load either to a function of the diameter of the indentation, or to a function of its depth. The hardness values so obtained vary with the indenter and the method of calculation. Various empirical conversions from one method of measurement to another have been proposed, none of which are satisfactory.
5.

for all materials. The importance of static indentation tests however lies in the fact that they all depend mainly on the plastic properties of the material and to a lesser extent on the elastic properties, and so can be analysed on a theoretical basis.

Dynamic Hardness.

In contrast to static indentation hardness we have dynamic hardness. In the simplest case a hard indenter is dropped onto the surface to be tested and the hardness is expressed in terms of the energy of impact and the size of the indentation produced. (Martel, 1895). Alternatively the hardness may be expressed in terms of the height of rebound of the indenter, as in the Shore Scleroscope (Shore, 1918).

Dynamic hardness values vary with the definition of hardness adopted, and with the velocity of impact of the indenter. For most materials the dynamic yield pressure, (the ratio of the energy of impact to the volume of the indentation) is similar in value to the static yield pressure (the ratio of the load to the projected area of the indentation). The dynamic yield pressure is usually slightly larger than the static yield pressure as it is more dependent on the elastic properties of the material, while both are dependent on its plastic properties.

In conclusion of this section on dynamic hardness testing the opinions of two experts are of value. Roudié (1930) stated that "elasticity and hardness are two inseparable
manifestations of molecular energy, which dynamic methods alone can define and measure." On the other hand, E. Meyer, whose work on indentation hardness is universally acclaimed, was of the conviction that dynamic effects should be eliminated from the concept of indentation hardness.

**Abrasive Hardness.**

The resistance of a material to mechanical wear is defined as abrasive hardness. An obvious method of measurement is the amount of material removed from the surface under specified experimental conditions. A knowledge of the resistance to mechanical wear of materials is of importance in a large number of applications, and there are as many methods of measurement of abrasive hardness as there are important applications.

Abrasion between surfaces depends on factors such as the coefficient of friction, surface conditions, testing speed and cold working, apart from the obvious experimental method. It is thus to be expected that measurements of abrasive hardness made under different conditions will not be comparable, and this is borne out in practice. It is quite impossible to define a method of measuring abrasive hardness which will be suitable for all the practical applications for which such a quantity is required. It is equally impossible to give a theoretical interpretation of the factors involved in an abrasive hardness test. This type of test is, however,
useful in certain restricted fields of pure scientific study; one such is the directional abrasive resistant properties of diamond.

Magnetic Hardness.

It is a matter of no small interest or importance that there is a direct correspondence between the magnetic and mechanical hardness of ferromagnetic materials. Such materials with a large magnetic coercive force are found to be mechanically hard; and if they are subsequently either heat treated or mechanically deformed so that the coercive force is changed, the mechanical hardness is changed in the same sense. Magnetic tests on these materials provide a non-destructive type of hardness test which may be very valuable. The reason for this correlation between mechanical and magnetic properties is not known, and is, the present author feels, not likely to be discovered until much more is known about how the structure and composition of metals affects their mechanical and magnetic properties.

There seems to be some correlation between electrical resistance and hardness, as they both increase if the material is mechanically deformed. The relationship is however fortuitous as raising the temperature destroys the agreement.

The above survey of hardness measurements serves to illustrate the range of properties which compounded in many different ways are referred to as hardness. Further
complications arise however when one considers the structure of the material under test. It may be crystalline or amorphous, isotropic or anisotropic, all of which possibilities raise new questions of interpretation. In the present work the discussion will be limited to polycrystalline metals, which are assumed to be isotropic.

When reference is made in scientific literature to hardness, static indentation hardness is implied, unless it is otherwise specified, as this has become the most widely used type of test. This is due to the fact that it has the advantages of speed, accuracy and simplicity to a greater degree than the other types of test.

It has thus become increasingly important to have some knowledge of the processes involved in a static indentation hardness test on a metal, and how the hardness depends on the mechanical properties of the metal. Tabor (1948, 1951 a) and b)) has given a theoretical interpretation of both static and dynamic indentation hardness tests explaining the hardness in terms of the elastic and plastic properties of the metal. He has shown how the hardness is linked to the strength of the metal and to its work-hardening capacity. The interpretation is well supported by practical tests on the one hand and theoretical analyses on the other. This interesting and timely work provides information whereby the application of hardness data to practical problems can be made with a greater understanding than hitherto. It also provides a basis upon
which further work on hardness can be explained.

The present work is an investigation of the processes involved in hardness testing as revealed by accurate measurement of the distortion of the metal surface caused by indenting. This is a new approach. The work deals with static indentation tests; and the results concern in some cases effects reported by other workers, and in other cases effects which have not previously been reported. Some of the results obtained support existing theories while others require an extension of these theories. It is of interest to the extent that all work which throws more light on the complex subject of hardness is of interest, and also in that it reveals some effects which warrant further study.

In the remaining part of this chapter the development of the static indentation test is traced in detail and in the succeeding two chapters the present theoretical approach is outlined and the work which bears directly on the results to be reported is surveyed. The next three chapters cover the experimental methods involved in the present work and the remaining chapters deal with the results obtained and some discussion as to their significance and theoretical interpretation.

\[
\text{B.H.N.} = \frac{3W}{\pi D^2 (0.5 - \sqrt{1 - 0.5^2})}
\]

where \( W \) is the load in kg., \( D \) is the diameter of the indenter.
1.2 STATIC INDENTATION TESTS.

In order to trace the development of the various types of static indentation hardness test, it will be convenient to classify the tests by the shape of the indenter employed. The tests will be described under the headings of spherical, conical and pyramidal indenters, as these are the shapes most generally used.

The Spherical Indenter.

In 1900 Brinell proposed a static indentation hardness test which has served as a basis for all subsequent tests of this type. In the Brinell test a hard spherical indenter is pressed normally, under a fixed load, onto the smooth surface of the material under examination. The load must be applied slowly to fulfil the conditions of a static test. After 30 seconds, when equilibrium has been reached, the load and indenter are removed and the diameter of the permanent indentation measured. The diameter of the indenter is standardised at 10 mm. and the testing load at 3,000 kg., though for soft materials this is reduced to 500 kg. to avoid too deep an indentation. The Brinell hardness number (B.H.N.) is defined as the ratio of the load to the surface area of the indentation. Thus

\[ B.H.N. = \frac{W}{\frac{\pi D}{2}(D - \sqrt{D^2 - d^2})} \]

where \( W \) is the load in kg., \( D \) is the diameter of the indenter
and a the radius of the indentation. Generally the S-N.R. is given by metal but varies with the load and the wear of the specimen. A satisfactory analysis is given by the following.

If the indentation, 

force acting on an application of radius \(a\) and depth \(h\).

The area of the ellipse is \(2\pi x dx\) and the force \(dP = 2\pi x dx\) to \(dP\) is integrated for the whole of the surface area the resultant horizontal force vanishes, and the resultant vertical force, which is equal to the applied load, is given by

\[
\sum dP = \pi x dx
\]
and $d$ the diameter of the indentation. Generally the B.H.N.
is not constant for a given metal but varies with the load and
the diameter of the indenter. Further, it is not a satisfactory
physical concept; for though at first sight it would seem to
be equal to the mean pressure over the surface of the
indentation, in fact it is not. The following analysis is
given by Tabor (1951 b).

If the true mean pressure is $P$ and there is no friction
between the indenter and metal, $P$ will act normally to the
surface of the indentation at all points. In fig. 1 consider
the forces acting on an annulus of radius $x$ and width $ds$.
The area of the annulus is $2\pi x\, ds$ and the force on it $P\cdot 2\pi x\, ds$.
If this is integrated for the whole of the surface area the
resultant horizontal force vanishes, and the resultant
vertical force, which is equal to the applied load, is given by

$$W = \int_0^a P \cdot 2\pi x \, dx = P\pi a^2$$

where $2a = d$ = the chordal diameter of the indentation. Thus
the true mean pressure is equal to the ratio of the load to
the projected area of the indentation. This result also
applies to conical and pyramidal indenters.

E. Meyer (1908) proposed the mean pressure as a measure
of hardness and it is now referred to as the Meyer hardness
number (M.H.N.). Thus, Meyer showed that indenters of
different diameters will give about the same value for $M.H.N.$
but different values for $a$, so that

$$M.H.N. = \frac{4W}{\pi d^2}$$
Both the Brinell and Meyer hardness numbers have the dimensions of stress and are normally expressed in kg./sq.mm.

Hardness values vary with the diameter of the indenter and the load, but it is found experimentally that geometrically similar indentations give equal hardness values. That is, if an indenter of diameter $D_1$ produces an indentation of diameter $d_1$ and another indenter of diameter $D_2$ produces an indentation of diameter $d_2$, the two indentations will be geometrically similar if

$$\frac{d_1}{D_1} = \frac{d_2}{D_2}$$

and the hardness values will be equal. This is illustrated diagramatically in Fig. 2. The principle of geometric similarity frequently occurs in the discussion of indentation hardness and will be referred to again in Chapter 2.

The empirical law connecting the load with the diameter of the indentation produced was put forward by Meyer (1908) and states

$$W = ad^n$$

where $a$ and $n$ are constants for the particular material. If $W$ is plotted against $d$ on logarithmic co-ordinates the resultant graph is a straight line whose slope is equal to the Meyer index $n$, and the value of $W$ for which $d$ equals one is numerically equal to $a$. Meyer showed that indenters of different diameters all give very nearly the same value for $n$ but different values for $a$, so that
Fig. 3. Brinell hardness and Meyer hardness values for annealed and work-hardened copper as the size of the indentation is increased. The Meyer hardness values lie on a monotonic curve. The Brinell hardness values first increase and then decrease for large indentations as a result of the increasing area of the curved surface of the indentation. The Brinell values thus make it appear that for large indentations the metal is softer than for small indentations.

Fig. 4. How angle at the apex of the diamond pyramid indenter was determined for use in the 136° diamond pyramid hardness test.
\[ A = a_1D_1 n^{-2} = a_2D_2 n^{-2} = a_3D_3 n^{-2} \]

where \( a_1, a_2 \) and \( a_3 \) are the values of a given by indenters of diameter \( D_1, D_2 \) and \( D_3 \) and \( A \) is a constant. The most general relation connecting \( W \) and \( d \) is

\[ W = \frac{Ad^n}{D^{n-2}} \]

The value of the Meyer index \( n \) varies from about 2.5 for annealed metals to 2.0 for fully work-hardened metals. Small loads give abnormally high values for \( n \), a fact which will be referred to again in Chapter 2.

A comparison of the Brinell and Meyer hardness numbers for copper is given in fig. 3. These results are for annealed and fully work-hardened copper and are quoted by Tabor (1951b). For the work-hardened copper the M.H.N. is essentially constant and independent of load. The B.H.N., however, falls with increasing load because of the increase of the curved surface of the indentation. This is equivalent to stating that the metal becomes softer on cold working, which is known to be incorrect. For the annealed material the B.H.N. is again misleading as it shows a maximum in the hardness curve, which is not given by the M.H.N., and is equivalent to assigning a work-softening characteristic above a certain value of strain.

From the above discussion it is apparent that the M.H.N. is the more satisfactory and fundamental concept. It is
employed throughout the present work and is referred to by
the symbol $H_m$.

Surikova (1945) have used a conical indenter for these

The Conical Indenter.

Ludwig (1908) first proposed a method of hardness
measurement which depended on the differential depth of
penetration of the indenter. The increase in penetration of
a diamond cone indenter was measured when a primary load was
replaced by a larger secondary load. This method eliminated
errors due both to mechanical defects in the instrument and
to surface imperfections of the specimen under test. Ludwig
employed a sharp pointed diamond cone of included angle 90°
and defined the hardness as the ratio of the load to the area
of the curved surface of the indentation. As for the B.H.N.,
the Ludwig hardness has no physical significance, the true
mean pressure between the indenter and the indentation being
given by the ratio of the load to the projected area of the
indentation. The Ludwig hardness is independent of load, as
all the indentations are geometrically similar, but varies
considerably with the included angle of the cone. This is
probably because friction becomes increasingly important as
the included angle decreases. (Hill, Lea, and Tupper, 1947).
The sharp pointed cone is used very little in hardness
measurements because the point is very susceptible to damage.
It is however a very useful research tool as it combines
geometrical similarity of indentations with circular symmetry

\[ R_b = 130 - \frac{d}{2} \]
of distortion. Recently Yakutovich, Vandyshev and Surikova (1948) have used a conical indenter for these reasons.

The Rockwell Test.

For general use the Brinell method of testing suffers from a number of disadvantages, namely large indentations, the necessity of surface preparation, slow rate of testing, and dependence on the operator's skill in measurement.

In order to overcome these defects the Rockwell test was designed. This test employs the differential depth method first suggested by Ludwig. A minor load is first applied, followed by a major load which is then removed and, with the minor load still applied, the additional depth of penetration into the material is measured. The hardness number is based on this depth and is measured directly on a dial gauge. There are two scales of Rockwell hardness utilising different major loads and indenters. For softer materials a major load of 100 kg. is used with a \( \frac{1}{16} \) inch diameter steel ball indenter. For harder materials a major load of 150 kg. is used with a sphero-conical "Brale" indenter. The "Brale" indenter consists of a diamond cone of included angle 120° with a spherical tip of radius 0.2 mm.

The minor load is standardised at 10 kg. The 100 kg. load and \( \frac{1}{16} \) inch steel ball indenter give hardness values on the Rockwell B scale,

\[
R_B = 130 - \frac{d}{2}
\]
where $d$ is the increase in depth of the indentation in microns. The 150 kg. load and Brale indenter give hardness values on the C scale

$$R_C = 100 - \frac{d}{2}$$

where $d$ is as above.

The Rockwell test has two great advantages. Firstly, the values are independent of the surface condition of the specimen, owing to the use of the minor load. Secondly, the hardness number is read directly from the dial gauge thus eliminating optical measurements of the indentation and speeding up the test considerably. The great disadvantage from the point of view of measuring the plastic properties of the metal is that the indentation is allowed to recover elastically before the depth is measured. When the major load is removed the indentation does not keep the same shape as the indenter but recovers by an amount depending on its elastic modulus. Hence the Rockwell test measures a mixture of the plastic and elastic properties of the metal and this complicates conversions of hardness numbers from one scale to another.

The Pyramidal Indenter.

Measurement of hardness by means of a square based diamond pyramid was first introduced by Smith and Sandland (1922). It was developed by Messrs. Vickers-Armstrong Ltd.,
and incorporated into their hardness tester. It is often referred to as the Vickers hardness test and hardness measurements are given as Vickers Hardness Numbers. As, however, there are a number of other hardness testers employing this indenter, Diamond Pyramid Hardness (D.P.H.) is a more satisfactory term and this will be used throughout the present work. The diamond pyramid hardness method follows the Brinell principle in that an indenter of definite shape is pressed into the material to be tested, the load removed, the diagonals of the indentation measured, and the hardness number defined as the ratio of the load to the surface area of the indentation. It also follows the Brinell test in the choice of the shape of the indenter. In Brinell testing, with a ball of diameter D, it is customary to use indentations of diameters between 0.25 D and 0.5 D. The average of these is 0.375 D and when tangents are drawn to a circle at the ends of a chord of length 0.375 D they include an angle of 136°. This angle was chosen as the angle between the faces of the pyramidal indenter, which is constructed as shown in fig. 4. The D.P.H. is defined by the formula:

\[
D.P.H. = \frac{2W\sin 2\theta}{d^2}
\]

where W is the load in kg., d is the diagonal of the indentation in mm., and \( \theta \) is the angle between the opposite faces of the indenter (\( = 136^\circ \)). The standard loads are 5, 10, 20, 30, 50, 100 and 120 kg. and are applied for 15 secs.
The indentations are measured with a metallurgical microscope.

There are two outstanding features of the diamond pyramid hardness test which call for some comment. The first is that since the indentations are geometrically similar for all loads the hardness is independent of the load for a homogeneous material. This is true for all loads above 1 kg. The second is that as the load can be varied from 1 to 120 kg, all the hardness variations met with in metals can be measured on the same hardness scale, from tin with a D.P.H. of 5 to cemented carbides with a D.P.H. about 1,500. It has a further advantage over the Brinell test in that the indentations are very much smaller, so permitting smaller specimens to be used and the hardness of thin hardened layers to be found. Although the diamond pyramid hardness test is ideal for research and laboratory work it is not well suited to routine testing. It is slow, requires careful preparation of the surface of the specimen and a personal error is involved in the measurement of the diagonals.

Hardness Conversions.

A situation sometimes arises when a hardness on one scale, say Brinell, is required when only say the Rockwell number can be obtained. Such a situation may arise for a number of reasons. Over the years this problem has led to the compilation of many hardness conversion tables connecting one scale with another. These tables have become standardised
and are now reasonably accurate. They apply, however, only to certain materials, and are all based on empirical results. The difficulty of compiling a general conversion table can best be appreciated by considering the factors involved in the various methods of testing. The Brinell, Rockwell and Diamond Pyramid tests will be taken as representative.

The B.H.N. is dependent on the load. As the load is specified, only one hardness value is obtained for each material; but this value is a function of the shape of the indentation, the work-hardening capacity of the metal, and the hardness of the ball indenter.

The Rockwell hardness is dependent mainly on the plastic properties of the metal, i.e. the yield stress and work-hardening capacity, but also, much more markedly than any other static test, on its elastic properties.

The Diamond pyramid hardness is independent of the load for homogeneous metals. The ratio of the D.P.H. to the yield stress of the material, however, depends on its work-hardening capacity.

These considerations, combined with the fact that the shapes of the indenters are quite different and so cause varying amounts of plastic strain, serve to indicate the difficulty of compiling a general hardness conversion table. It is possible to construct such a table for one particular material, and to explain it in terms of the known mechanical properties of the material; but the application of such a
20.

table must be strictly limited to the particular material for which it was constructed.
CHAPTER 2.

PHYSICAL SIGNIFICANCE OF STATIC
INDENTATION HARDNESS.

In this chapter, following the review of the development of static indentation hardness measurements, the present state of the theory of indentation hardness is reviewed. The physical processes involved are discussed and the relation of hardness to other physical properties of the metal is indicated. The treatment closely follows that given by Tabor (1951 b).

Before considering the indentation processes, it is necessary to discuss the relevant properties of the metals under examination. This is best done by considering the elastic and plastic properties of an ideal metal, and then proceeding to consider how the properties of real metals differ from these, and how these variations affect the indentation processes.

2.1 PLASTICITY OF METALS.

The stress-strain curve for a cylinder of ideal elastic-plastic metal in tension is shown in fig. 5. The linear strain $E$ is plotted against the true stress $Y$, and it is observed that initially there is a slight increase in length which is proportional to the applied stress. Hooke's law is obeyed over this range, and if the stress is removed the
Fig. 5.

Fig. 6.

