APPLICATION OF GRIBOV CALCULUS
TO TWO-BODY PROCESSES

by

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ABSTRACT

A new model for two-body high energy scattering is presented as part of an investigation into the phenomenology of the non planar structure of Reggeon-particle scattering. The model is a modification of the weak cut reggeized for Pion-Nucleon scattering and is developed in form of a correlation modified quasi eikonal where the Reggeon number of Pomeron are allowed to change the projection of the nucleon spin. A correlation parameter - the ' - has its origin in Gribov's theory, provides an indication about the failure of the traditional weak cut reggeized and restores its most profound shortcoming - the prediction of an incorrect phase behaviour of the helicity isovector amplitude in the reaction $\pi^+p \rightarrow \pi^0\Lambda$ - while retaining the model's attractive simplicity. The vertical Reggeon-calculus depend in general on the angle between the momenta of the exchanged reggepoles. By parametric dependence we take into account the effective contribution of inelastic intermediate states in the unitarity expansion Regge-particle scattering amplitude. We obtain a reasonable phase energy description of the isovector amplitude, demonstrate in detail the mechanism by which the correct phase behaviour is restored. The spin-structure of is investigated and observables of $\pi N$ scattering between 6 and 200 GeV/c within a range of momentum transfers are being produced.
PREFACE

The work described in this Thesis was carried out under the supervision of Dr. K. J. M. Moriarty in the Mathematics, Royal Holloway College, between October 1973 and September 1976.

The material presented in the text is original except in so far as explicit reference is made to the work been submitted for a Degree in this, or any other, University.

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I. INTRODUCTION

There is overwhelming evidence (1) that the simple power law energy behaviour predicted by the Regge formula at energies is a good approximation to the real world of two body scattering. A particularly striking example of this has been provided by the recent data from Fermilab (2) on the Pion Nucleon charge exchange reaction $\pi^-p \rightarrow \pi^-n$ at 20 and 200 GeV/c. The effective J-plane singularity moves on an only slightly curved trajectory which approximates the straight line excellently. The data, however, show a small bump at the nonsense wrong signature zero (NWSZ) around $t/t_{max}$ and the slight deviation from a straight line further out in $t/t_{max}$ are the mild reflections of correction terms to pole exchange. These corrections arise as cuts in the complex angular momentum plane (3). They are treated by means of a version of the absorption model (4). Within such an approach one understands that, in the case of the relative size of the helicity nonflip compared to the pole needs to be particularly large to account for the part of the helicity nonflip isovector amplitude. In addition, this zero has to occur before the zero of the relative size of the helicity nonflip isovector amplitude with sufficient separation so as to account for the correct phase structure which is manifest in the phase of the recoil nucleon in $\pi^-p \rightarrow \pi^-n$ (5). None of the traditional absorption models can account for the features of the data.

Cuts have, due to their smaller slope ($\alpha'_\text{cut} < \alpha'_\text{pole}$) the tendency to "take over" further out in $t/t_{max}$ and allow for a (Lack of shrinkage). The mild deviation of the effective trajectory from a straight line constitutes, therefore, a relatively weak rise and fall of the symmetric part of the elastic polarization (6) in $\pi N$ scattering seems to imply a rather key role in the determination of elastic as well as inelastic polarization of $\pi N$ scattering is played by the phase of the phase is the major source of uncertainty in an otherwise model-independent amplitude analysis. Thus the key role is to obtain a self consistent description of the $\pi N$ system by constructing simultaneously the isoscalar amplitude.

$\#_{\text{cut}}$
The elastic amplitude enters the absorption model in form of strong dominantly elastic rescattering in the initial state. This is supposed to effectively take account of the many competing inelastic channels which open up "head on collisions". The elastic amplitude replaces, in the high energy approximation of the absorption model, and final state wave functions of a complex optical potential whose imaginary part is meant to simulate the competing channels. This high energy version of the distorted-wave Born approximation of low energy nuclear absorption and final state wave functions of a complex optical potential whose imaginary part is meant to simulate the competing channels. This high energy version of the distorted-wave Born approximation of low energy nuclear absorption by Sopkovich (7) and has been applied successfully to correct the too exaggerated peak close to $f$ and the too slow fall-off with momentum transfer predicted by the peripheral or one-particle exchange model production of quasi-two-body reactions. (8). Note that in the OPE model the couplings are constant and these factors for the couplings in the Born approximation exactly simulate, at fixed energies, the effects of absorption.

Traditionally the elastic amplitude has been taken from experiment, equal in initial and final states, diagonal imaginary with no /t/ dependent phase and has been parameterized by a Gaussian. This produces the desired appearance as a grey absorbing disc reducing the lower partial waves and leaving the higher ones unaffected, very sharply the forward peak of the /t/ distribution.