Fig. 7.
cylinder assumes its original length, i.e. the path 0A is reversible. When the stress reaches a certain critical value the metal deforms in a non-reversible way, the stress at which this occurs being the elastic limit or yield stress $Y_0$. As the yield stress is constant for an ideal metal the stress-strain curve is a straight line BC parallel to the strain axis. If, at some point D, the stress is reduced, the cylinder contracts along the line DO$^1$, having suffered a permanent plastic deformation of amount 00$^1$. If the stress is now reapplied the deformation will proceed along 0$^1$DC. All real metals have stress-strain curves differing widely from the ideal, but it is possible to obtain a fair approximation to the ideal case by the following method.

The stress-strain curve for a typical metal is shown in fig. 6. As the deformation proceeds there is a steady increase in the stress at which plastic deformation occurs. If the stress is removed at the point D the specimen will contract elastically along the line DO$^1$. On reapplying the stress the deformation will follow the curved dotted line shown in fig. 6. Thus if the stress-strain curve of a specimen that has already undergone considerable plastic deformation is plotted, it has the form shown in fig. 7, where the yield stress varies very little with the strain. A heavily work-hardened metal, then, has plastic properties similar to those of an ideal metal.
When an indenter is pressed into the surface of a metal the stresses are not simply tensile or compressive in nature. The treatment must thus be extended to include combined stresses.

It can be shown experimentally that hydrostatic pressure does not affect the yield stress of metals. It is therefore assumed that only the non-hydrostatic parts of the combined stresses are operative in causing plastic flow. Consider a metal specimen subjected to principal (orthogonal) stresses $P_1$, $P_2$ and $P_3$. These can be split into a hydrostatic component $\frac{1}{3}(P_1 + P_2 + P_3)$ and three reduced stresses $P_1 - \frac{1}{3}(P_1 + P_2 + P_3)$, $P_2 - \frac{1}{3}(P_1 + P_2 + P_3)$ and $P_3 - \frac{1}{3}(P_1 + P_2 + P_3)$. Only these reduced stresses can cause plastic flow and they are found experimentally to do so when the sum of their squares is a constant, that is when

$$\frac{1}{3}(P_1 - P_2)^2 + (P_2 - P_3)^2 + (P_3 - P_1)^2$$

is a constant. For uniaxial tension $P_2 = P_3 = 0$ and yielding occurs when $P_1 = Y$ and hence the value of the constant is $2/3 Y^2$. The equation now becomes

$$(P_1 - P_2)^2 + (P_2 - P_3)^2 + (P_3 - P_1)^2 = 2Y^2$$

This equation, originally derived by Huber (1904) and independently by Mises (1913), is known as the Huber-Mises criterion of plasticity, and is supported by a large body of experimental data.

An alternative criterion, proposed by Tresca (1864), assumes that plastic deformation occurs when the maximum
Fig. 8.

Fig. 9.

Fig. 10.
shear stress reaches a certain value. For the principal stresses \( P_1, P_2 \) and \( P_3 \), the shear stresses are \( \frac{1}{2}(P_1 - P_2), \frac{1}{2}(P_2 - P_3) \) and \( \frac{1}{2}(P_3 - P_1) \). If \( P_1 > P_2 > P_3 \) the maximum shear stress is \( \frac{1}{2}(P_1 - P_3) \). As this determines the onset of plastic deformation we have the plastic region. If, however, a slip line turns to a fixed direction as the Tresca criterion. In plane strain \( (P_3 = 0) \) the Huber-Mises criterion reduces to \( P_1 - P_2 = \frac{2}{\sqrt{3}} Y \) which in this case is identical with the Tresca criterion except for the value of the constant.

A further criterion due to Haar and von Karman (1909) assumes that two of the principal stresses are equal and finds \( P_1 - P_2 = Y \) and \( P_2 = P_3 \) as the criterion for plastic flow. There is no physical basis for the assumption that the two principal stresses are equal, but this criterion is useful for solving certain problems which would be intractable using any other criterion. No attempt will be made here to discuss the mathematical treatment of plastic deformation problems. The physical principles involved in their solution will, however, be indicated for the case of two-dimensional plastic flow. Fig. 8 shows graphically how two stresses \( P \) and \( Q \) can be combined to produce a hydrostatic component \( p \) and a shear stress \( k \), where \( p = \frac{1}{2}(P + Q) \) and \( k = \frac{1}{2}(P - Q). \) When plastic deformation occurs the stresses at any point may be expressed in terms of a shear stress \( k \), which is constant for an ideal
metal, and a hydrostatic pressure $p$. The lines of maximum shear stress are referred to as slip lines and they constitute two sets of orthogonal curves. Detailed mathematical treatment shows that if the slip lines are straight, $p$ is constant throughout the plastic region. If, however, a slip line turns through an angle $\phi$ relative to a fixed direction $p \pm 2k\phi$ is a constant along the slip line. (The positive and negative signs refer to the two families of slip lines.) The solution to a problem in plane strain can be given in terms of the associated slip line field.

The only indentation problems for which rigorous solutions have been found are two-dimensional in character. Bishop, Hill and Mott (1945) determined the slip line field for an infinitely long wedge indenting a block of ideal rigid-plastic metal. Their solution is shown in fig. 9. Friction is neglected, and if this is allowed for the extent of the flow pattern is increased. This analysis has been extended by Hill (1950) to find the critical size of the specimen for such an indentation process. This analysis reveals a direct correspondence between the pressure over the indentation, i.e. the hardness, and the yield stress of the metal.

Ishlinsky (1944) has been able to derive the solutions for a flat circular punch and a ball penetrating an ideal rigid-plastic metal, using the Haar-Karman criterion of
plasticity. The solution of the latter is shown in fig. 10 and it is to be noticed that it does not deal with the displacement of the metal. This is an important omission and, together with the fact that the Haar-Karman criterion is not strictly valid, reduces the value of the solution considerably.

Thus at present a rigorous analysis of the indentation process, even for an ideal metal, is not possible. A more empirical approach is called for, and this will be outlined in the succeeding sections of this chapter. When dealing with a concept as complex as hardness the empirical approach often cannot be avoided. In this case it provides considerable qualitative information about the processes involved in a hardness test and the relation of hardness to the other physical properties of the metal.

2.2 BALL INDENTERS.

Ideal Metals.

If a hard ball is pressed onto the flat surface of an ideal metal, both ball and metal deform elastically according to the classical equations of Hertz (1881). The radius of the circle of contact is given by

\[ a = \left(\frac{3}{16} Wgr \left( \frac{1 - \sigma_1^2}{E_1} + \frac{1 - \sigma_2^2}{E_2} \right) \right)^{\frac{1}{3}} \]

where \( W \) is the load applied, \( E_1 \) and \( E_2 \) are the Young's moduli of the indenter and metal respectively and \( \sigma_1 \) and \( \sigma_2 \) are the
corresponding values of Poisson's ratio. The pressure over the circle of contact is not uniform, being a maximum at the centre and zero at the edge.

As the load on the indenter is increased the shear stress first exceeds the yield stress at a point below the centre of the contact region and plastic flow occurs. It can be shown by applying the Tresca or Huber-Mises criterion to the problem (Timoshenko, 1934) that the point at which yielding occurs is about half the radius of the circle of contact below the contact surface. O'Neill (1934) and Holm, Holm and Shobert (1949) assumed incorrectly that deformation would first take place at the edge of the circle of contact. Brittle materials deform in this way as their deformation is dependent on the maximum tensile stress while for metals it is dependent on the maximum shear stress. The mean pressure over the contact region at the onset of plastic deformation is 1.1 times the yield stress. If the mean pressure just exceeds this value and the load is removed the resulting indentation will be very small as most of the metal has only been deformed elastically.

If the load is increased further the mean pressure increases and the plastic region spreads until all the metal surrounding the indenter is in a state of plasticity. The stage at which this occurs is not easily defined but is generally considered to be reached when the yield pressure varies little with further increase in indentation size.
This transition to the fully plastic stage is very important from both the theoretical and practical points of view. If the load is smaller than the critical value, the hardness reading obtained will be too small, and the value of the Meyer index $n$, referred to in Chapter 1, will be too large. Only indentations in the fully plastic range can be analysed theoretically at present, and then only approximately as outlined above; so the mechanism of the indentation below the range of full plasticity is of interest theoretically. The concept of full plasticity is not accepted by Holm, Holm and Shobert (1949) though as they were working with graphite their arguments cannot be directly applied in the case of metals. The yield stress of the indented metal at a linear strain curve may be constructed. Experimentally the agreement is quite good between the stress-strain curve compiled from hardness data and that found by tensile testing.

If the load is increased above the critical value for full plasticity, the indenter sinks further into the metal and the mean pressure stays constant at about 2.8 times the yield stress.

Real Metals.

The above treatment assumes that the material does not work-harden, whereas in practice all metals do work-harden to some extent. Because of this the effective value of the yield stress may become considerably greater during the indentation process than it was at the onset of plastic deformation. Thus there are two factors involved in ball indentation tests; the transition from the onset of plastic
deformation to the fully plastic stage, and the increase in the yield stress as the indentation increases in size. Most indentation tests are made within the range of full plasticity; hence the main factor influencing the increase in the mean pressure, and so the hardness, is the work-hardening characteristic of the metal.

The increase in the yield stress of the metal depends upon the deformation caused by the indentation. It is found empirically that if a representative yield stress $Y_r$ is defined by the formula

$$P = 2.8 Y_r$$

where $P$ is the mean pressure over the indentation, $Y_r$ is equal to the yield stress of the indented metal at a linear strain of 20% per cent., where $d$ and $D$ are the diameters of the indentation and indenter respectively. (Tabor, 1951 b). Thus from the hardness measurements the stress-strain curve of the metal may be constructed. Experimentally the agreement is quite good between the stress-strain curve compiled from hardness data and that found by tensile testing.

If the stress-strain curve of a metal can be represented by the formula

$$Y = b E^x$$

where $Y$ is the stress, $E$ the strain, and $b$ and $x$ are constants, Tabor has shown that the Meyer index $n$ should be related to the work-hardening index $x$ by the formula

$$n = 2 + x$$
The experimental agreement in this case is quite good, especially in view of the fact that no stress-strain curve has exactly the assumed shape.

When equilibrium has been reached in an indentation process the whole of the load is supported by the elastic stresses in the metal. If the load is removed the indentation will recover and its shape will change. If the load is now reapplied the indenter and indentation deform elastically so that they just fit over the diameter of the original indentation, and the load is again supported by the elastic stresses in the metal. If the load is further increased the stresses exceed the elastic limit and further plastic deformation occurs, there is an increase in the size of the indentation, and of the amount of work-hardening, until equilibrium is again reached.

2.3 PYRAMIDAL AND CONICAL INDENTERS.

It is instructive to compare the processes involved in hardness tests employing sharp pointed indenters alongside those involved in the ball test. This section will be concerned mainly with the pyramidal indenter as most of the experimental work has been done with this indenter; the same principles can, however, be applied to all sharp pointed indenters.

For any sharp pointed indenter there is no initial elastic deformation of the surface, as the region of full
plasticity is reached for the smallest loads. The indentations are all geometrically similar and so, for an ideal metal, the mean pressure is independent of the size of the indentation, and of the load. It is found experimentally for 136° diamond pyramid indentations in a fully work-hardened metal, that the mean indentation pressure $P$ remains constant at about 3.2 times the yield stress of the metal. The D.P.H. is defined as the ratio of the load to the surface area of the indentation, while the mean pressure $P$ is calculated from the projected area. The ratio of the projected to the surface area for a diamond pyramid indentation is 0.927, hence

$$D.P.H. \approx 0.927xP \approx 3.2x0.927xy \approx 3Y.$$  

By means of this formula the yield stress of a fully work-hardened metal may be derived from its hardness value.

Tabor (1951 b) has applied the above treatment to metals that work-harden, in the following way. A representative yield stress $Y_r$ is defined by the equation

$$D.P.H. = 3Y_r.$$  

By a comparison of the stress-strain curves and D.P.H.-strain curves for various materials it is deduced that the deformation produced by the indentation is equivalent to an additional linear strain of $8\%$. Thus the yield stress, as measured by the D.P.H. is that for a strain $8\%$ greater than
the specimen has undergone. Although it is not strictly valid to add linear strains the results are in good agreement with the stress-strain curves of the metals.

The fact that the hardness is independent of the load for pyramid and cone indentations, is convenient in that the load for these tests need not be specified. It is not possible, however, to derive any information as to the work-hardening properties of the metal from these tests, as it is from the ball test.

In the literature the words indentation and impression are often used rather loosely. In the present work indentation refers to the pit in the metal made by the indenter, while impression refers to the whole of the distortion, both the indentation and the flow pattern.

3.1 RIDGING AND SINKING.

The distortions of the metal surface around hardness test indentations have for some time been classified as ridging or sinking. Riding denotes a distortion of the metal above its original level close to the indentation, while sinking denotes a distortion below the original level.

Figs. 11 and 12 are photographs and diagrams of ridging and sinking impressions. As shown in fig. 12b, it is generally assumed that at some distance from the indentation, for a sinking impression, the metal is distorted above the initial level. It has long been established that annealed metals give sinking impressions, while work-hardened metals give the
SURFACE DISTORTIONS.

In this chapter a review of the previous work on the specific aspects of the static indentation hardness test which are covered by the present work will be given. Each section corresponds to a chapter in Part 3 and these are arranged in the same order for convenience.

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3.1 RIDGING AND SINKING.

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Deformation around the indentation produced by a spherical indenter:
(a) 'piling-up' which is observed with highly worked materials, (b) 'sinking-in' which is observed with annealed materials. The effects have been exaggerated to show more clearly the deformation relative to the original level (dotted line).

Fig. 12.
ridging type. For ball and cone indenters this merely means that the rim of the indentation is above or below the initial surface level, this is not the case, however, for a pyramidal indenter. If a pyramid is forced into a work-hardened metal the metal will flow up the faces of the indenter but will be constrained at the corners. Hence the ridging will be considerably higher at the centre of the sides of the indentation than at the corners, and thus the indentation will be wider at the centre of the faces than at the corners. This causes the characteristic barrel shaped indentation.

A similar argument follows for the sinking type impression, but in this case the sinking is a maximum at the centre of the faces, giving rise to a pincushion shaped indentation. Fig. 13 illustrates these types of indentation. The distortions around hardness test indentations are often referred to in the literature as flow patterns, and this nomenclature will be followed in the present work.

Investigators have always experienced some difficulty in measuring flow patterns. A number of methods have been employed with varying degrees of success.

O'Neill and Cuthbertson (1931) attempted to measure the ridging by means of the convexity of barrel shaped diamond pyramid indentations. This method is unsatisfactory as, apart from the inherent difficulties of accurate measurement, it has been shown (Tolansky and Nickols, 1952) that elastic recovery can affect the shape of diamond pyramid
Fig. 14. Spherometer measurements of Brinell impressions. (a) Sinking in. (b) Pilling up. (c) Flatting.

- $a$ is obtained from spherometer readings;
- $b$ is calculated from $a$ and $d$ (depth of impression measured with a traveling microscope) and the radius of the steel ball resting in the impression.
- $c = d - k$ (sinking in Fig. 14(a) and 14(c));
- $d - k$ is obtained from spherometer readings;
- $e = b - a$ - measured depth of impression;
- $f (b - a)$ = depth of impression, calculated from measured diam, assuming 8-mm normal pressure;
- $g = a - b$ = flatting, i.e., difference between depth calculated from diam and measured depth.

From Nuthley and Summers, Heat Treatment and Forging, December 1923.

Fig. 15. Workhardened.

Fig. 16. Plastic Flow.
indentations, in some cases causing ridging to be associated
with pincushion shaped indentations. Norbury and Samuel (1923) measured the position of the
rim of the indentation relative to the main surface level,
and were able to correlate this with the work-hardened state
of the metal. The method is shown diagramatically in
fig. 14. They found that the Meyer index \( n \) was connected
to the height of the ridging, or the depth of the sinking,
expressed as a percentage of the depth of the indentation,
by a straight line connecting:
\[
\begin{align*}
    n &= 2.0 & \text{30\% ridging,} \\
    n &= 2.3 & \text{0\% ridging,} \\
    n &= 2.6 & \text{30\% sinking.}
\end{align*}
\]
These results are of some interest, but it must be borne in mind that the
measurements were of the position of the rim of the hardness
indentation, and not of the flow pattern.

A probe method was used by Foss and Brumfield (1922) to measure both indentations and flow patterns, and a similar
method has recently been employed by Yakutovich, Vandyshev
and Surikova (1948). A probe, attached to an accurate dial
gauge, is lowered until it makes electrical contact with the
surface under study. By means of a traversing mechanism and
the dial reading, a section of the surface may be found.
This method is satisfactory for measuring large indentations
(chordal diameter greater than 1mm.), but it is not
sufficiently accurate for indentations smaller than this.
Krupkowski (1931) calculated flow patterns for ball and
cone indentations in work-hardened copper, from measurements
of the diameter and depth of the indentation, and the depth of penetration of the indenter. He assumed that the flow patterns had the shape shown in fig. 15, and that the volume of the flow pattern was equal to that of the indentation, neither of which assumptions are true in general. However, his results showed that the flow pattern increased in height and decreased in extent, as the metal was successively cold worked. He obtained a value of work-hardening of 4% reduction in area, which marked the transition between the production of the ridging and sinking types of hardness test impression.

Tolansky and Nickols (1949 a & b, 1952) applied multiple beam interferometric methods to the measurement of hardness test impressions. These methods reveal the shape of the whole surface, and can measure flow patterns whose heights are as small as 0.05 microns. Tolansky and Nickols measured flow patterns around indentations in tungsten carbide, various steels, duraluminium, and single crystals of tin. They found that pinocushion shaped diamond pyramid indentations could be caused by elastic recovery and so were not necessarily indicative of sinking. They also reported that the flow patterns caused by various sized diamond pyramid indentations in steel were geometrically similar. Their measurements on tin crystals reveal a ridging in one direction on the crystal face and a sinking in another direction. The ridging and sinking directions are connected with the crystallographic
axes of the crystal. From a more detailed study of the flow patterns of tin and bismuth crystals, using a double cone indenter which reveals directional hardness, Williams (1953) has shown that the ridging and sinking directions remain fixed as the indenter is rotated, and that the variation of hardness value observed, and the flow pattern shape, can be explained by the critical shear stresses of the slip planes in the crystal.

Reference was made in Chapter 2 to the theoretical analysis of Bishop, Hill and Mott for an infinitely long wedge indenting an ideal plastic-rigid body. The associated distortion of the surface, or flow pattern, was shown in fig. 9. Dugdale (1953) has performed some wedge indentation experiments on work-hardened metals and his results agree quite well with the theoretical predictions. He finds, however, that the flow patterns are not rectilinear as predicted, but curved at D and C and rather more extended than the theory suggests. He ascribes these effects to the slight work-hardening capacity of the cold worked material, and does not consider that friction plays a vital part in influencing the flow pattern shape.

A qualitative explanation of the formation of flow patterns has been given by Tabor (1954) and this is illustrated in fig. 16. In a work-hardened metal the ridge is formed because the material has to be displaced and part of the displaced metal flows up the face of the indenter (fig. 16a).
The mechanism for a fully annealed metal is, however, strikingly different. When the indenter first begins to sink into the metal, the metal adjacent to the indenter becomes markedly work-hardened relative to the undeformed metal farther away (fig. 16b). When the indenter sinks in farther it carries with it the surrounding metal which acts as an enlarged indenter, and deforms the metal still farther away from the indenter (fig. 16c). Thus the displaced metal appears to move away from the indenter. This results in a depression of the metal immediately adjacent to the indenter and a slight elevation at some distance. (fig. 16d).

Heyer (1937) studied the problem of ridging and sinking in terms of the stress distribution around the indentation. He found that in most cases sinking was associated with relative weakness of the metal in shear, as measured by the ratio of its shear to its compressive yield stress. Ridging was associated with a high value of this ratio. He also reported that relatively large underlying shear deformations were associated with sinking impressions, while relatively large surface compressions were associated with ridging.

From the work reported in this section it may safely be concluded that the shape of the flow pattern is very largely dependent on the work-hardening capacity of the metal. For an ideal metal with no work-hardening capacity the flow pattern is a compact ridge. As the work-hardening capacity
increases the ridging becomes more extended until it assumes
the characteristic shape generally referred to as sinking.

3.2 VOLUME OF FLOW PATTERN.

In view of the ease with which, having measured the
shape of a flow pattern, its volume may be found and compared
with the volume of the indentation, it is noteworthy that
only two sets of workers have reported such volume comparisons.
Yakutovich, Vandyshew and Surikova (1948) found that for cone
indentations in Aluminium, Copper, Duralumin, Armco iron,
and two alloy steels, the volumes of the flow patterns,
measured with a probe instrument, were equal to the volumes
of the indentations to within 4%. This seems to be in direct
contradiction to the work of Tolansky and Nickols (1952) who
reported that for a cone indentation in a nickel-chromium-
molybdenum alloy steel, the volume of the flow pattern,
measured interferometrically, was only 14% of the volume of
the indentation. In this calculation no account was taken
of the fact that the indenter did not have a sharp pointed
tip, or of the elastic recovery of the indentation. If
these effects had been taken into account the figure might
have been increased by a factor of two but even this could
not account for the large discrepancy between the two
volumes.

These two sets of results cannot be strictly compared
as the former set of workers measured indentations whose
volumes were some $10^{-3}$ c.c. Tolansky and Nickols, however, measured indentations of volume some $3 \times 10^{-7}$ c.c. Thus this apparent loss of volume of metal may be an effect which is only noticeable for small indentations (chordal diameter 0.2 mm.)

3.3 SHALLOWING.

It has already been pointed out that the deformation associated with a hardness test indentation is both plastic and elastic in nature. The plastic properties of a material determine to a great extent its hardness behaviour; however, the elastic properties are involved in a number of second order effects which may not be omitted from a full discussion of the hardness test. When equilibrium has been reached in the indentation process the load is supported by the elastic stresses in the metal. If the indentation is examined after the load has been removed, it is found to have a different shape to that of the indenter. This change of shape, which is usually observed as a decrease in depth of the indentation, is referred to in the literature as shallowing or recovery. Shallowing has been attributed to the release of the elastic stresses in the metal specimen. Recovered ball indentations are found to be practically spherical but with a radius of curvature greater than that of the indenter, whilst recovered cone indentations have a greater included angle than the conical indenter.
Foss and Brumfield (1922) in a careful mechanical exploration of recovered ball indentations found that they were symmetrical and of spherical form but that their radii of curvature might, for hard metals, be as much as three times as large as that of the indenter.

Norbury and Samuel (1928) found that for a given metal the product of the proportion of the depth under load recovered on removal of the load, and the diameter of the indentation, was practically independent of the size of the indentation. That is

\[ \frac{h_0 - h}{h_0} \times d = \text{Constant} \]

where \( h_0 \) is the depth under load, \( h \) the recovered depth, and \( d \) the diameter of the indentation.

Tabor (1948) argued that if the recovery were elastic it should be reversible. That is to say, if the indenter is replaced in the recovered indentation and the original load applied, the surfaces should deform elastically, and on removing the load again the diameter and curvature of the recovered indentation should be unchanged. In order to test this he made indentations with hard steel balls of various diameters, and a range of loads, in a series of metal specimens. The diameters of the indentations and their radii of curvature were measured after 1, 2, 3 and 5 cyclic applications of the load. The indentations were found to remain essentially unaltered in diameter and curvature after this treatment.
Thus the recovery of indentations may be taken to be truly elastic. Tabor went on to show that Hertz's classical equations for the elastic deformation of spherical surfaces could be applied to this process. The equations depend on the diameter of the indentation, the load, the ratio of curvatures of the indenter and indentation and the elastic properties of the indenter sample.

![Diagram: Pressure distribution over circle of contact for a metal deformed by a spherical indenter. Full line, elastic deformation (Hertz); broken line, plastic deformation (Ishlinsky).]

Fig. 17.
Thus the recovery of indentations may be taken to be truly elastic. Tabor went on to show that Hertz's classical equations for the elastic deformation of spherical surfaces could be applied to this process. The equations connect the diameter of the indentation, the load, the radii of curvature of the indenter and indentation and the elastic properties of the indenter and metal. Thus for ball indentations the recovery can be explained on the basis of simple classical theory.

Various workers have investigated the shape of recovered ball indentations and found them to be spherical. Thus Batson (1918) and Foss and Brumfield (1922) were able, after finding the indentations spherical, to measure their radii of curvature. An explanation of the fact that recovered ball indentations are spherical has been proposed by Tabor (1951 b) on the basis of the Hertz classical elastic equations and some recent theoretical work by Ishlinsky (1944).

The Hertzian analysis gives a unique solution for the pressure distribution between two solids, which will deform a flat surface to a portion of a sphere or a spherical surface to a sphere of different radius. This pressure distribution is shown in fig. 17 as a full line. Ishlinsky's analysis of the ball indentation test was referred to in Chapter 2.

Using the Haar-Karman criterion of plasticity he determined
analytically the pressure between a spherical indenter and the indentation under conditions of full plasticity. This is shown as a dotted line in fig. 17 and is essentially the pressure distribution operative during the plastic deformation of a metal by a spherical indenter. The pressure distribution involved in the plastic formation of the indentation is similar to that involved in the elastic deformation of spherical surfaces. Hence when the load is removed the stresses which are released will deform the indentation in a manner similar to that involved in the elastic deformation of spherical surfaces, and the recovered indentation will be very nearly spherical. This would not be the case if the two pressure distributions were widely different. Since it is known that recovered ball indentations are essentially spherical, the plastic pressure distribution must be very close to that given by Ishlinsky. Thus, even though the Haar-Karman criterion is not valid physically it does give results which are in quite good accord with experiment.

3.5 CRITICAL SPECIMEN SIZE.

If the ratio of the size of the test specimen to the size of the indentation is reduced a point will be reached when the hardness test is no longer valid. This is revealed by a drop in the hardness value and a slight distortion of the specimen. The specimen then no longer acts as a semi-infinite
block. This critical specimen size for a valid hardness test is of importance both theoretically and practically.

Moore (1909) stated that the centre of a ball indentation should be at least 2.5 times the diameter of the indentation from the edge of the specimen, and that the specimen thickness should be at least seven times the depth of the indentation. Hankins and Aldous (1934) measured the variation of hardness with specimen size for both ball and diamond pyramid indentations. They came to similar conclusions to Moore concerning the width of the specimen, but found that the critical thickness for ball indentations varied considerably with the material. Six times the indentation depth was required for mild steel, fifteen times for copper and twenty times for spring steel. The critical thickness for diamond pyramid indentations was found to be 1.5 times the diagonal of the indentation for most materials. These conclusions are generally accepted and are embodied in the standard specifications.

Hill (1950) approached the problem theoretically. He considered the case of an infinitely long wedge or punch indenting an ideal plastic rigid body and calculated the effect of the width and thickness of the specimen. The values obtained for critical width and thickness are of limited application owing to the two-dimensional case considered. The interesting conclusion, from the point of view of the present work, is the mechanism of deformation
This is illustrated in Fig. 18. The dotted lines in Fig. 18 show the actual dimensions of an engine. Dugdale (1901) and Biltgren (1924) have measured the theoretical predictions and found that the agreement is well with the experimental results.

Fig. 19 shows the variation of load in a block of plastic. An impression on occasion plastic deformation of the indenter. Variation of the load depends on the increase in the indenter's dimensions. The high load for 3,000 kg. load and 10 mm. steel balls after impact.
involved when the indentation is near an edge. This is illustrated in fig. 18. The solid block bounded by the dotted line ON is found to rotate about the line KLM as shown by the arrow. Values of the critical width and the dimensions of the block are given for a series of indenter angles. Dugdale (1953) indented various work-hardened metals with wedge indenters and verified Hill's values for critical width. He did not however study the mode of deformation. Green (1951) measured the deformation of a block of plasticine caused by indenting it with a flat punch and found that the mode of deformation agreed well with the theoretical predictions.

3.6 DEFORMATION OF INDENTER.

An indenter will always deform elastically, and on occasion plastically during the indentation process. If plastic deformation occurs spurious hardness numbers will result. Various workers have measured the plastic deformation of indenters, in particular spherical indenters, and a body of experimental fact has been built up. All determinations of the permanent deformation of ball indenters depend on measurements of their polar diameter. This will decrease if the ball is permanently deformed. Strubeck (1901) detected the deformation and Batson (1918) and Hultgren (1924) measured the decrease in the polar diameter of 10 mm. steel balls after indenting various materials with a 3,000 kg. load.
They found that the deformation depended on the hardness of
the indenter and the material indented, and that cold-worked
balls did not deform so readily or to such a large extent as
normal balls. Mailänder (1925) considered that the indenter
should be 1.7 times as hard as the test piece, hard steel
balls not giving reliable results on specimens whose B.H.N.
was greater than 450 to 500. From purely theoretical plastic
considerations Tabor (1951 b) derived a critical value of
2.5 for the ratio of the B.H.N. of the indenter to that of
the metal. Thus the balls used in Brinell hardness tests,
which have a hardness number of some 900 kg./sq.mm., should
not be used on metals whose hardness number is greater than
400 kg./sq.mm. Graphs of the B.H.N. as measured with a
hard steel ball (B.H.N. 900) and a Tungsten Carbide ball
(B.H.N. 1,500) are shown in fig. 19. The difference in
values is due to the plastic deformation of the steel ball.
PART 2

CHAPTER 4

APPARATUS AND EXPERIMENTS

4.1 HARDNESS TESTING EQUIPMENT

Hardness Testers

The machine used for making most of the indentations was a Penetrascope portable hardness tester. This instrument is capable of applying loads from one to 30 kg. by means of a hydraulic thrust unit. The load is read directly from a dial gauge. The specimen can be examined microscopically prior to indentation. The load was applied for a uniform period of 15 seconds and the indentations were measured by means of the optical microscope incorporated in the instrument. The Penetrascope was three indenters to be described below.

For loads greater than 30 kg. a Vickers hardness tester was used. This was also adapted to take all three indenters and could apply loads up to 120 kg. for a period of 15 seconds,

Indenters

The diamond pyramid indenter used in the present work was examined interferometrically to determine the quality of its faces. The four interferograms, which were taken separately, have been combined to give Fig. 20. It is obvious that the faces are not optically flat, as the fringes are not parallel straight lines, however, they are quite flat
CHAPTER 4.

APPARATUS AND SPECIMENS.

4.1 HARDNESS TESTING EQUIPMENT.

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over the area near the tip of the anode which is used during the indentation process. The polish marks are three of the flaws that the "moisten" indentation 100 times hardness can be found to measure. A diamond ball, it against quite all ball. The diamond ball indicator for the basis work lies in its Newton's Rings formula is 60.5 am. The advantage of the property of producing indentations in the hardest metals without undergoing plastic deformation itself. Thus the hardest metals are amenable to study under the same conditions as prevail for softer metals.
over the area near the tip of the indenter which is used during the indentation process. The polish marks on three of the faces are very slight and on the fourth are only some 0.05 microns deep. Under high magnification it was found that the four faces did not meet at a point but a ridge or "roof top". This ridge was 1.2 microns long, but as all the indentations made with the indenter have diagonals at least 100 times as large as this, the ridge has no effect on the hardness readings. The angles between the pairs of opposite faces were measured with an optical goniometer. They were found to be $135^\circ 25'$ and $135^\circ 27'$, the accuracy of the measurement being about $2'$.

A diamond ball indenter was used in much of the present work. Its shape was checked interferometrically by matching it against an uncoated glass flat. The fringes were not quite circular indicating a very slight non-sphericity of the ball. The polish of the ball as revealed by the fringes is quite good, and its radius of curvature, as given by the Newtons Rings formula, is 0.388 mm. The advantage of the diamond ball indenter for the present work lies in its property of producing indentations in the hardest metals without undergoing plastic deformation itself. Thus the hardest metals are amenable to study under the same conditions as prevail for softer metals.
The steel ball indenters used for some of the tests were hardened steel ball bearings. They were found interferometrically to be very closely spherical. Their D.P.H. was about 950 and their radii were checked after indentation to determine whether plastic deformation had occurred. The determination of the radius of curvature by Newton's rings is a more accurate method of detecting plastic deformation than is the measurement of the reduction of the polar diameter as used by previous workers (see section 3.6). An increase in the radius of curvature was found when the balls were used to indent any metal of hardness greater than 300 D.P.H., the deformed radius of curvature tending to increase slightly with load. On indenting a specimen of D.P.H. 950 the radius of curvature of a typical ball was increased from 0.397 mm. to 0.605 mm. In all cases the shape of the recovered region was very closely spherical and its edge was clearly marked. The increased sensitivity of measurement may account for the lower value of the critical hardness than had previously been reported; however, fig. 19 clearly shows a difference in hardness values at 300 B.H.N. which can now be attributed to the plastic deformation of the steel ball indenter.

The conical indenter used in the present work was the normal type of Braille penetrator used in the Rockwell Test. It was specially polished and its shape was checked interferometrically. The spherical end was found to be fairly
satisfactory, though not as good as the diamond ball; the polish was good. The sides of the cone were found to be good in both shape and polish.

4.2 SPECIMENS.

A list of all the specimens used in the present work, together with previous heat or mechanical treatment, is given in Table 1, together with the chapters in which the results from each specimen are included. The analyses of six of the steel specimens are given in Table 2.

Specimens 2 and 3 were work-hardened by known amounts in a conventional rolling mill. No grain orientation was produced as the reductions in thickness were quite small. Transverse sections of the rods were ground and polished for testing.

The heat treatment of the specimens was in accordance with normal practice.

The preparation of the surfaces of polycrystalline metals for interferometric examination involves slight variations from normal metallographic polishing technique. The overall flatness of the surface is of great importance for interferometric study. For this reason microscratches produced in the final stage of polishing, i.e. with alumina on a wet rotating cloth lap, are relatively unimportant. The steel specimens were polished and etched lightly a number of times in order to remove, as far as possible, the work hardened
# Table 1

<table>
<thead>
<tr>
<th>Number</th>
<th>Specimen</th>
<th>Results in Chapters</th>
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<tbody>
<tr>
<td>1</td>
<td>Aluminium</td>
<td>9,10</td>
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<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
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</tr>
<tr>
<td>4</td>
<td>Silver</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>Hard Brass</td>
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<td>6</td>
<td>Silver Steel</td>
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<td>7</td>
<td>Mild Steel Oil Quenched</td>
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<td>8</td>
<td>Alloy Steel Tempered 1 hr. at 150°C</td>
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<tr>
<td>9</td>
<td>Stainless Steel</td>
<td>8,9,10</td>
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<tr>
<td>10</td>
<td>Nickel Steel</td>
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<tr>
<td>11</td>
<td>Mild Steel Normalised 950°C</td>
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<td>12</td>
<td>Alloy Steel Oil Quenched 835°C</td>
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<td>13</td>
<td>Alloy Steel Water Quenched 820°C</td>
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<tr>
<td>14</td>
<td>As 13 Tempered at 100°C</td>
<td>7,8</td>
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<td>15</td>
<td>As 13 Tempered at 150°C</td>
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<td>16</td>
<td>As 13 Tempered at 200°C</td>
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<td>17</td>
<td>Hardened Steel Slip Gauge</td>
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<td>Tungsten Carbide D.P.H. 1,340</td>
<td>9</td>
</tr>
<tr>
<td>19</td>
<td>Tungsten Carbide D.P.H. 1,810</td>
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# Table 2

<table>
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<tr>
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<th>Ni</th>
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<td>0.2</td>
<td>-</td>
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<td>0.5</td>
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<td>8.0</td>
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<tr>
<td>10</td>
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<td>0.36</td>
<td>0.63</td>
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<td>0.03</td>
<td>0.71</td>
<td>-</td>
<td>-</td>
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<tr>
<td>11</td>
<td>0.13</td>
<td>-</td>
<td>1.2</td>
<td>0.22</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
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<td>0.04</td>
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<td>2.50</td>
<td>2.45</td>
<td>0.10</td>
</tr>
<tr>
<td>13</td>
<td>0.41</td>
<td>0.15</td>
<td>0.61</td>
<td>0.02</td>
<td>0.02</td>
<td>0.56</td>
<td>1.27</td>
<td>0.31</td>
</tr>
</tbody>
</table>
In some cases one constituent of the alloy tended to pull out, revealing long wave undulations for the reason that it was necessary for the sides to be as right as possible. As such, three different methods were used in polishing a clamp as shown in Fig. 21. The faces AB and DE were in contact while the faces CB and EF were polished together as one face. By this means the faces up to their convex edges were polished accurately. In order to "rocking" the specimen in a clamp, the types of clamps shown in Fig. 22.
layer. In some cases one constituent of the alloy tended to polish away preferentially; this tendency was clearly revealed by the interference fringes. Electrolytic polishing was found to give quite a smooth surface with comparatively long wavelength undulations. The presence of these undulations made electrolytic polishing quite unsuitable for the present work.

For the experiments to be described in Chapter 11 it was necessary to use specimens with two flat polished sides at right angles to each other. It was essential that these sides be as flat as possible up to their edge of intersection. As such an edge cannot be produced with normal polishing methods the specimens were mounted in a clamp as shown in fig. 21. The faces AB and DE were in contact while the faces CB and EF were polished together as one face. By this means the faces CB and EF were polished very nearly flat right up to their edges at B and E. Then the polished faces CB and EF were clamped in contact and the faces AB and DE were polished in the same way. By this means sufficiently accurate corners were produced.