Absorption and reggeization of the OPE model has led the Imperial College group to construct, in connection with the strong exchange degeneracy and couplings which are determined by a higher symmetry scheme: a very crude parameter-free reggeized weak cut absorption model for Meson-Baryon scattering with considerable predications. (4). Although this model has failed to predict the correct phase behaviour of the isovector helicity NWSZ input of the basic exchange seems to be strongly supported by the FNAL data.

The weak cut absorption originally proposed by Cohen-Tannoudji et al. and Arnold et al. (4) has qualitatively correct polarization in $^{3}\pi^p \rightarrow ^3\pi^p$. Its quantitative prediction, however, is drastically wrong. The reason for this is the wrong strength and wrong phase of the cut amplitude. Absorption is essentially a convolution in momentum transfer, the NWSZ input pole changes sign in the region of integration there will be a cancellation in the integral and the amplitude will tend to be small. It is in fact too small to obtain the zero of the imaginary part of the isovector amplitude which is much further inwards in /t/ than predicted by the pure NWSZ pole. If one were to increase the strength factor to pull the zero further in, this would completely destroy the already displaced dip position.
a zero in the imaginary part implies, due to the wrong cut phase, a nearby zero of the real part. But to polarization of \( \pi^+ p \rightarrow \pi^+ n \) the zero of the imaginary part has to occur before the real one and both sufficiently enough apart.

The lack of strength of the weak cut model in other reactions such as \( \gamma p \rightarrow \pi^+ n \) and \( \eta p \rightarrow \pi^0 \) is also theoretical side, the intuitive basis of the Sopkovich formula has lead to serious doubts. This concerns, truncation of rescattering in the initial and final states to on-mass shell states. The weak absorption model the contribution of inelastic diffractive intermediate states. These intermediate states are, however, a c\( s\)-channel unitarity. Furthermore, the very existence of cuts is due to the presence of the third order double spectral function. These functions cause fixed pole singularities at wrong signature points for the partial wave and cannot be made to vanish by a superconvergence relation. These fixed poles would, due to unitarity, be essential singularity violating the Froissart bound were it not for cuts specially invoked for these reasons prevent this happening. In the presence of the third order double spectral function it is not unlikely that the occurring fixed poles are strong and enter the Regge residues multiplicatively. They then would cancel the unstructured pole amplitude.

In combining both points of view, namely the necessity to incorporate the inelastic diffractive intermediate factor \( \lambda \) and the absence of NWSZ in the input pole, the Michigan group Henney et al (4) have constructed absorption model and also successfully fitted a great amount of data. In particular the dip structure of the \( \eta p \rightarrow \pi^0 \) is now being produced by pole-cut interference. The polarization, however, has been equally as wrongly present cut model. The relatively large strength of the Michigan cut actually once produced the crossover position paid for with over absorption, which could be restored by eikonalization (9), while losing the crossover position.

The failure of traditional absorption-irrespective whether weak or strong - to account for positive polarization (both versions result in an approximately 90% negative peak) has stimulated a great number of successful previous work and without NWSZ input poles (10). Despite their different appearance they all have one factor in common: completely ad hoc and consists essentially in a broadening of the J-plane discontinuity due to the addition of J-plane singularity, a circumstance which led these models into strong conflict with duality. (11c) Duality property of two body amplitudes, is definitely a property of \( \phi \) exchange. The zero of the imaginary part...
amplitude already occurs in the lower energy resonances. All phase modified model amplitudes when FESR compare wrongly in their \( |t| \) dependence with the phase shift FESR integral. Ironically the only alternative scheme - Barger and Phillips' \( g + g' \) pole model (12) - has predicted positive \( \pi N \) inelastic polarization symmetry of elastic \( \pi N \) polarization, though not the double zero. It is in excellent agreement (13) with the experimental data and in addition is compatible with FESR and local-average duality. (11 a)

Polarization generated by two different trajectories, however, changes rapidly with a fixed power of the energy. In the case of a cut this power is proportional to the momentum transfer consequently resulting in a mild \( |\varphi| \) within the range of the diffraction cone. The \( g + g' \) model, in fact, changes the shape of the polarization that already at 18 GeV/c it shows the tendency to approach the unwanted shape of the absorption model. (9) The resonances have yet been identified along the \( g' \) trajectory causes further doubt about the validity of such although it seems to serve as a surprisingly good parameterization.