Hand grinding and polishing was found to produce convex polished surfaces on the softer materials. This is due to "rocking" of the specimen during the hand grinding process. In order to reduce this rocking the specimens were mounted in a clamp. Two types of clamp are shown in fig. 22. In
effect they increase the surface area of the specimen and so reduce the extent of the rocking movement. The clamps were made of metal of comparable hardness to that of the specimens, and the specimens prepared in this way were sufficiently flat for accurate measurement. Above, mention was made of multiple beam interferometry. The measurement of surface features by this method is a well established experimental technique and has been fully discussed by Tolansky (1948). It was first applied to the study of the surface deformation of metals by Tolansky and Nichols (1949, a and b, 1952). They showed that it was a very powerful method for revealing and measuring such distortions. More recently Williams (1953) has employed the method to measure the distortions in cast tin and bismuth crystals.

The main advantages of this system of measurement are that the whole of the surface deformation is revealed and can be easily assessed and measured, the measurement is absolute, and it is extremely simple in operation. It is necessary for the metal under examination to have a flat, highly reflecting surface; this is quite easily produced on most polycrystalline metals by the accepted techniques of metallographic polishing. The accuracy of the method is inherently high, and the distortions caused by the normal loads employed in hardness testing lie just within the range most suitable for measurement.
MULTIPLE BEAM INTERFEROMETRY.

In the survey of methods of measurement of surface distortions of metals given in section 3.1 above, mention was made of multiple beam interferometry. The measurement of surface features by this method is a well established experimental technique and has been fully discussed by Tolansky (1948). It was first applied to the study of the surface deformation of metals by Tolansky and Nickols (1949, a and b, 1952). They showed that it was a very powerful method for revealing and measuring such distortions. More recently Williams (1953) has employed the method to measure the distortions in cast tin and bismuth crystals.

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Fig. 23.

Fig. 24. x 85
Multiple beam interferometry is the chief optical method employed in the present work and this chapter will be devoted to a brief discussion of the relevant theory, experimental set up, and methods of measurement.

5.1 PRODUCTION OF FRINGES.

If two highly reflecting surfaces are brought close together and illuminated with normal parallel monochromatic light, interference fringes are observed localised in the wedge formed by the two surfaces. These are multiple beam fringes of equal thickness of the wedge and are referred to as Fizeau fringes. In reflection they appear as sharp dark fringes on a bright background, and in transmission as sharp bright fringes on a dark background. In order to produce Fizeau fringes over a polished metal surface an optically flat piece of glass, on which has been deposited a slightly transparent highly reflecting layer, is placed in contact with the metal surface and illuminated with parallel monochromatic light. A metallurgical microscope is adapted, as shown in fig. 23, so that the image of the light source is formed in the back focal plane of the objective 0. The light emerges from the objective as a parallel beam and falls normally on the interferometer gap, and hence the value of \( \frac{1}{n\lambda \cos \theta} \) is constant for each fringe. The objective 0 forms an image of the interferometer, and so the fringes, in the image plane of the microscope. The presence of the optical flat which is necessary to produce the fringes limits the power of the
lenses employed, and so the lateral magnification and resolution. Fizeau fringes formed in the air wedge between a well-polished stainless steel surface and a high reflecting optical flat are shown in fig. 24. The thickness of the wedge is constant along each fringe and is either half a wavelength greater or less than the thickness along the adjacent fringe. Thus the fringes act as contour lines of one surface with respect to the other, whose height interval is half the wavelength of the light used. The fine structure of the fringes is due to the polishing marks on the metal surface. In this case the undulations cover about a fifteenth of an order or 180 Å.

Localised multiple beam fringes obey the formula

$$n \lambda = 2 \mu t \cos \theta$$

where $n$ is the order of interference and is an integer, $\lambda$ is the wavelength of the light, $\mu$ is the refractive index of the material between the interferometer plates, $t$ is the thickness of the interferometer gap, and $\theta$ is the angle of incidence.

With $\lambda$ and $\theta$ constant, Fizeau fringes of equal thickness are given. If $\theta$ is kept constant and $\lambda$ varied a new set of fringes is produced. These have been termed fringes of equal chromatic order by Tolansky (1945). The order $n$, and hence the value of $\frac{t}{\lambda}$, is constant for each fringe.

Experimentally the fringes are produced with a white light source in place of the monochromatic source used for Fizeau fringes. A spectrograph is positioned with its slit in the
image plane of the objective and, if the spectrograph slit is fine, equal chromatic order fringes appear in the focal plane. In reflection these are dark bands crossing a bright continuous spectrum. For a given value of \( \mu \) and \( \theta \) each wedge thickness \( t \) will have a set of fringes, and as the value of \( t \) is varied the whole fringe system moves to the red or violet end of the spectrum.

5.2 INTENSITY DISTRIBUTION.

For an interferometer whose two component surfaces are parallel and have reflection and transmission coefficients of \( R \) and \( T \) respectively, the intensity distribution in the transmitted system is given by the formula

\[
\text{Intensity} = I = \frac{T^2}{(1-R)^2} \frac{1}{1+\frac{4R}{(1-R)^2}} \frac{\sin^2 \frac{\delta}{2}}{\frac{\delta}{2}}
\]

where \( \delta = \frac{2\pi}{\lambda} \cdot 2 \mu t \cos \theta \).

This is referred to as the Airy distribution, and represents sharp maxima on a dark background.

In the reflected system however the treatment is more complex. The first beam has a phase change with respect to the second beam which is quite different from that between any other two successive beams. The theory of reflection fringes has been discussed fully by Hamy (1906) and Holden (1949). The experimental conditions for good reflection fringes are more critical than in transmission. The reflectivities of
the surfaces of the interferometer must be critically matched for the best results. The absorption of the front surface of the interferometer is very critical. The absorption $A$ is defined as $1 - R - T$ and if it is high the contrast of the fringes is very poor. This effect is most noticeable in the higher range of reflectivities but it is not confined to this range. If the reflecting layer on the front surface of the interferometer imparts a phase change to the light reflected from it, the fringes assume an asymmetrical maximum-minimum shape which can impair the accuracy of measurement. For high reflectivities it is essential to use an objective lens with a numerical aperture of about 0.3 in order to collect all the beams which contribute to the intensity distribution. If the numerical aperture is too small, the fringe quality suffers and secondary fringes appear.

5.3 PHASE LAG.

Brossell (1947) has shown for the transmitted fringe system that if the interferometer plates are inclined at a small angle $\phi$ to each other there is an additional retardation of the nth beam with respect to the first beam, given by

$$\frac{4}{3} n^3 \phi^2 t.$$  

Thus the path difference between the first and nth beam is $2nt - \frac{4}{3} n^3 \phi^2 t$. If the retardation exceeds $\frac{\lambda}{2}$ all successive beams will oppose the Airy summation and the fringe definition will suffer. For a given reflectivity the critical values of wedge angle and gap for the additional
retardation not to exceed $\frac{\lambda}{2}$, can be found, and the experimental conditions varied accordingly. In topographical studies the wedge angle is usually fixed and the gap becomes the only variable. Although Brossell only considered the transmitted system his results are applicable to all types of multiple beam reflection fringes.

In the present work the interferometer gap was reduced to a minimum by placing the optical glass in contact with the metal specimen. This, combined with the low reflectivities used ($R = 0.6$) ensured that the additional phase lag was less than $\frac{\lambda}{2}$. 33 mm., 25 mm. and 16 mm. objectives were used whose numerical apertures were 0.1, 0.15 and 0.30 respectively. The light sources were a high pressure mercury arc filtered to transmit the green line (wavelength 5461 A.U.) for the Fizeau fringes, and a carbon arc source for the equal chromatic order fringes.

5.4 **FIZEAU FRINGES.**

In the introduction to this chapter the three main advantages of using the multiple beam interferometric method for surface studies were given. Apart from these, however, is the fact that it does not affect the surface under study at all, i.e. by scratching etc., as all mechanical methods are inclined to do. The method is absolute and measures the surfaces in terms of light waves thus eliminating all mechanical errors. The chief advantage of being able to see
the structure of the single object at once is that irregularities in the surface are immediately obvious. An asymmetrical flow pattern around a ball indentation in a polished silver specimen is shown in Fig. 25. With a method that barely reveals a section of the surface one discovery and measurement can be made in a reasonable time and under the microscope.

The Linmark instrument is an adaptation of the Wilton instrument used to measure surface roughness. It is an adaptation of the Linmark instrument and in use the fringes are transferred to the surface under examination. The fringes have a one-to-one correspondence with the depth of the surface and the depth that is not sufficient.

Multiple fringes are recorded in the present work whenever possible. The method is sensitive and adaptable.

Method of Measurement.

For measurement purposes the Pleissn fringes are treated as contour lines of the surface, and a section may be plotted as shown in Fig. 25. The section given is that along the diameter of a ball indentation in steel.
the structure of the whole surface at once is that unsymmetrical flow patterns are immediately obvious. An unsymmetrical flow pattern around a ball indentation in a polished silver specimen is shown in fig. 25. With a method that merely reveals a section of the surface the discovery and measurement of such an asymmetry would require considerable time and labour.

The Linnik interferometer (Räntzeh, 1945) has been used to measure the surface distortions of metals. This instrument is an adaption of the Michelson interferometer for surface measurements. The disadvantages of the instrument compared with multiple beam interferometry, for the present work, are that the magnification is fixed and quite high; that the fringes have a $\cos^2$ intensity distribution, i.e. the intensity maxima are as broad as the minima, which restricts accuracy; and that the depth of focus is only about one micron, which is not sufficient for many flow patterns.

Multiple beam Fizeau reflection fringes were used in the present work wherever possible and were found to be very sensitive and adaptable.

Method of Measurement.

For measurement purposes the Fizeau fringes are treated as contour lines of the surface, and a section may be plotted as shown in fig. 26. The section given is that along the diameter of a ball indentation in steel. The height
The difference between the fringes is 0.013 millimeters and the vertical magnification 2500. The original surface level, shown with a broken line in the upper image, starts from the position magnified 360 out 1.

3.5 Results

From this, it is evident that fine heights can be accurately given if we use those patterns measured in the miliometer. The order of the fringes in the previous reduces the intensity of each corresponding to an integral value of \( \lambda \) which falls within the visible spectrum.

If \( \lambda \) is increased by an amount \( \Delta \lambda \), \( \lambda \) increases by \( \Delta \lambda \) given by...
difference between the fringes is 0.273 microns and the vertical magnification 23,000. The original surface level, shown with a broken line in fig. 26, is interpolated from the position of the undistorted fringes. The horizontal magnification is 45; this is variable and can be as high as 300 but is rarely above this.

5.5 **EQUAL CHROMATIC ORDER FRINGES.**

From the above discussion of measurement of flow patterns with Fizeau fringes it is apparent that flow patterns whose heights are less than \( \frac{3 \lambda}{2} \) or 0.75 microns cannot be measured accurately by this method as there are too few fringes to give an accurate section. Equal chromatic order fringes are used to measure these smaller flow patterns.

The formula governing the formation of equal chromatic order fringes is

\[ n \lambda = 2 \mu t \cos \theta . \]

In the present work \( \mu = 1 \) and \( \theta = 90^\circ \) and the formula reduces to

\[ n \lambda = 2t \]

or

\[ t = \frac{n \lambda}{2} \]

For a given value of \( t \) there will be a series of values of each corresponding to an integral value of \( n \). Consider one such value of \( \lambda \) which falls within the visible spectrum. If \( t \) is increased by an amount \( \Delta t \), \( \lambda \) increases by \( \Delta \lambda \) given by
Fig. 27.

Fig. 28. x150
\[ \Delta \ell = \frac{n}{2} \Delta \lambda \]

Consider now the section of the surface under study along the line which is imaged on the slit of the spectroscope. Suppose the interferometer thickness is constant at the top of the slit, decreases and then increases, due to a "hill" on the surface under test, and then regains its original constant value. The equal chromatic order fringes will be parallel to the spectroscope slit at the top of the field, then deviate to the violet end of the spectrum as \( t \) decreases, and return to their original positions at the bottom of the field. This is shown diagramatically in fig. 27. Hence each equal chromatic order fringe is a section of the surface under test along the line which is imaged on the slit of the spectroscope. The shape of all the fringes is the same but their magnification varies.

Method of Measurement.

One fringe is selected and measured on a comparator taking as co-ordinates the vertical distance from the bottom of the spectrum \( y \) and the horizontal distance \( x \) from a spectral line of known wavelength \( \lambda_0 \). The wavelength \( \lambda \) at any point distant \( x \) from the spectral line is then \( \lambda_0 + Nx \), where \( N \) is the dispersion of the spectrograph. Consider the fringe passing through a point distant \( x \) from the spectral line

\[ \lambda = \lambda_0 + Nx = \frac{2t}{n} \]

For the adjacent fringes on either side \( \lambda_1 = \lambda_0 + Nx_1 = \frac{2t}{n+1} \)
\[ \lambda_2 = \lambda_0 + N x_2 = \frac{2t}{n-1} \]

Therefore
\[ N(x_2 - x_1) = \frac{4t}{n^2 - 1}. \]

In the present work, \( n \) is always greater than 20 and therefore \( n^2 \) is much greater than one, hence
\[ N(x_2 - x_1) = \frac{4t}{n^2} = \frac{2t}{n} \times \frac{2}{n} = \lambda x \frac{2}{n} \]

Therefore
\[ \frac{\lambda}{x_2 - x_1} = \frac{N}{2} \]

Now if the fringe under consideration deviates and passes through a point distant \( x + \Delta x \) from the spectral line at some other \( y \) co-ordinate, we have
\[ \lambda_0 + N(x + \Delta x) = \frac{2(t + \Delta t)}{n} \]

where \( \Delta t \) is the increase in the value of \( t \).

Therefore
\[ N \Delta x = \frac{2 \Delta t}{n} \]
\[ \therefore \Delta t = \frac{Nn}{2} \Delta x \]
\[ \therefore \Delta t = \left( \frac{\lambda}{x_2-x_1} \right) \Delta x. \]

By means of this formula it is possible to convert from fringe displacements directly to surface measurements and hence the section of the surface may be measured.

Flow patterns whose heights are as small as 500 A.U. can be measured with equal chromatic order fringes, this limit being set by the depth of the polish marks on the metal surfaces. The vertical magnification attainable with these fringes is
some 100,000 times with the experimental set up used in the present work. Equal chromatic order fringes over the diameter of a ball indentation in steel are shown in fig. 28. The traces of the green and two yellow lines of mercury, which are superimposed on the plate for measurement purposes, are also shown.

Diameter is of great importance in the production of good quality multiple beam reflection fringes. It has for some time been recognised that silver films, which are the best metallic films available for this work, have four important disadvantages in this respect.

Firstly, the absorption and reflection properties of silver films deteriorate on exposure to the atmosphere. Secondly, these absorption properties are subject to unpredictable variation. Thirdly, the absorption of silver films is anomalously high for lower reflectivities. Fourthly, there is a phase change on reflection at the silver surface, which has been shown by Holden (1949) to be responsible for the production of asymmetrical fringes for lower reflectivities, which affect local accuracy of measurement of fringe displacements.

Multilayer dielectric films do not suffer from these disadvantages, and indeed possess properties which in other ways are superior to those of silver for the interferometric examination of metal surfaces. A survey of their production and properties will be given, and their particular applications to interferometry will be discussed.
Reference was made in Chapter 5 to the fact that the absorption of the high reflecting coating on the front surface of the interferometer is of great importance in the production of good quality multiple beam reflection fringes. It has for some time been recognised that silver films, which are the best metallic films available for this work, have four important disadvantages in this respect.

Firstly, the absorption and reflection properties of silver films deteriorate on exposure to the atmosphere. Secondly, these absorption properties are subject to unpredictable variation. Thirdly, the absorption of silver films is anomalously high for lower reflectivities. Fourthly, there is a phase change on reflection at the silver surface, which has been shown by Holden (1949) to be responsible for the production of asymmetrical fringes for lower reflectivities, which affect local accuracy of measurement of fringe displacements.

Multilayer dielectric films do not suffer from these disadvantages, and indeed possess properties which in other ways are superior to those of silver for the interferometric examination of metal surfaces. A survey of their production and properties will be given, and their particular applications to interferometry will be discussed.

An extensive review on multilayer films is given by Cotton and Woods (1931), and any account of their properties by Ferriol and Gouy (1930). The principle of multilayer films is most readily appreciated by considering a single film. If a single film having an optical thickness at \( \lambda \), where \( \lambda \) is the metrical thickness, and \( \lambda \) the wavelength of light in vacuum, is deposited on glass, the reflectivity of the glass surface is increased if the refractive index \( n \) of the film is greater than that of the glass. The light reflects at the dielectric-air surface suffers a phase change of \( \pi \) and is in phase with that reflected at the dielectric-glass surface which has traversed an additional optical path \( 2nt = \frac{2n\lambda}{\lambda} \), this being equivalent to a phase change of \( \pi \). Since the light reflected at these surfaces is in phase it combines to produce an increase of reflectivity, giving the film a thicker.

Multilayer dielectric films do not suffer from these
An extensive bibliography on multilayer films is given by Cotton and Rouard (1950) and Kuhn (1951), and an account of their properties by Jacquinot and Dufour (1950). An account of their production and application to interferometry is given by Turnbull and Belk (1952).

6.1 PRINCIPLE OF MULTILAYER FILMS.

The principle of multilayer films is most readily appreciated by considering a single film. If a single film having an optical thickness \( nt = \frac{\lambda}{4} \), where \( t \) is the metrical thickness and \( \lambda \) the wavelength of light in vacuo, is deposited on glass, the reflectivity of the glass surface is increased if the refractive index \( n \) of the film is greater than that of the glass. The light reflected at the dielectric-air surface suffers a phase change of \( \pi \) and is in phase with that reflected at the dielectric-glass surface which has traversed an additional optical path \( 2nt = \frac{\lambda}{2} \), this being equivalent to a phase change of \( \pi \). Since the light reflected at these surfaces is in phase it combines to produce an increase in reflectivity. When the film is \( \frac{\lambda}{2} \) thick it will have a low reflectivity, because the extra optical path for the light reflected at the dielectric-glass surface is \( \lambda \), corresponding to a phase change of \( 2\pi \). As the thickness increases further, the reflectivity rises until it is a maximum again for a thickness of \( \frac{\lambda}{4} \). If a film has a refractive index lower than that of the glass substrate, the
Fig. 29.

where \( n_1 \) is the refractive index of the film and \( n_2 \) that of the substrate, \( \theta \) the angle of incidence, and \( \lambda \) the wavelength at which the interference occurs.

By using two refractive index microscopes, various reflectivities were obtained. To ensure that the waves are in phase, for an odd number of quarter wave layers, the film was deposited on a glass slide, and separated by a metal slide, for an even number of quarter wave layers. The reflectivities are shown in Fig. 30.

Fig. 30.

The relative phases of the reflected light for two quarter wave films deposited on glass, and also for a three layer film, are shown in Fig. 31.

Fig. 31.
light reflected at the upper and lower surfaces is out of phase, for a thickness of $\frac{\lambda}{4}$, and an anti-reflection surface is formed.

The reflectivity of a single quarter wave film is given by

$$R_{\text{max}} = \left[ \frac{n_2 - n_1}{n_2 + n_1} \right]^2$$

where $n_1$ is the refractive index of the film and $n_2$ that of the substrate. The reflectivity is a maximum for the wavelength at which the film is a quarter wave thick.

By using a number of films of alternately high and low refractive index and each of optical thickness $\frac{\lambda}{4}$, various reflectivities up to a maximum of 97% may be attained. To ensure that the light reflected from each interface is in phase, for an odd number of films the refractive index order on a glass substrate must be $G$ (Glass), $H$ (High), $L$ (Low), $H$, and for an even number of films $G, L, H, L, H$. Fig. 29 shows the relative phases of the reflected light for two quarter wave films deposited on glass, and also for a three layer film. In the order $GHL$ the light reflected at the air-cryolite (AL) and cryolite-zinc sulphide (LH) boundaries is out of phase, whereas for the order $GLH$ it is in phase. The reflectivities are 14% and 40% respectively.

6.2 **PRODUCTION OF MULTILAYER FILMS**

Thin dielectric films are produced by thermal evaporation in vacuum, the conditions being similar to those required
for metals. The main difficulty lies in the control of the thickness of each layer as the material is deposited. It is necessary to measure the thickness during evaporation as accurately and simply as possible, without breaking the vacuum. The most accurate and convenient methods utilise the optical reflecting properties of the evaporated films.

Control by the colour change of the reflected light was employed in the present work, and the details of the method are discussed below.

Evaporation Conditions.

The evaporation plant is of the vertical type, the evaporating chamber consisting of a Pyrex bell jar 18 ins. high. The evaporation pressure of $10^{-4}$ mm. Hg. is produced by an oil diffusion pump backed by a rotary pump.

A list of the dielectric materials available for the production of multilayer films is given in Table 3.

**TABLE 3.**

<table>
<thead>
<tr>
<th>Substance</th>
<th>Refractive Index</th>
<th>Melting Point °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZnS</td>
<td>2.37</td>
<td>1,020</td>
</tr>
<tr>
<td>Sb$_2$S$_3$</td>
<td>3 to 4</td>
<td>550</td>
</tr>
<tr>
<td>PbCl$_2$</td>
<td>2.2</td>
<td>501</td>
</tr>
<tr>
<td>TiO$_2$</td>
<td>2.6</td>
<td>1,640</td>
</tr>
<tr>
<td>Cryolite</td>
<td>1.36</td>
<td>1,000</td>
</tr>
<tr>
<td>MgF$_2$</td>
<td>1.38</td>
<td>1,396</td>
</tr>
</tbody>
</table>
A suitable material must evaporate at a temperature below the melting point of the filament without decomposing, have a suitable refractive index, and a low optical absorption. The film produced should be highly resistant to atmospheric attack and durable. Of the high refractive index materials listed in Table 3 titanium dioxide tends to decompose upon evaporation and antimony sulphide, although it has a very high refractive index, is too absorbent. Lead chloride has a low melting point and is easily evaporated, but the film produced is attacked by atmospheric moisture and is readily scratched. Zinc sulphide is found to be the most suitable material and this was used in the present work. Magnesium fluoride and cryolite are equally good as low refractive index materials; the latter was used throughout the present work.

The filaments, which contain the dielectric materials, are placed close together directly under the specimens to be coated, with a piece of glass between them to prevent material from one filament contaminating that in the other during evaporation. The filaments are made from molybdenum sheet, in the form of "V" shaped boats, their size being limited by the heating current available.

Evaporation Procedure:

Prior to the evaporation of multilayers the glass flats to be coated are thoroughly cleaned. They are washed in
soap and water, then gently dried and polished with high grade cotton wool until they give no breath figures. When the pressure in the evaporating chamber has been reduced below 0.5 mm Hg, a high tension discharge is passed for some minutes, in order to clean the specimens further by ionic bombardment. After cleaning, the pressure in the chamber is reduced by the diffusion pump to $10^{-4}$ mm. Hg. for the evaporation.

The method of thickness control originated by Banning (1947) is very simple and gives quite accurate results. The control is effected by the visual observation of the change of colour of the light reflected from the film. As the thickness of the film increases the wavelength for which the film has its maximum reflectivity changes and so the colour of the film, as seen in reflection, changes. For a zinc sulphide film the colour becomes white when the optical thickness is $\frac{\lambda}{4}$ for green light, and as the thickness approaches $\frac{\lambda}{2}$ the film becomes antireflecting and the reflection colour changes to a deep magenta. At $\frac{3\lambda}{4}$ the film is again highly reflecting and the colour is greenish-white. Cryolite becomes magenta at $\frac{\lambda}{4}$, white at $\frac{\lambda}{2}$ and magenta again at $\frac{3\lambda}{4}$. Table 4 lists the colours of different thicknesses of zinc sulphide and cryolite films as seen in reflection at near normal incidence.
### TABLE 4.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Optical Thickness For Green Light</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue-white</td>
<td>yellow</td>
</tr>
<tr>
<td>white</td>
<td>magenta</td>
</tr>
<tr>
<td>yellow</td>
<td>blue</td>
</tr>
<tr>
<td>magenta</td>
<td>white</td>
</tr>
<tr>
<td>blue</td>
<td>yellow</td>
</tr>
<tr>
<td>greenish-white</td>
<td>magenta</td>
</tr>
<tr>
<td>yellow</td>
<td>blue</td>
</tr>
<tr>
<td>magenta</td>
<td>greenish-white</td>
</tr>
</tbody>
</table>

The specimens to be coated are placed, together with a monitor plate, vertically above the filaments and as far away as possible to ensure a uniform deposit. The filament-specimen distance in the present work was 10 ins. The monitor plate, consisting of a rectangular piece of glass, can be partially or wholly shielded from the evaporating material by a movable shutter. This is operated, through a Wilson seal, from beneath the baseplate. As the first layer is evaporated the shutter is moved to expose a small section of the monitor plate and this is viewed through the top of the chamber, throughout the evaporation. The light source found most suitable is a 60 watt opal bulb. When the colour of the monitor in reflection is as required the
filament current is switched off. A fresh section of the monitor is exposed for each successive layer. In this way any number of layers of any thickness can be deposited.

It has been shown by Polster (1952) that if zinc sulphide is evaporated at a speed greater than 300 Å per minute it forms comparatively large micro-crystals which cause appreciable scattering and absorption. This sets a limit to the speed of evaporation of zinc sulphide, although no such precaution is necessary for cryolite.

The weakness of this method of thickness control lies in the personal equation of the observer, and the fact that some of the colour changes are difficult to observe. In order to overcome this a photoelectric control unit was constructed similar to that reported by Dufour (1948). In this method the intensity of monochromatic green light reflected from the monitor is measured photoelectrically and this serves as an indication of the thickness of the film. It was found that the control by colour change was quite satisfactory for numbers of layers up to five and that only above this was the more accurate photoelectric method required. In the present work three-layer films were used almost exclusively and these were quite satisfactorily produced with the visual control method described above.
6.3 PROPERTIES OF MULTILAYER FILMS.

As the present work is concerned only with the interferometric examination of metal surfaces the properties of multilayers and their advantages for this application only will be discussed. Their application to multiple beam interferometry in general has been discussed by Tolansky (1952) and Belk, Tolansky and Turnbull (1954). The metal surfaces examined in the present work have reflectivities of 70% or slightly less and require approximately similar reflectivities on the optical flat to produce the most satisfactory fringes. For this reason the merits of relatively low reflectivity multilayer films will be compared with those of silver films of similar reflectivity.

Of the disadvantages of silver films mentioned in the introduction to this chapter the first named was that of the deterioration of the reflection and absorption properties of silver films with exposure to the atmosphere. This is a very important effect and limits the use of low reflectivity silver films to a few days after their production. In the case of multilayer films of zinc sulphide and cryolite, however, no such deterioration is apparent after some months of contact with the atmosphere. This reduces the number of evaporations considerably and means that apart from accidents such as scratching the films can be used for very long periods.

Multilayer films made of other dielectric materials do not
have such outstanding advantages in this respect.

The absorption of all multilayer films is low, and for low reflectivities is much lower than that of silver films which have anomalously high absorptions in the low reflectivity range. This results in marked improvements in both intensity and contrast of reflection fringes. Jarret (1952) reports that for a seven-layer film (four layers of zinc sulphide and three of cryolite) the reflectivity is 94% and the absorption 1%, while a silver film of similar reflectivity has an absorption of 5% (Tolansky, 1946). As the number of layers is reduced the reflectivity and absorption decrease. The absorption for low reflectivity multilayer films is negligible and the visibility of the resulting fringes as near perfect as the optical system employed will allow. The absorption arises mainly in the zinc sulphide layers and seems to depend on purity, vacuum conditions, and rate of deposition; attention to these points may lead to a further improvement in multilayer film properties.

The reproducibility of a range of reflectivities and absorptions is a further advantage of multilayer films. A list is given in Table 5 of the reflectivities of the range of multilayer films, and their respective numbers of layers, for zinc sulphide and cryolite on a glass substrate of refractive index 1.5.
In the centre column of Table 5 G refers to glass, H to high and L to low refractive index material. This table of values which are reproducible to within a few per cent, covers the whole range required for multiple beam interferometry. Having selected the required reflectivity it can be produced by evaporating the requisite number of layers in the correct order. Thus all elements of trial and error are removed from the evaporation process.

Holden (1949) showed that there was an anomalous phase change on reflection at the surface of a silver film which gave rise to asymmetrical reflection fringes. The normal dark reflection fringe is bordered by a bright edge. This occurs particularly in the range of reflectivities required for the examination of metal surfaces and affects the local

<table>
<thead>
<tr>
<th>Number of Layers</th>
<th>Refractive Index Order</th>
<th>Reflectivity %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GH</td>
<td>31</td>
</tr>
<tr>
<td>2</td>
<td>GLH</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>GHLH</td>
<td>67</td>
</tr>
<tr>
<td>4</td>
<td>GHLHLH</td>
<td>72</td>
</tr>
<tr>
<td>5</td>
<td>GHLHLH</td>
<td>87</td>
</tr>
<tr>
<td>7</td>
<td>GHLHLHHLH</td>
<td>94</td>
</tr>
<tr>
<td>9</td>
<td>GHLHLHHLHHLH</td>
<td>97</td>
</tr>
</tbody>
</table>
accuracy of fringe measurement. The phase change on reflection at a multilayer surface, however, is $\pi$; and it can be shown theoretically and verified experimentally that this gives rise to symmetrical dark fringes on a bright background. This property of multilayer films of giving a known phase change on reflection, has been utilised to verify Holden's work on the intensity distribution of reflection fringes (Belk, 1954).

As the component films are only the correct thickness for one wavelength the reflectivity of multilayer films varies considerably with the wavelength. The reflectivity-wavelength characteristics of a three and seven-layer film are given in fig. 30. They both have their maximum reflectivity in the green and fall off towards the red and violet ends of the spectrum. These wavelength-reflectivity characteristics may be very readily found by means of the fringes of equal chromatic order. The white light transmission fringes of equal chromatic order, over a wavelength range 3700 - 5800 A., for a pair of seven-layer reflectors deposited on glass, are shown in fig. 31. The reflectivity is a maximum at 4700 A. as the sharpest fringe occurs at about this wavelength. As the reflectivity decreases the fringe definition deteriorates, but it is a matter of considerable interest that the fringe definition has only fallen off a little over a range of $\pm$ 500 A from the maximum. The rapid deterioration into a
continuum beyond 5500 Å is striking; this indicates that although these particular films will give excellent fringes in the blue they are almost transparent in the red. It might be anticipated that this variation of fringe sharpness with wavelength would rule out the use of multilayer films for the examination of surfaces by equal chromatic order fringes. This is not the case, however, as if a film of reflectivity $R_1$ is matched against a surface of reflectivity $R_2$ the fringe width is determined by the combination $\sqrt{R_1R_2}$. For all the polished metal surfaces used in the present work the value of $R_2$ was constant over the visible spectrum, and even though the value of $R_1$ was variable the combination $\sqrt{R_1R_2}$ was of sufficient value to give sharp equal chromatic order fringes over the whole of the visible spectrum.

Normally the optical thickness of the component layers of a multilayer reflector is a quarter of a wavelength. The same maximum reflectivities are given for layers of thickness any odd multiple of this. The absorption is increased and the wavelength-reflectivity characteristic is completely changed. The peak in the reflectivity distribution becomes much sharper about the maximum value. Thus a film whose component layers are three quarters of a wavelength optical thickness might have a high reflectivity in the green and be completely transparent in the red and blue. This property was first reported by Banning (1947) and has been adapted to reveal surface features when employing the
In order to overcome this difficulty second order filters have been used (component layers in optical thickness).
microscope for multiple beam studies. If a specimen with a structured surface is examined interferometrically the microscopic details of the surface features are frequently obliterated. The fringes contour the surface features but the features themselves are not visible. The cause of this masking of surface detail is the reflectivity of the matching surface, as the fraction of the light which penetrates to the specimen surface and is diffracted back is swamped by the intense first beam directly reflected from the optical flat. The surface under study can only modify the intensity in the narrow fringe regions. In order to overcome this difficulty second order multilayer films have been used (component layers $\frac{3\lambda}{4}$ optical thickness). First fringes are obtained with the correct wavelength. On changing to a wavelength band for which the multilayer film is effectively transparent, the surface can be seen with no fringes, without any change in the optical set up. An application is shown in figs. 32 and 33. Hardness test indentations were made on a crudely polished steel specimen having on it several residual scratches. The fringes given by matching this with a second order multilayer film whose maximum reflectivity was at about 5,500 A., and illuminating with mercury green light are shown in fig. 32. By changing to a red filtered carbon arc source fig. 33 is obtained. There are no fringes and a high definition microscope image of the surface is given.
The final advantage of multilayer films over silver films for the interferometric study of metal surfaces is their much greater resistance to scratching. The metal specimen is placed in contact with the high reflecting layer, for reasons given in Chapter 5, and any subsequent movement of the specimen will tend to scratch the layer. This is very noticeable in the case of silver films whose mechanical properties are quite poor in this respect. Multilayer films are very much more robust for this work and can even be gently wiped with cotton wool to remove dust particles, a treatment which would completely destroy a thin silver layer.
CHAPTER 7.

FLOW PATTERN SHAPE.

The impressions which occur in hardness testing are predominantly of the ridging type. In the present work the flow patterns measured were all very similar in section to that shown in fig. 26. The ratio of the diameter of the flow pattern to that of the indentation varied, as did the maximum height of the flow, but by means of suitable multiplying factors most of the flow patterns could be superimposed on each other with accuracy. This similarity of flow pattern shape for various indenters was striking. This flow pattern shape is similar to that reported by Bugdale for wedge indentations in work-hardened metals.

In order to be able to compare the relative extents of flow patterns a parameter, which is referred to as the "half width", was defined. The definition is illustrated in fig. 34. $A$ is half the maximum height of the flow pattern above the undistorted level. $BC$ is the horizontal distance of $A$ from the edge of the indentation. The half width $w$ is defined by the formula

$$w = \frac{2x_{BC}}{d}$$

where $d$ is the chordal diameter of the indentation. The value of the half width will be larger for a flow pattern whose extent is greater, relative to the diameter of the indentation. The half width has proved to be a useful
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$$W = \frac{2 \times BC}{d}$$

where $d$ is the chordal diameter of the indentation. The value of the half width will be larger for a flow pattern whose extent is greater, relative to the diameter of the indentation. The half width has proved to be a useful
measure of flow patterns held essentially constant throughout the present work.

Furthermore it was found that for pyramid indentations, in steel, over a range of loads the flow patterns were "similarly practically repeated at an increasing scale". The values of \( a \) for these impressions would be the same. Values \( a \) and \( b \) indicate the experimenter's interpretation of the data:

\[ \frac{D}{2} \]

---

**Fig. 34.**

Values of \( a \) and \( b \) indicated the experimenter's interpretation of the data. In the first place, values of the half-width and Meyer Index of three steel specimens were found. The results are given in Table 6 and it will be seen that as the work-hardening capacity, i.e., the Meyer Index increases, the half-width also increases.
measure of flow patterns and will be employed throughout the present work.

Tolansky and Nickols (1952) found that for pyramid indentations in steel and for a range of loads the flow patterns were "almost identically repeated but on increasing scales". The values of \( W \) for these impressions would be the same, and this has been verified by the present author. Values of \( W \) determined experimentally for a range of materials and indenters do vary considerably and in this chapter the experimental results will be described and their theoretical interpretation indicated.

7.1 VARIATION WITH WORK-HARDENING CAPACITY.

From purely geometric considerations it is to be expected that the flow pattern shape will be dependent on the geometric shape of the indenter. In order to avoid this complication the results in this section are all measurements of diamond pyramid impressions. Measurements were made of the flow patterns along the perpendicular bisectors of the sides of the indentations. This is the line along which the maximum flow height occurs.

In the first place values of the half width and Meyer index \( n \) of three steel specimens were found. The results are given in Table 6 and it will be seen that as the work-hardening capacity, i.e. the Meyer index increases, the half width also increases.
Thus an ideal metal would be expected to have a very compact ridging type of flow pattern. This is in accordance with the work of Krupkowski (1931) who found indirectly that the height of the flow pattern decreased and its extent increased, as the metal was work-hardened.

In order to verify this general conclusion specimens of silver and steel were heat treated at successively higher temperatures, and values of Meyer index, D.P.H., and half width, were found. The results are given in Table 7.

**TABLE 6.**

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Meyer Index</th>
<th>Half Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2.16</td>
<td>0.39</td>
</tr>
<tr>
<td>8</td>
<td>2.22</td>
<td>0.45</td>
</tr>
<tr>
<td>17</td>
<td>2.37</td>
<td>0.65</td>
</tr>
</tbody>
</table>

**TABLE 7.**

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Heat Treatment</th>
<th>Meyer Index</th>
<th>Half Width</th>
<th>D.P.H.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>As rolled</td>
<td>2.10</td>
<td>0.4</td>
<td>202</td>
</tr>
<tr>
<td></td>
<td>400°C for 1.5 hrs.</td>
<td>2.34</td>
<td>0.45</td>
<td>186</td>
</tr>
<tr>
<td></td>
<td>850°C for 20 mins.</td>
<td>2.36</td>
<td>1.07</td>
<td>105</td>
</tr>
<tr>
<td>3</td>
<td>As Rolled</td>
<td>2.32</td>
<td>0.5</td>
<td>80.0</td>
</tr>
<tr>
<td></td>
<td>200°C for 1.5 hrs.</td>
<td>2.66</td>
<td>1.03</td>
<td>42.0</td>
</tr>
<tr>
<td></td>
<td>400°C for 1.5 hrs.</td>
<td>2.58</td>
<td>1.28</td>
<td>33.1</td>
</tr>
<tr>
<td></td>
<td>600°C for 1.5 hrs.</td>
<td>2.53</td>
<td>-</td>
<td>19.4</td>
</tr>
<tr>
<td></td>
<td>800°C for 1.5 hrs.</td>
<td>2.82</td>
<td>-</td>
<td>18.8</td>
</tr>
</tbody>
</table>
Fig. 35.