The circumstance that at an early stage of the development of the absorption model the introduction of strong factors into the Born term of the OPE model could simulate exactly the effect of absorption shows how sensitive empirical factors in order to suit a first intuitive guess. On the theoretical side there is the necessity to include absorption model contributions from inelastic diffractive intermediate states; on the phenomenological side, reggeized absorption model is too weak and has a wrong phase and energy behaviour. A source of strength available by the Michigan approach through a vague enhancement coefficient in connection with an unfavourable canceling the NWSZ in the Regge pole amplitude. All phase modified models have taught us that the imaginary helicity nonflip isovector exchange should be more absorbed than the real part. This, as we saw, however, with duality. The FNAL data on the other hand suggest that the energy dependence of cuts should be rather

The result of a closer examination into the origin of the phase problem may be summarized in the two major
traditional absorption cut:

1. the relative cut pole phase is too close to \( 180^\circ \) at \( |t| = 0 \)
2. although the absorptive cut rotates with increasing \( |t| \) away from the pole, it does so too slow by comparison with the fast following pole. Thus, the pole catches up with the cut at the critical
phase difference of 180° already at very small /t/ and there it causes the polarization to change its sign from positive to negative.

As the phenomenological investigation performed in this thesis has shown, it is possible to obtain a realistic description of \( \pi^-p \rightarrow \pi^-n \) including the elastic polarization of \( \pi^-p \) scattering by simulating the inelastic intermediate states in the following way:

1. The size of the effective interaction region of the Reggeon involved in the cut has to shrink considerably smaller value in comparison with the size of the Reggeon in the Born term. The same holds for the elastic amplitude.

2. In addition, the elastic amplitude has to have a /t/ dependent phase.

3. At last there is an overall damping form factor which renormalizes the shape of the cut.

The combined effect of this prescription on the phase - while keeping the attractive NWSZ - is that:

1. The cut is strengthened in forward direction.

2. The initial phase angle at /t/ = 0 has been rotated in anti-clockwise direction resulting in a purely imaginary cut term.

3. The cut's traditional slow rotation becomes accelerated.

The new cut is now able to reverse the sense of rotation of the helicity nonflip pole so strongly that the part of the amplitude moves into the vicinity of the actual crossover position.

When extrapolating the amplitudes to FNAL energies one can stabilize the helicity nonflip phase angle with considerably stable polarization over a wide range in energy. By doing this the logarithmic energy denominator vanishes. This promises agreement with duality. In fact, the phase modification has not been

1. by broadening the \( \Im \)-plane singularity structure but by enhancing the ratio between the slope of the \( ^* \)Pomeron and the effective interaction size which results in a larger phase and greater strength of the cut. The t
problems further out in $\frac{1}{t}$ and for large energies remain, however, causing a slower fall off of the differential cross section and a spread between both effective trajectories at 6 and 200 GeV/c. Thanks to the NWSZ pole, however, the discrepancy is not so severe as it otherwise would be.

This procedure of modifying the cut sounds as ad hoc as many modification attempts of the past. But a great deal of it disappears if we see it in connection with Gribov's theory, from which it emerges quite naturally. Gribov's theory, as the most general frame for the whole strong interaction physics, is based on the assumption of the absence of singular forces. This convergence property in conjunction with analyticity led to Gribov's Reggeon diagram technique (14).

The fundamental postulate in connection with multiperipheral kinematics which governs the multiparticle production of two-body reactions is that the two-body amplitude through unitarity, generated a two-dimensional field theory of high energy and subtransfer reactions.

The following picture of two body scattering emerged: During the scattering process the two colliding hadrons act as Reggeons to produce and absorb respectively the energy momentum of Reggeons in form of quasi particles. The Reggeons, each other while they diffuse over one time and two space dimensions. The sum over all such interaction diagrams is the non-relativistic theory for asymptotic energies. Each diagram corresponds to a term in a Rayleigh-Schrödinger expansion of the amplitude in powers of $(\ln s)^{-1}$. The rules for evaluating the diagrams are reminiscent of Feynman rules.

Compositeness enters the theory - emphasising its multiperipheral origin - via unitarity when the colliding hadrons are constituents in a cascade of decays. The interaction of the corresponding constituents from both hadrons causes the production of multiperipheral showers which correspond in turn to the exchanged reggeons. For clarification of Gribov's Reggeon diagram technique (14) can indeed serve as a powerful tool in practical calculations for vacuum and number exchange at attainable energies. The vertices of the theory are, however, unknown and are expected not to be because of their non-planar nature. These vertices describe transitions between external particles and reggeons, and reggeons. The diagrams of the expansion of the scattering amplitude have been systematically computed by Tei selecting the importance of their contributions which have been determined according to powers of $(\ln s)^{-1}$. 
The phenomenology of two-body reactions investigated by means of unenhanced diagrams has been performed by Ter-Martirosy an and collaborators. (18) A Gaussian model of the two particle → several reggeon transitions is allowed for a closed eikonal expression of the scattering amplitude. A more general exponential parameterization, however, though considered by Ter-Martirosyan has not yet been carried out.