Fig. 36.
For steel the annealed condition gives an extended ridging effect, and no sinking. This effect was also found by Norbury and Samuel (1928). The half width varies inversely with the Vickers hardness and changes much more rapidly than the Meyer index for small amounts of work-hardening. For silver similar relations hold, but sinking occurs in the specimens treated at 600°C and 800°C, hence no values of half width can be quoted. Fig. 35 is the Fizeau interferogram of the specimen treated at 600°C. Sinking can be seen distinctly near the indentation, but there is a definite piling-up effect with its maximum some 1.5 D from the centre of the indentation. For the specimens treated at 200°C and 400°C very slight sinking close to the edge of the indentation is visible, similar to that shown in fig. 26.

The results on the silver specimens show that the maximum of the flow pattern moves away from the indenter as the work-hardening capacity of the specimen is increased by progressive tempering. Even in the case of distinct ridging the maximum height of the flow pattern is at a small distance from the edge of the indentation. As the work-hardening capacity is increased the sinking adjacent to the indenter becomes more apparent and the maximum of the flow pattern moves away from the indenter. Thus the characteristic sinking type of impression is merely a very spread out ridging with its flow height reduced in proportion. The measurements of the sinking impressions support this view,
and in this way the transition from the ridging to the sinking type of impression becomes much more easy to visualise.

Finally, some indentations were made in the copper and silver specimens, numbers 2 and 3 in Table 1 which had been cold-worked by known amounts. They were annealed then cold rolled, their percentage reductions in thickness being, for silver 1.6, 4.5, 9.3 and 18.2, and for copper 2.0, 3.9, 8.5 and 18.4. For both pyramid and ball indentations in these specimens all the flow patterns were of the ridging type. As the work-hardening increased the maximum height of flow increased and the extent of the flow decreased. Slight sinking close to the edge was visible for the smallest reduction in thickness only. As the materials had a fairly large grain size the flow patterns were sufficiently unsymmetrical, to be unsuitable for measurements of half width. Thus sinking only occurs for very small amounts of work-hardening. The critical value of the work-hardening for the transition from the ridging to the sinking type of impression in copper is given by Krupkowski (1931) as \( 4\% \), by indirect measurements on cone and ball indentations. Direct measurement shows that this value is too large, the actual value being something less than \( 2\% \), for both copper and silver.

7.2 VARIATION WITH INDENTATION SHAPE.

Ball indentations made under successively increasing loads cause increasing amounts of strain-hardening, as the
indentations are not geometrically similar. Indentations that are not geometrically similar would be expected to give flow patterns which differed from each other, and this is found to be the case. Unlike pyramidal impressions, the half widths of ball impressions vary with the load. Fig. 36 shows graphs, for four steel specimens, of half width $W$ against the chordal diameter $d$. They are plotted in this way so that the half width may be compared with the geometric shape of the indentation, which is specified by its diameter and not the load applied. The graphs all show a decrease of $W$ as the diameter increases and two of them show an initial increase. For all specimens the half width of the impressions made above a certain load decreases with further increase of load. The numbers of the graphs in fig. 36 refer to the specimen numbers given in Table 1.

There are two main mechanisms involved in ball indentation tests; the transition to fully plastic deformation, and the variation of strain pattern with the shape of the indentation. In order to study the former of these, ball indentations were made in specimen number 17 whose hardness was 950 D.P.H. The loads employed were 2, 5, 10, 20, 40, 80 and 120 kg. and the flow patterns were measured either by Fizeau or equal chromatic order fringes. The very shallow 2 kg. indentation gave no detectable flow pattern. The 5 kg. indentation gave a flow pattern which was confined very closely to the
Fig. 37. $\times 500$

Fig. 38.
indentation. The fringes of equal chromatic order over this indentation are shown in fig. 37. The fringes reveal the shape of the indentation clearly, and the flow pattern appears as a small hump close to the edge of the indentation. The 10 kg. indentation gave a measurable flow pattern, and the half widths for this and the larger indentations are plotted in fig. 38. The critical load for full plasticity is also given and the half width is seen to reach its maximum at about this value. This is significant as the increase in extent of the flow pattern below full plasticity must be linked with the mechanism involved in the indentation process. The critical load for full plasticity comes in a similar position for specimens 13, 14 and 15 whose graphs were shown in fig. 36.

7.3 DISCUSSION.

The main experimental conclusions which have to be explained are, the decrease in the half width as the load is increased in the fully plastic range, and the increase in the half width up to a maximum at the critical load for full plasticity. The only analytical work that has been done on flow pattern shapes is that of Bishop, Hill and Mott (1945) and Dugdale (1953), already referred to in Chapter 3. This work was concerned with wedge indentations; and the connection between wedge and ball indentations is not sufficiently close to warrant any definite comparisons to be drawn.

For an ideal rigid-plastic metal under frictionless
wedge indentation, the half width is 0.5 for a flat punch, and this increases as the angle of the wedge is decreased. If the metal has even a small work-hardening capacity the results are considerably different from this. Dugdale's results concern wedge indentations in real metals and show that the half width decreases and then increases again as the angle of the wedge is decreased. These results may be applied to the present work in the manner indicated below. A shallow ball indentation may be compared with a very large angle conical indentation, which will have some similarity with a large angle wedge indentation. As the depth of the ball indentation increases the angle of the equivalent cone decreases. It is possible that for a ball indenter the half width decreases with increase of load for the same reason that it decreases for smaller angle wedge indentations. The equivalent wedge angle, however, only reaches a sufficiently small angle for the subsequent increase in half width to be observed for specimen 8, whose graph is given in fig. 36. Davies (1949) showed experimentally that plastic flow was initiated at a point beneath the centre of the indentation. As the load is increased the volume of material undergoing plastic deformation is assumed to increase until a state is reached when the plastic region has become large enough for fully plastic deformation to occur. The size of the plastic region is only known qualitatively for any given value of load,
and the only problems to which even approximate solutions can be given theoretically, are those concerning the onset of plasticity, and the fully plastic state. It is therefore of some interest that the shape of the indentation and flow pattern give some indication of the extent of the plastic region during the intermediate process.

From the measurements reported the following process seems to occur in the initial stages of penetration. Plastic flow commences beneath the centre of the indenter. When the plastic region has spread a little, penetration occurs. There is no flow pattern associated with the smallest indentation, the whole of its volume being stored in the bulk of the metal. For larger loads the flow pattern begins to appear, first very close to the indentation but spreading until when full plasticity is reached it occupies a diameter about twice that of the indentation. (This varies with its work-hardening capacity.) Thus full plasticity refers to a state when the plastic region is considerably bigger in area than the indentation.

Some of the other results in the present work have a bearing on the fully plastic state and will be commented on under the relevant heading. A summary of all the information about the various processes involved in hardness testing will be given in the final chapter.
89.

CHAPTER 8.

VOLUME OF FLOW PATTERN.

In section 3.2 a brief review was given of the measurements of flow pattern volume made by Yakutovich, Vandyshhev and Surikova (1948) and Tolansky and Nickols (1952). In view of the large discrepancy in their results a systematic study of the ratio of the volume of the flow pattern to that of the indentation was undertaken for a series of steel specimens and is reported in this chapter. The possible explanations of the discrepancies in these volumes are also discussed.

8.1 METHOD OF MEASUREMENT.

The method of measurement of flow patterns is given in Chapter 5. In order to calculate the volume of a flow pattern it must be circularly symmetrical. This necessitates the use of circularly symmetrical indenters and in the present work the diamond cone and ball referred to in section 4.1 were used. Diametral sections of the flow patterns were taken in two directions at right angles and the average shape was found. The volumes of the flow patterns were calculated on the basis of the following simple mathematical treatment. The volume swept out by rotating an element of width \( r \) and height \( y_r \), as shown in fig. 39, about \( OY \) is \( 2\pi r x_r y_r \), where \( x_r \) is the distance of the centre of the element from \( OY \).
The total
back ScD
and the
total
value of
of the
pattern
to be
reason

called

are

measured and its movements are plotted electrically and
plotted on semilog paper. For ball indentations
the volume were calculated from measurements of

disasters
were calculated.

calculation

3.3 RESULTS

The graphs

as a parabola,
the volume

specific

volume are

Fig. 39.

Fig. 40.
The total volume swept out by all such elements is $2\pi x r^2 y r$.

Each flow pattern was split up into about fifteen elements and the volume was calculated from the above formula. The value of $x_1$ was made equal to $\frac{D}{2} + \frac{r}{2}$ where $D$ is the diameter of the indentation, in order that the whole of the flow pattern might be covered.

In general the sides of the indentation were too steep to be measured by multiple beam interferometry. For this reason they were measured with a mechanical profile instrument called a Talysurf. A very lightly loaded spherical diamond stylus of radius one micron is moved across the surface to be measured and its movements are amplified electrically and plotted on curvilinear graph paper. For ball indentations the volumes were calculated from measurements of their diameters and depths. The volumes of the conical indentations were calculated by a method similar to that used for the calculation of flow pattern volumes.

8.2 RESULTS.

The results of the measurements are listed in Table 8, together with the specimens and loads employed. The specific volume is defined as the volume of the flow pattern expressed as a percentage of the volume of the indentation. Thus, if the volumes of the flow pattern and indentation are equal the specific volume is 100. Experimentally the values of specific volume are considerably below 100, the average value being 47.
<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Indenter</th>
<th>Load kg.</th>
<th>Volume of Indentation $\times 10^{-1}$ cm$^3$</th>
<th>Specific Volume</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Cone</td>
<td>10</td>
<td>4.63</td>
<td>60</td>
<td>○</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.0</td>
<td>1.04</td>
<td>78</td>
<td>△</td>
</tr>
<tr>
<td>8</td>
<td>Ball</td>
<td>1.0</td>
<td>0.55</td>
<td>45</td>
<td>□</td>
</tr>
<tr>
<td>9</td>
<td>Ball</td>
<td>20</td>
<td>11.5</td>
<td>48</td>
<td>■</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>3.15</td>
<td>40</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.0</td>
<td>0.96</td>
<td>46</td>
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<td>0.404</td>
<td>43</td>
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<td>Ball</td>
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<td>12.75</td>
<td>61</td>
<td>◆</td>
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<tr>
<td></td>
<td></td>
<td>15</td>
<td>3.52</td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Ball</td>
<td>10</td>
<td>9.26</td>
<td>72</td>
<td>X</td>
</tr>
<tr>
<td>13</td>
<td>Ball</td>
<td>30</td>
<td>3.10</td>
<td>64</td>
<td>•</td>
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<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.327</td>
<td>33</td>
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</tr>
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<td>14</td>
<td>Ball</td>
<td>30</td>
<td>2.47</td>
<td>34</td>
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<td></td>
<td></td>
<td>10</td>
<td>0.297</td>
<td>26</td>
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</tr>
<tr>
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<td>2.95</td>
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<td></td>
<td></td>
<td>10</td>
<td>0.432</td>
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<td></td>
</tr>
<tr>
<td>16</td>
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<td>3.33</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>0.383</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Cone</td>
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<td>2.70</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ball</td>
<td>120</td>
<td>9.97</td>
<td>63</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>80</td>
<td>3.14</td>
<td>43</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.962</td>
<td>44</td>
<td></td>
</tr>
</tbody>
</table>
This indentation illustrates the state of affairs referred to in section 7.3 when the plastic region is large enough to permit this indentation to be formed.

It does not disperse into the indenation of volume graph shown.

Fig. 41. x 270

Fig. 42.
The specimens numbered 13, 14, 15 and 16, had been quenched and tempered at successively higher temperatures. Details of the heat treatment are given in Table 1. The graph of specific volume against tempering temperature for these specimens is shown in fig. 40. For three of the specimens the specific volume is greater for the 30 kg. indentation than for the 10 kg.; however, for both loads the specific volumes decrease and then increase with increase in tempering temperature.

The equal chromatic order fringes over a shallow ball indentation in stainless steel are shown in fig. 41. The fringes reveal that there is no flow pattern associated with this indentation, hence its specific volume will be zero. This indentation illustrates the state of affairs referred to in section 7.3 when the plastic region is large enough to permit penetration but not large enough for a flow pattern to be formed.

It was suggested in section 3.2 that the volume discrepancy might be dependent on the volume of the indentation. In order to explore this possibility the specific volumes of the indentations listed in Table 8 were plotted against the volume of the indentations on a logarithmic scale. The graph is shown in fig. 42, on which are also plotted Yakutovich, Vanyush and Surikova's value of specific volume of 100 represented by $\Delta$, and the value for the indentation shown in fig. 41, represented by $\nabla$. Superimposed on the
graph are values of the average specific volume for the various ranges of indentation volume. These average values clearly reveal the tendency of the specific volume to increase with the indentation volume. A curve is also drawn through the points in fig. 42 which indicates that for large indentations values of specific volume of the order of 100 may be expected, while for very small indentations the specific volume will approach zero. The accuracy with which all the flow pattern volumes could be measured was estimated at 10%. The indentations could be measured more accurately than this, and so the values of specific volume can be guaranteed to within ±5. Thus the variation of specific volume must be classed as a real effect. The spread of the points in fig. 42 is quite wide, owing to the variety of properties of the steels used, but the trend is significant and must be accounted for by any proposed explanation of the volume discrepancies.

8.3 DISCUSSION.

The discrepancy between the volumes of the flow pattern and indentation is unexpected. However, the discrepancy has been shown experimentally to exist, and an explanation must be sought. There is the possibility that a slight distortion of the specimen might occur thus ensuring that its volume remained constant. The distortion might take the form of
a very slight uniform expansion of the specimen, or a bulge on its underside or edge whose volume was equal to that of the missing section. Both these possibilities are theoretically very unlikely. The work of Hill, already referred to in section 3.5, showed that no deformation of the specimen occurred unless its size was comparable with that of the indentation. This conclusion has been shown to be valid for a practical hardness test by the present author, the results being reported in Chapter 11. As the indentations used in the present work were very much smaller than the specimens, a large scale distortion of the specimen cannot have occurred. The appearance of a localised distortion at some distance from the indentation is extremely unlikely for a polycrystalline specimen, and even if such a distortion were present it would have been detected. Furthermore if distortion of the specimen were responsible for the observed volume discrepancies, the specific volume would be expected to decrease with increasing indentation size, which is contrary to the variation found experimentally.

If the volume of the specimen does not stay constant its average density must increase. As the accurate measurement of a density change of the order of 1 in $10^6$ is extremely difficult, no direct measurement of the density change of an indented specimen was attempted. Any increase in density of the specimen must be localised in the region affected by
the indentation, and may be confined to the plastically deformed metal adjacent to the indenter. The density changes in plastically deformed metals were measured by Bridgman (1949) who found that they were of the order of one part in $10^3$ or $10^4$ and varied in sign with the metal. In order to account for the observed volume discrepancies on this basis the volume of metal undergoing considerable plastic strain must be some $10^3$ times that of the indentation, even if the density change is of the correct sign. According to Heyer (1937) the volume undergoing measurable plastic strain is some 40 times the volume of the indentation, so an increase in the density of the plastically distorted metal cannot account for the observed volume discrepancies.

There is a possibility of the metal which was compressed elastically during the indentation process, remaining under stress when the indenting load is removed. The retained stresses would need to be compressive in nature to account for the volume discrepancies, and would be considerably smaller than those in operation during the formation of the indentation. Nadai (1931) who has made a study of all aspects of plastic deformation stated:

"If a hardened steel ball is pressed into a piece of soft iron or ductile metal, the material in the neighbourhood of the permanent indentation, after removal of the load cannot remain unstressed. In the region which has been permanently deformed by the ball, besides the normal stresses, acting
parallel to the load, considerable compressive stresses also must have acted in the lateral direction. The latter will cause stresses in the elastic neighbourhood of the indentation and a part of these will continue to exist after the compressive load has ceased to act. The plastically distorted part behaves, under the compression of the ball, like a hard wedge driven into a tree trunk."

The presence of stresses of this type in cold-worked rods and bars, of which only parts have been plastically deformed, can be demonstrated experimentally. Assuming that the volume of action of these stresses is $10^3$ times that of the indentation, and that the specific volume is 50, the average compressive hydrostatic stress, which accounts for the volume discrepancy, is 8 kg./sq.mm. for iron. This stress is some 50 times smaller than the stress acting over the contact surface during deformation, and residual stresses larger than this have been measured in cold worked bars by Hehn (1918) and Sines and Carson (1952). Thus it is possible to account quantitatively for the volume discrepancies on the basis of these residual stresses.

It can be shown both practically and theoretically that the whole of the surface around a ball indentation is displaced elastically below the initial surface level while the load is applied. When the load is removed the surface recovers its initial shape and the ridging or sinking becomes apparent. In view of this very large elastic compression
occurring and account for the conception.

Because of the information above, the indentation procedure was used. Indentations were made deep, with diameters that were viewed parallel to the length of the throw and with a constant shear stress in the material. The fringes were photographed before, during, and after the experiment to show the isochromatics due to cylindrical indenter and wedge indentation shown in Fig. 44 and the changes of the fringe pattern due to the cylindrical indentation in a metal. However, to indicate that the stresses beneath an indentation in any material are present also in another...
occurring during loading the small compression required to account for the volume discrepancy can be more easily conceived. Because of the difficulty of obtaining any direct information about the proposed residual stresses, the indentation process was studied photoelastically. Indentations were made in a rod of Catalin \( \frac{1}{4} \)" thick and 1" deep, with cylindrical and wedge shaped indenters. The rods were viewed parallel to the indenter axis and perpendicular to the length of the rod, with circularly polarised monochromatic light. The fringes produced in this way are called isochromatics and they follow the lines of constant shear stress in the material. The fringes were photographed before, during, and after loading. The rod was effectively stress free before loading and figs. 43 and 44 show the isochromatics during full load and after unloading, for a cylindrical indenter. Similar results were given for a 90° wedge indenter. The residual stresses are clearly shown in fig. 44 and their distribution is similar in form to that for full load. The difference between a cylindrical indentation in Catalin and a hardness indentation in a metal is apparent. They are sufficiently alike, however, to indicate that if residual stresses are present beneath an indentation in Catalin they may be expected to be present also in metals.
Stresses remaining after cold-working or quenching can be released by suitable heat treatment. For steels the treatment required is a low temperature tempering.

Indentations were made in a specimen of nickel steel (number 8 in Table 1) which, subsequent to measurement of the impressions, was vacuum tempered at $500^\circ C$ for 30 minutes and $700^\circ C$ for 30 minutes. No change in the shape either of the indentations or the flow patterns could be detected. Had a positive result been obtained the existence of the retained stresses would have been demonstrated; the negative result however does not disprove their existence. On tempering, the small compressed volume could easily be absorbed into the specimen, on release of the stresses, without a significant change of its shape.

The final problem associated with the proposed explanation of the volume discrepancies is the method of accounting for the variation of the specific volume with volume of indentation and metal. As the majority of the indentations listed in Table 8 were made with the diamond ball indenter, a tentative explanation will be given for ball indentations.

Although the specific volume increases continuously with the indentation volume, the actual value of the volume discrepancy increases also. Thus the volume of metal compressed by the residual stresses increases with the indentation volume; as, however, it is not proportional to the indentation
volume the value of the specific volume increases. A diagramatic representation of the variation of the flow pattern volume with the volume of the indentation is shown in Fig. 45. The specific volume at any load is given by

\[ \frac{\text{specific volume}}{\text{load}} \times 100 \]

and although the discrepancy between the flow pattern and indentation volumes also increases.

The indentation volume is determined by using the mean pressure of the retained load. Although the stresses involved in the indentation process increase the residual stresses do not; and cannot therefore account for so large a proportion of the volume of the indentation.

Apart from the limitation of size of the residual stresses their distribution changes considerably as the size of the indentation increases, and this may account for part of the observed variation in specific volume.

Fig. 45.
volume the value of the specific volume increases. A diagrammatic representation of the variation of the flow pattern volume with the volume of the indentation is shown in fig. 45. The specific volume at any load is given by $\frac{BC}{AC} \times 100$ and although the discrepancy between the flow pattern and indentation volumes increases, the specific volume also increases.

The initial deformation shown in fig. 41 for which the specific volume is zero has already been dealt with in Chapter 7. As the load is increased and the flow pattern begins to appear the specific volume will increase. After full plasticity has been reached the specific volume is determined by the distribution of the stresses around the indentation. As the load is further increased the mean pressure over the indentation increases. The value of the retained stresses, however, cannot exceed a certain value related to the yield stress of the metal. Thus although the stresses involved in the indentation process increase the residual stresses do not; and cannot therefore account for so large a proportion of the volume of the indentation. Apart from the limitation of size of the residual stresses their distribution changes considerably as the size of the indentation increases, and this may account for part of the observed variation of specific volume.

Quenched specimens always contain residual stresses and
a specimen which is already stressed internally will not be able to retain so much additional stress as an originally stress free specimen. This may account for the high values of specific volume given by specimen 13 shown in fig. 40. Both the 10 and 30 kg. indentations have larger specific volumes than those in the same steel which had received a low temperature tempering prior to indentation. The value of the residual stresses must be dependent on the yield stress of the metal, and if this is small the residual stresses will be small and the specific volume will be large. The soft normalised mild steel specimen 11 gave a high value of specific volume which could be accounted for in this way.

The interpretation of the volume discrepancies is not simple but a certain amount of correlation between theory and experiment is apparent. However, considerably more work will be required before a complete explanation of the phenomenon can be given.

The interpretation of the volume discrepancies is not simple but a certain amount of correlation between theory and experiment is apparent. However, considerably more work will be required before a complete explanation of the phenomenon can be given.

\[
\text{Shallowing} \quad a = \frac{n_0 - h}{n_0}
\]

where \(n_0\) is the depth of the indentation under load and \(h\) its depth after elastic recovery. The shallowing coefficient will be unity if the indentation recovers completely, and zero if it does not recover at all.

For a shallow ball indentation it is easily shown that
The previous work on the shallowing of indentations, reported in section 3.3 was concerned with large indentations. Ball indentations with depths as small as 0.05 microns can be measured by multiple beam interferometry, and the shallowing behaviour of these indentations proves to be of some interest. A few of the larger indentations to be referred to in this chapter were measured with the Talysurf instrument described briefly in section 8.1. The majority, however, were measured by multiple beam interferometric methods.

Hitherto the radius of curvature of the recovered indentation has been used as a measure of its shallowing. In view of the fact that recovered ball indentations may not be spherical some other measure is needed. The definition of shallowing adopted in the present work, is the proportion of the depth of the indentation under load, recovered when the load is removed. Thus

\[ s = \frac{h_0 - h}{h_0} \]

where \( h_0 \) is the depth of the indentation under load and \( h \) its depth after elastic recovery. The shallowing coefficient will be unity if the indentation recovers completely, and zero if it does not recover at all.

For a shallow ball indentation it is easily shown that
where \( r_1 \) is the radius of the indenter, \( h \) the recovered depth of the indentation and \( d \) its chordal diameter. In the derivation of this formula the indenter is assumed to be rigid. However, all indenters do deform elastically during the indentation process and a correction can be applied for this. The radius of curvature of the area of contact between the indenter and metal is given by Timoshenko (1934) as

\[
R = \frac{E_1 + E_2}{E_1 \frac{1}{r_1} + E_2 \frac{1}{r_2}}
\]

where \( r_1 \) and \( r_2 \) are the radii of curvature of the indenter and indentation respectively, and \( E_1 \) and \( E_2 \) are their Young's moduli. Thus we may define the true shallowing \( S_0 \) as

\[
S_0 = 1 - \frac{8Rh}{d^2}
\]

where \( R = \frac{E_1 + E_2}{E_1 \frac{1}{r_1} + E_2 \frac{1}{r_2}} \).

It can be shown that the relation between the apparent shallowing \( S \) and the true shallowing \( S_0 \) is

\[
S_0 = \frac{ES}{E+1 - S} \quad \text{where} \quad E = \frac{E_1}{E_2}
\]

Values of true and apparent shallowing for a number of specimens and indenters are given in Table 9, from which some general conclusions may be drawn. Firstly, the true shallowing for Aluminium is greater than 14% of the depth.
TABLE 9.

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>D.P.H.</th>
<th>Indenter</th>
<th>Load kg.</th>
<th>S</th>
<th>$S_0$</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
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<td>1.0</td>
<td>0.186</td>
<td>0.146</td>
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<td>8</td>
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<td>0.456</td>
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<td>□</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/32&quot; Steel</td>
<td>3.0</td>
<td>0.248</td>
<td>0.141</td>
<td>□</td>
</tr>
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<td></td>
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</tr>
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<td>20</td>
<td>0.217</td>
<td>0.183</td>
<td>▽</td>
</tr>
<tr>
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<td>0.164</td>
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<td>0.196</td>
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<td>0.308</td>
<td>0.182</td>
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</tr>
<tr>
<td></td>
<td></td>
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<td>10</td>
<td>0.484</td>
<td>0.319</td>
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</tr>
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<td></td>
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<td>0.624</td>
<td>0.454</td>
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<td>2.0</td>
<td>0.690</td>
<td>0.527</td>
<td>▽</td>
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<td></td>
<td>1.0</td>
<td>0.743</td>
<td>0.590</td>
<td>▽</td>
</tr>
<tr>
<td>17</td>
<td>950</td>
<td>Diamond</td>
<td>120</td>
<td>0.338</td>
<td>0.293</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/32&quot; Steel</td>
<td>80</td>
<td>0.366</td>
<td>0.320</td>
<td>○</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>40</td>
<td>0.472</td>
<td>0.422</td>
<td>○</td>
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<tr>
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<td></td>
<td></td>
<td>20</td>
<td>0.524</td>
<td>0.474</td>
<td>○</td>
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<td>950</td>
<td>1/32&quot; Steel</td>
<td>20</td>
<td>0.724</td>
<td>0.567</td>
<td>●</td>
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<td>18</td>
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<td>Diamond</td>
<td>20</td>
<td>0.481</td>
<td>0.367</td>
<td>◊</td>
</tr>
<tr>
<td>19</td>
<td>1810</td>
<td>Diamond</td>
<td>20</td>
<td>0.640</td>
<td>0.526</td>
<td>◊</td>
</tr>
</tbody>
</table>
In one case the true shallowing is some 72% of the depth under load, which corresponds to the indentation reported by Foss and Brumfield (1922) whose radius of curvature was three times that of the indenter. The shallowing increases as the load is reduced, but there is little connection between the shallowing of diamond and steel ball indentations in the same metal.

Tabor (1951 b) showed that Hertz's equation for the elastic deformation of spherical surfaces may be applied to the recovery of ball indentations. We have

\[
d = 2.22 \left( \frac{W_{r}}{g} \right) \frac{r_{1}r_{2}}{r_{2}^{2}-r_{1}^{2}} \left( \frac{1}{E_{1}} + \frac{1}{E_{2}} \right) \}
\]

where \( W = \) load and \( g \) is the acceleration due to gravity.

\[
\therefore d^3 = [2.22]^3 \frac{g}{2} \left( \frac{1}{E_{1}} + \frac{1}{E_{2}} \right) (W \cdot \frac{r_{1}r_{2}}{r_{2}^{2}-r_{1}^{2}})^{-1}
\]

Now Meyer's law states that

\[
W = \frac{A}{r_{1}^{n-2}} d^n
\]

where \( A \) is a constant for the metal.

\[
\therefore d^3 = \left( \frac{Wr_{1}^{n-2}}{A} \right)^{\frac{3}{n}}
\]

which is much more complex than the formula for the apparent shallowing. Typically only the apparent shallowing can be treated analytically.

\[
\therefore W^{-1} = k \frac{r_{1}r_{2}}{r_{2}^{2}-r_{1}^{2}}
\]

where \( k = (2.22)^3 \frac{A^{\frac{3}{n}} g}{2r_{1}^{3-n}} \left( \frac{1}{E_{1}} + \frac{1}{E_{2}} \right)\)
Now \( S = \frac{h_0 - h}{h_0} = 1 - \frac{h}{h_0} \).

As the indentation and indenter are spherical

\[ d^2 = 8r_1h_0 = 8r_2h \]

\[ \frac{h}{h_0} = \frac{r_1}{r_2} \]

\[ 1 - \frac{h}{h_0} = 1 - \frac{r_1}{r_2} = S = \frac{K}{3n-1}r_1 \]

Substituting for \( K \)

\[ S = \left[ (2.22)^3 \frac{3}{n} \frac{r_1}{r_2} (\frac{1}{E_1} + \frac{1}{E_2}) \right] \]

where \( x = \frac{3-n}{n} \)

\[ S = xW^{-x} \]

On calculating the true shallowing from this we have

\[ S = AW^{-x} + BW^{-2x} + CW^{-3x} \]

which is much more complex than the formula for the apparent shallowing. Owing to this complexity only the apparent shallowing can be treated analytically.

Theoretically if \( \log S \) is plotted against \( \log W \) a straight line of slope \( \frac{n-3}{n} \) should result, where \( n \) is the
The results already given in Table 9 are plotted in this way Fig. 46. It is possible to draw straight line graphs through the points, but in general two straight lines of different slopes are required for each set of points.

At a particular value of a, the curves indicate a metal increase of curvature modulus to be the theoretical modulus. As the product of the load and the value of a can be taken to imply a change in the value of a, it is well known that above the critical load for full plasticity the value of n is constant, but below this load it increases to the value of 3 for purely elastic deformation. For n = 3, x = 0 and for n = 2, x = 0.5. Thus as the load is reduced through the critical value for full plasticity,
Meyer index. The results already given in Table 9 are plotted in this way in fig. 46. It is possible to draw straight line graphs through the points, but in general two straight lines of different slopes are required for each set of points.

At constant load the value of $S$ will be dependent only on the value of the constant $X$. This in turn depends upon the hardness of the metal, the radius of curvature of the indenter, and the elastic properties of the indenter and metal. From the theoretical treatment $S$ is expected to increase as the hardness of the metal increases, the radius of curvature of the indenter increases or the Young's modulus of the indenter or metal decreases. This is found to be the case experimentally. Thus qualitatively the theoretical treatment is verified.

The variation of shallowing with load is, however, more complex. Basically the theoretical relationship is followed but in the majority of cases $x$ and $X$ have two separate values. As $x$ and $X$ both depend on the value of $n$ the change in their values can be taken to imply a change in the value of $n$. It is well known that above the critical load for full plasticity the value of $n$ is constant, but below this load it increases to the value of 3 for purely elastic deformation. For $n = 3$, $x = 0$ and for $n = 2$, $x = 0.5$. Thus as the load is reduced through the critical value for full plasticity,
x will decrease slowly or zero. This is illustrated in Fig. 47. Above the critical load for full plasticity $W_c$, the slopes of both the $\log d$, $\log W$ and $\log S$, $\log W$ graphs are constant. Below this value of load, however, the value of $n$ increases to 3, and the value of $x$ decreases to be explained on a further graph in fig. 46. The graph is expected to be a straight line.

For the diamond load for full plastic load at which the diagram intersects the horizontal axis $W_c$ is the critical load for full plastic deformation. In the 100 kg. and higher part of the shallowing load with load can be used.

A comparison of the values of the Meyer index $n$ derived from the Meyer analysis, and the shallowing index $x$, is given in Table 10.
x will decrease slowly to zero. This is illustrated in fig. 47. Above the critical load for full plasticity $W_c$ the slopes of both the log. $d$, log. $W$ and log. $S$, log. $W$ graphs are constant. Below this value of load, however, the value of $n$ increases to 3, as shown on the upper graph, and the value of $x$ decreases to zero. The results already described can be explained on this basis.

For the diamond ball indentations in specimen 9 the value of $W_c$, as determined from the Meyer analysis, is 1.2 kg.; that is, it comes just at the left hand side of the graph in fig. 46. The graph of log. $S$ versus log. $W$ is straight but is expected to bend if extended into the lower range of loads. For the diamond ball indentations in specimen 17 the critical load for full plasticity is 50 kg., which is very close to the load at which the two straight line portions of the shallowing diagram intersect. For the $\frac{1}{4}$" steel ball indentations in specimens 8 and 9 the critical load is well above 100 kg. and so the points plotted in fig. 46 represent the curved part of the log. $S$, log. $W$ graph. Thus the variation of shallowing with load can be explained.

A comparison of the values of the Meyer index $n$ derived from the Meyer analysis, and the shallowing index $x$, is given in Table 10.
TABLE 10.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Specimen Number</th>
<th>Indenter</th>
<th>Method</th>
<th>Meyer Analysis</th>
<th>Shallowing</th>
</tr>
</thead>
<tbody>
<tr>
<td>▼</td>
<td>9</td>
<td>Diamond</td>
<td></td>
<td>2.33</td>
<td>2.24</td>
</tr>
<tr>
<td>o</td>
<td>17</td>
<td>Diamond</td>
<td></td>
<td>2.29</td>
<td>2.28</td>
</tr>
<tr>
<td>▼</td>
<td>9</td>
<td>¼'' Steel</td>
<td></td>
<td>2.24</td>
<td>2.15</td>
</tr>
<tr>
<td>□</td>
<td>8</td>
<td>¼'' Steel</td>
<td></td>
<td>2.22</td>
<td>2.43</td>
</tr>
</tbody>
</table>

The agreement is quite good for the specimens whose indentations were made above the critical value for full plasticity, but is not so good for those whose indentations are below the fully plastic range. One reason for this is that below the fully plastic range the recovered ball indentations are not spherical. Evidence of this will be given in the next chapter. As the indentations are not spherical, Hertz's theory and the shallowing equation based on it are bound to break down.
CHAPTER 10.

SHAPE OF BALL INDENTATIONS.

Reference was made in section 3.4 to the measurements of Batson, and Foss and Brumfield, on recovered ball indentations, and the explanation of their spherical shape given by Tabor. All the indentations measured by these workers were large both in depth and diameter. The results of depth measurements of shallow ball indentations were given in chapter 9. The shape of these indentations can also be measured by multiple beam interferometric methods, and in this chapter the results of such measurements will be presented, together with a discussion of their significance.

Fig. 48 is an interferogram of a typical shallow ball indentation. The dark fringes contour the indentation, revealing its shape and depth. In this case the depth is about one hundredth of the diameter. If the diameter of the fringes $d$ is plotted against a sequence of natural numbers $n$, the resulting graph will represent the shape of the indentation. The diameters of the fringes over the indentation shown in fig. 48 are plotted in this way in fig. 49a, where the shape of the indentation is compared with a spherical surface of the same diameter and depth. It is obvious from this figure that the indentation is not spherical, but the deviation is more clearly revealed by plotting the square of the diameter of the fringes against a sequence of natural numbers. This would
Fig. 49.
be a straight line graph for a shallow, spherical, indentation. The diameters of the fringes over the indentation shown in fig. 48 are plotted in this way in fig. 49b, and the deviation from the straight line is quite marked. This curve is typical of many of the shallow ball indentations measured. Diamond and steel ball indentations in materials ranging from Aluminium to Tungsten Carbide, exhibited this effect in some degree.

10.1 METHOD OF MEASUREMENT.

In order to compare quantitatively the extent of the deviation from sphericity of the indentations, some measure of the deviation of a curve from a fixed shape is required. Such a measure is most simply constructed if the reference curve is a straight line. For this reason the non-sphericity of the indentation is defined in terms of the deviation of the $d^2$, $n$ curve from a straight line. In order to specify the extent and shape of the non-sphericity the deviations of the curve from both the tangent at the origin, and the straight line connecting the extreme ends of the $d^2$, $n$ curve, are required. The proposed definitions are illustrated in fig. 50, in which the curve is shown as a full line and the two reference lines are shown broken. The deviation from the tangent at the origin is defined as $C_n = \frac{\Delta n_1}{n_1}$, and the deviation from the straight line connecting the ends of the curve is defined as
The following formula was found to be a very close approximation to the experimental $d^2$, $n$ curves obtained:

$$n = m_1 d^2 - (m_1 - m_2) d^2 \left( \frac{d^2}{D^2} \right)^r$$

where $m_1$ and $m_2$ are the slopes of the two straight reference lines, and $r$ is a constant for each indentation. From this equation it can be shown that

$$c_n = \frac{m_1 - m_2}{m_2}$$

$$c_d = \frac{m_1 - m_2}{m_2} \times \frac{r}{r + 1} \times \frac{1}{(1 + r)^\frac{1}{r}}$$

The ratio of the two measures of non-sphericity is thus

$$x = \frac{c_n}{c_d} = \frac{r + 1}{r} \frac{1}{(1 + r)^\frac{1}{r}}$$

and depends only on $r$. The value of $r$ determines the shape of the curve if the bounding lines are given, i.e. $m_1$ and $m_2$ constant. $d^2$, $n$ curves for fixed values of $m_1$ and $m_2$ and values of $r$ of 1, 2 and 4 are shown in fig. 51, from which it is seen that as $r$ increases the curve remains coincident with
Fig. 52.

Fig. 53.
the tangent at the origin for a greater part of its length; that is, a greater proportion of the centre of the indentation is spherical.

10.2 RESULTS.

Values of $C_n$ and $C_d$ were found for a number of diamond and steel ball indentations in metal specimens of various hardness values, and the values of $r$ were calculated from their ratios. The results are listed in Table 11. The values of $C_n$ and $r$ are plotted against the load, for the indentations listed in Table 11, in figs. 52 and 53. The key to the symbols used to represent the various metals is given in the last column of the Table. The black points on the graphs refer to steel and the white points to diamond ball indentations.

A number of general conclusions may be drawn from these results, which are capable of precise physical interpretation. The main conclusions are as follows:

(1) The value of $C_n$ decreases to zero as the load is increased, for the same material and indenter.

(2) The value of $C_n$ increases with the D.P.H. of the specimen, for the same indenter and load.

(3) The value of $C_n$ is always greater for a steel ball indenter than for a diamond ball indenter of the same radius of curvature, for the same material and load.
<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>D.P.H.</th>
<th>Indenter</th>
<th>Load kg.</th>
<th>$C_n$</th>
<th>$r$</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26</td>
<td>1/4&quot; Steel</td>
<td>1.0</td>
<td>0.067</td>
<td>1.22</td>
<td>▲</td>
</tr>
<tr>
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<td>240</td>
<td>1/4&quot; Steel</td>
<td>5.0</td>
<td>0.376</td>
<td>1.58</td>
<td>▼</td>
</tr>
<tr>
<td>8</td>
<td>260</td>
<td>1/32&quot; Steel</td>
<td>3.0</td>
<td>0.193</td>
<td>1.03</td>
<td>■</td>
</tr>
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<td></td>
<td></td>
<td>Diamond</td>
<td>3.0</td>
<td>0.104</td>
<td>0.85</td>
<td>□</td>
</tr>
<tr>
<td>12</td>
<td>607</td>
<td>1/32&quot; Steel</td>
<td>10</td>
<td>0.281</td>
<td>1.51</td>
<td>▲</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Diamond</td>
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<td>0.161</td>
<td>1.30</td>
<td>△</td>
</tr>
<tr>
<td>17</td>
<td>950</td>
<td>1/32&quot; Steel</td>
<td>20</td>
<td>0.371</td>
<td>2.00</td>
<td>●</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Diamond</td>
<td>10</td>
<td>0.426</td>
<td>0.95</td>
<td>○</td>
</tr>
<tr>
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<td>0.230</td>
<td>1.45</td>
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</tr>
<tr>
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<td>0.092</td>
<td>9.3</td>
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</tr>
<tr>
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<td></td>
<td>80</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>1610</td>
<td>Diamond</td>
<td>20</td>
<td>0.277</td>
<td>1.63</td>
<td>◇</td>
</tr>
</tbody>
</table>

The value of $r$ increases as the load is increased, for the same indenter and load.

The spherical portion at the centre of the indentation increases in size with the load, for the same metal and indenter.

The spherical portion of the indentation decreases in size as the hardness of the specimen is
(4) The value of \( r \) increases as the load is increased, for the same material and indenter.

(5) The value of \( r \) decreases as the D.P.H. of the specimen increases, for the same indenter and load.

(6) The value of \( r \) is always greater for a steel ball indenter than a diamond ball indenter of the same radius of curvature, for the same material and load.

As \( C_n \) measures the extent of the non-sphericity, and the value of \( r \) indicates its shape, the above experimental conclusions are interpreted as follows:-

(1) The non-sphericity of the indentation decreases as the load is increased, for the same indenter and metal, until at a certain load the indentation becomes spherical.

(2) Indentations made with the same load and indenter are less spherical on harder metals.

(3) A steel ball indentation is always less spherical than the corresponding diamond ball indentation.

(4) The spherical portion at the centre of the indentation increases in size with the load, for the same metal and indenter.

(5) The spherical portion of the indentation decreases in size as the hardness of the specimen is
Fig. 54.

Fig. 55.
increased, for the same indenter and load.

(6) The spherical portion of the indentation is larger in size for a steel ball indenter than for a diamond ball indenter of the same radius of curvature, for the same material and load.

An interesting point arises from a comparison of the \( d^2 \), \( n \) curves for the 10, 20 and 40 kg. indentations in specimen 17. These curves are shown superimposed in fig. 54. The growth of the central spherical region and the decrease in the non-sphericity as the load is increased, are clearly revealed. The interesting point, however, is that all the curves have the same tangent at the origin. That is, the radius of curvature of the central part of all three indentations is the same. This has some bearing on the pressure distribution between the indenter and metal, and the growth of the plastic region, and will be referred to again in the next section.

10.3 DISCUSSION.

In view of the effect of the transition to fully plastic deformation on the flow pattern shape and shallowing of ball indentations, it is natural to seek to link the non-sphericity of the indentations with the same phenomenon. The critical load for full plasticity for diamond ball indentations in specimen 17 has already been given in Chapter 7 as 50 kg., and on reference to figs. 52 and 53 it is seen that this is the
load at which the indentations in this specimen assume a spherical shape. All the non-spherical indentations listed in Table 11 were found to have been made below the fully plastic range, as revealed by the Meyer analysis. Thus, recovered ball indentations are only spherical if they are made with loads larger than the critical load for full plasticity.

Tabor (1951 b) proposed the following formula for determination of the critical load for full plasticity $W_c$.

$$W_c = 2610 \frac{Y^2 r^2}{(E_1/E_2)^2}$$

where $Y$ is the yield stress of the metal, $r$ is the radius of curvature of the indenter, and $E_1$ and $E_2$ are the Young's moduli of the indenter and metal. This formula is only valid for work-hardened metals. On the basis of this equation, and the result reported in paragraph (1) above, that as the load is reduced below the critical load for full plasticity the indentations become progressively less spherical, the other experimental conclusions can be explained.

(2) The critical load for full plasticity increases with the hardness, or yield strength, of the metal; hence the constant load becomes progressively less than the critical load and the non-sphericity increases.
(3) As the Young's modulus is increased the critical load decreases and so the non-sphericity is reduced.

(4) The spherical portion of the indentation must be connected with the plastic region as the size of both increases with the load.

(5) Decrease of hardness at constant load is equivalent to increase of load at constant hardness, hence these results are in accord with those in paragraph (4).

(6) The spherical portion of the indentation is larger for a steel than a diamond ball indenter. There is no reason to expect this, or the converse, to be true on the basis of the above treatment.

There are three experimental facts upon which any attempt to construct a pressure distribution between the indenter and metal, below the fully plastic range, must be based. In the fully plastic range the indentation is spherical and the approximate pressure distribution is known; the centre portions of all indentations made below the fully plastic range have the same radius of curvature; as the load is reduced the mean pressure over the indentation decreases.

The pressure distribution for fully plastic deformation is essentially that shown in fig. 55a, as reported in section 3.4. The maximum pressure is \( \frac{3}{2}Y \) where \( Y \) is the yield stress.
of the metal. For an indentation made below the fully plastic range the mean pressure will be smaller. An attempt to construct such a pressure distribution is shown in fig. 55b, where the maximum is taken as 3Y and the shape at the centre is the same as that for fully plastic deformation. The centre of an indentation made under this pressure distribution will be expected to be spherical, the average pressure will be lower than for the distribution shown in fig. 55a, and the non-sphericity will be due to the reduction in value of the pressures with respect to the yield stress of the metal.

was also mentioned, together with its experimental verification by Green and Boulade. However, none of this work gives any definite information as to the type of deformation occurring when a hardness test is made with a conventional indenter near to the edge of the specimen. This deformation is the subject of the present chapter.

11.1 PREPARATION OF MEASUREMENT.

In order to investigate the deformation pattern experimentally, specimens with two flat polished sides at right angles to each other were used. The preparation of these specimens was described in Chapter 4. An indentation was made at right angles to, and near the edge of one face, and the two faces were examined interferometrically. The specimens used were of hard brass and silver metal, numbers 5 and 6 in Table 1. Conical and pyramidal diamond indentors
The purpose of the work reported in this chapter is to investigate the deformation of the specimen which occurs when an indentation is made near an edge. The survey of work on critical specimen size given in section 3.5 included the work of Moore, and Hankins and Aldous, which was concerned with the hardness value as such and not with the deformation produced. Hill's theoretical work on wedge indentation was also mentioned, together with its experimental verification by Green and Dugdale. However, none of this work gives any definite information as to the type of deformation occurring when a hardness test is made with a conventional indenter near to the edge of the specimen. This deformation is the subject of the present chapter.

11.1 METHOD OF MEASUREMENT.

In order to investigate the deformation pattern experimentally, specimens with two flat polished sides at right angles to each other were used. The preparation of these specimens was described in Chapter 4. An indentation was made at right angles to, and near the edge of one face, and the two faces were examined interferometrically. The specimens used were of hard brass and silver steel, numbers 5 and 6 in Table 1. Conical and pyramidal diamond indenters
were used, of which the latter is the better approximation to the infinitely long wedge whose deformation pattern has been studied theoretically. The indentations were made with specified loads at various distances from the edge of the specimen. The distance of the centre of the indentation from the specimen edge, and the diameter of the indentation, were measured with the microscope incorporated in the hardness tester. Interferograms of a typical test are shown in fig. 56. The pyramid indentation, with its sides parallel and perpendicular to the edge, is shown in fig. 56a, and the distortion produced on the side of the specimen is shown in fig. 56b. The deformation of the side of the specimen is in the form of a "hill" whose maximum height is at the specimen edge and opposite the centre of the indentation. The central section of the hill along the line CD is shown in fig. 57; this was compiled from the shape of the surface before and after indentation and so represents the distortion caused by the indentation and not the final shape of the surface. Its rectilinear shape and large depth, in comparison with the depth of the indentation, are of interest. The section of the hill along the line AB is shown in fig. 58, where it is compared with the width of the indentation. The extent of the hill is about equal to the total extent of the flow pattern around the indentation.
Fig. 59.
From sections similar to that shown in fig. 57 the angle $\Theta$ and maximum height of flow $h$ may be found for a series of indentations and plotted against the distance of the centre of the indentation from the edge of the specimen. If the width of the indentation along a line perpendicular to the edge of the specimen is $a$, and the perpendicular distance of the centre of the indentation from the edge of the specimen is $x$, the value of $\frac{x}{a}$ will determine the distortion caused by the indentation. Geometrical considerations indicate that for a constant value of $\frac{x}{a}$ the angle $\Theta$ should be constant and the maximum height $h$ proportional to $x$.

11.2 RESULTS.

Each series of tests was made at constant load, i.e. constant $a$, for differing values of $x$, and the values of $\Theta$, $h$ and $DE$ were measured for each test. The results are listed in Table 12 and the values of $\Theta$ and $h$ are plotted against $\frac{x}{a}$ in fig. 59. The accuracy of detection of a distortion is about half a wavelength $\lambda$, and the accuracy of measurement of distortions is about a tenth of a wavelength. The wavelength of the light employed in all the present work was 0.546 microns. The graphs shown in fig. 59 reveal the rapid increase in $\Theta$ and $h$ as the value of $\frac{x}{a}$ is reduced. The critical width for pyramid indentations in brass, i.e. the value of $\frac{x}{a}$ for which $h = 0$, is about twice that for steel.
TABLE 12.

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Metal</th>
<th>Indenter</th>
<th>Load (kg)</th>
<th>$x$</th>
<th>$\theta$ (minutes)</th>
<th>$h$</th>
<th>DE (microns)</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Brass</td>
<td>Cone</td>
<td>10</td>
<td>1.31</td>
<td>57</td>
<td>31.6</td>
<td>526</td>
<td>△</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>2.09</td>
<td>8.0</td>
<td>5.2</td>
<td>620</td>
<td></td>
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<tr>
<td>5</td>
<td>Brass</td>
<td>Pyramid</td>
<td>10</td>
<td>1.60</td>
<td>35</td>
<td>18.7</td>
<td>498</td>
<td>□</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>2.13</td>
<td>12.7</td>
<td>6.23</td>
<td>462</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>2.93</td>
<td>3.4</td>
<td>2.53</td>
<td>698</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>4.06</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Steel</td>
<td>Pyramid</td>
<td>5</td>
<td>1.29</td>
<td>37</td>
<td>6.65</td>
<td>168</td>
<td>♠</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>1.38</td>
<td>20</td>
<td>3.32</td>
<td>158</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Steel</td>
<td>Pyramid</td>
<td>10</td>
<td>0.95</td>
<td>129</td>
<td>37.3</td>
<td>270</td>
<td>□</td>
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<tr>
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<td>10</td>
<td>1.32</td>
<td>32</td>
<td>8.5</td>
<td>250</td>
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<td></td>
<td>10</td>
<td>1.68</td>
<td>10.5</td>
<td>3.68</td>
<td>321</td>
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<td>10</td>
<td>1.81</td>
<td>12.5</td>
<td>3.98</td>
<td>300</td>
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<td></td>
<td></td>
<td>10</td>
<td>2.27</td>
<td>-</td>
<td>-</td>
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</tbody>
</table>
For brass the values of \( h \) and \( \theta \) are smaller for the cone than the pyramid indentations, for the same value of \( \frac{x}{a} \). This is understandable, as for a cone indentation only one point is at a distance of \( x - \frac{a}{2} \) from the specimen edge, whereas for a pyramid indentation the whole of the side is at this distance. Thus for a given value of \( \frac{x}{a} \), a pyramid indentation will be expected to cause a larger distortion than a cone indentation. The values of \( h \) for the 5 kg. indentations in steel are less than those for the 10 kg. indentations as expected; and if multiplied by \( \sqrt{2} \) they fall close to the 10 kg. curve. The values of \( \theta \) for all the pyramid indentations in steel lie on a single curve. In all cases the values of \( h \) measured were very small, the maximum being some 12 microns. As there is no significant hardness variation associated with the tests it may be concluded that the distortion is quite appreciable before any substantial variation in hardness becomes apparent.

An interesting result arises from an examination of the surface distortion around the indentation. In fig. 56 a the square shaped indentation is surrounded by a rather irregular flow pattern. The straight fringes between the indentation and the edge of the specimen are much closer together than they are at the same distance from the edge, at some distance from the indentation. This indicates a tilt in the surface between the indentation and the specimen edge. This tilt was present in all the tests reported. The angle of tilt
may readily be measured by means of the fringes, and this was done for a number of the indentations reported above. In every case the angle of tilt was equal to the angle $\theta$ of the hill on the side of the specimen. This means that the block of metal between the indentation and the edge of the specimen has rotated as a solid body. The depth of the block is some 20 times the depth of the indentation and its width and length are of the order of the diameter of the indentation.

This mode of deformation closely resembles that given by Hill (1950) for an infinitely long wedge indenter indenting an ideal plastic-rigid body. The diagram from Hill's paper is given in fig. 18 and has already been briefly referred to in section 3.5. The sets of orthogonal lines ADEPC, BDFG, etc., are plastic slip lines. They are lines of maximum shear stress and their shape defines both the pressure distribution and the mode of deformation. Hill states, "At a certain penetration the plastic region (whose conjectured boundary is indicated by broken curves in fig. 18) just contains a complete slip line HJKLM, extending from the axis of symmetry to the lateral surface. Up to this moment the plastic material below the indenter has been rigidly constrained. After this moment the rigid corners of the specimen are assumed to be displaced sideways by sliding over the slip line KLM." He goes on to discuss in detail the deformation for a knife edge, a flat die, and a wedge of semi-angle about 50°.
He finds for the wedge that the critical width is about 4.5 times the width of the impression and that M is 2.14 H below the corner, where H is the distance AB in fig. 18. He concludes by pointing out the difference between the two and three dimensional problems and states that in general the critical distances will be smaller in the latter case owing to the greater freedom of flow around a three dimensional indentation.

Even though the resemblance between the conditions assumed by Hill and those which apply in the practical case is slight, the mode of deformation in both cases is essentially the same. This similarity is striking in view of the work-hardening properties of the specimens, and the difference between the conditions for two and three dimensional plastic flow. This shows that the theoretical analyses of hardness testing problems do give results which are applicable to practical testing, even though the assumptions involved would be expected to restrict the application considerably.
CHAPTER 12.

SUMMARY OF RESULTS.

The results in the last five chapters have all been discussed in relation to other theoretical and practical work and it only remains in this chapter to link the results together and give an overall picture of the fresh information reported. Four main points have been covered. The surface distortions accompanying all stages of ball indentation have been fully described; the variation of flow pattern shape has been studied and the transition from ridging to sinking illustrated; the presence of residual stresses beneath the indentations has been discussed; and the mode of deformation when an indentation is made near the specimen edge has been measured. These results are all of interest in the quest for a better understanding of the hardness test, and the first topic, which is mentioned in four of the chapters of results, calls for special mention here.

The process of ball indentation may conveniently be split into four sections, namely, initiation of plastic deformation, partial plasticity, critical load and full plasticity.

The initiation of plastic deformation is characterised by very shallow, highly non-spherical indentations. They have no associated flow pattern and from their shape it would appear that plastic deformation does commence below the
centre of the indentation and not at the edge as has sometimes
been supposed. Thus the critical load marks the attainment
of a certain shape by the pressure distribution and this is
accounted for by the onset of a new type of deformation
mechanism namely, fully plastic deformation.

The partial plastic range is characterised by indentations
whose centres are spherical and of a constant radius of
curvature, for the same metal and indenter. The flow
patterns associated with the indentations spread, relative to
the indentation diameter, with increase in load, and their
volumes increase slowly, though they remain considerably
smaller than those of the corresponding indentations.

The critical load, which is about 200 times the load for
the initiation of plastic flow, can be defined in terms of the
impression shape with a precision which was lacking in its
original definition in terms of mean indentation pressure (see
section 2.2). It is the load at which the extent of the flow
pattern, relative to the indentation diameter, is a maximum.
Also at this load the indentations become spherical and begin
to obey Hertz's law of elastic recovery. In accordance with
current opinion, in the present work the critical load for
full plasticity has been identified with the critical load for
the validity of Meyer's law. In view of its effect on the
other properties mentioned above, the distinction between full
and partial plasticity is seen to be a very real one affecting
all aspects of the hardness test, and not only the mean pressure.

Above the critical load the shape of the pressure distribution
is constant, as shown by the closely spherical recovered
indentations, but below it the pressure distribution varies with the load. Thus the critical load marks the attainment of a certain shape by the pressure distribution and this is accounted for by the onset of a new type of deformation mechanism namely, fully plastic deformation.

The region of full plasticity is characterized by spherical indentations that obey Hertz's law of elastic recovery. The extent of the flow patterns decreases with increase of load, and their volumes slowly approach those of the associated indentations.

Briefly summarising, the present work has verified, and somewhat extended, Tabor's theory of ball indentation; it has shown that the work-hardening capacity of the metal, rather than the shape of the indenter, determines the flow pattern shape; and has shown that the method of treating the indentation process as a problem in plasticity is sound.
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