The success with which Gribov's reggeon-diagram technique has been applied in the eikonal approximation to the system is very impressive. The region of applicability in /t/ of the model, however, is limited to the diffraction region, which is manifest in its failure to compare with the experimental data in the following three cases:

1. Inelastic polarization of $\pi^- p \rightarrow \pi^- p$
   - The eikonal or optical approximation which has no extra parameter than the one as already introduced and Regge pole (without NWSZ) generates at 11 GeV/c a negative 65% spike around /t/ = 0.6, whereas compatible with vanishing polarization. This large negative spike is also characteristic for all absorption models, irrespective of whether they are weak or strong. Glebov et al. (19) include in the calculations however the $\rho \otimes P'$ cut and give the Pomeron pole a slope of $\alpha' = 0.6$. A large negative spike is obtained in this way helps to improve the polarization (see also (19)) such that it does not change sign before /t/ \approx 0.4 and agrees with the data for /t/ \approx 0.4. The contribution of the $\rho \otimes P'$ however decreases rapidly with increasing energy and the model predicts for 200 GeV/c a very small negative spike up to /t/ \approx 0.2. The polarization changes sign and drops rapidly to its 90% spike at 200 GeV/c, in contrast to what the data at low energy lead us to expect.

2. Elastic polarization of $\pi^0 p \rightarrow \pi^0 p$
   - The model fails to produce the double zero. The polarization changes sign instead and grows in magnitude for /t/ > 0.6.

3. Elastic and inelastic differential cross section
   - Both cross sections are too small in magnitude for /t/ > 0.6 in particular the elastic one, which is minimum at /t/ \approx 1.2 in contrast to the data, and the inelastic underestimates the dip at /t/ \approx 0.6.

In addition, the growth of total cross section at Serphukov energies was not predicted by the optical model.

A correct description of total cross section for various two-body processes at Serphukov energies was achieved (20) with modified eikonal approximation by taking account of the formation of particle beams in the interaction process.
such shower corrections to the eikonal approximation, the so-called quasi-eikonal model\(^*\) \(^{(21)}\) could occur in energy, while the "old problems" in /t/ of \(\pi N\) scattering still remain.

A close analysis by Eremyan \(^{(22)}\) of the structure of the \(\pi N\) scattering amplitudes demonstrated the importance of a correct description of both helicity amplitudes of the isovector exchange within the frame of the quasi-eikonal modification of the quasi-eikonal model, \(^{**}\) Eremyan \(^{(23)}\) has given a successful description of the \(\pi N\) system range up to /t/ = 2.00 (GeV/c)\(^2\). The modified quasi-eikonal model introduces in particular a slow /t/ dependence of shower enhancement coefficients in connection with a more complex parameterization of the P, P', and G

The present thesis describes a further modification of the quasi-eikonal model by giving the Gribov vertex an exponential parameterization, as suggested by Ter-Makrosyan \(^{(16,18)}\). This then corresponds to the prescription for the modified absorption cut and takes into account the effective contribution of inelastic interaction of the system. The optical approximation treats multiple scattering as the independent re-scattering of the colliding hadrons of structure. We, however, show that this independence is responsible for the poor description of the phase isovector amplitude. Gribov's analysis of the Mandelstam cut implies that the diffractively dissociated hadrons remain mutually correlated in the exchange of the reggeons. The vertices are non-planar. They allow for the temporary association of the reggeons and depend on the angle between the momenta of the reggeons exchanged by the constituents.

A Gribov vertex in the second order unenhanced diagram corresponds to the t-channel partial wave amplitude of the particle-particle→reggeon→reggeon and is the residue of the fixed pole at the first nonsense wrong signature putting the reggeons on their mass shell. There exists a super-convergent sum rule through which the vertices are given as a contour-integral over the corresponding absorptive part of the off-mass-shell particle-reggeon scattering amplitudes, which are appropriate sub energy plane.

The first approximation of the reggeon-particle scattering amplitude by its Born term establishes the link to the eikonalized absorption model. Indeed, the fits to the pion-nucleon system produced by the first generation of the reggeon-diagram technique to two-body phenomenology are very similar to those of the eikonal model.

\(^*\) QEM

\(^{**}\) MQEM
As we see Eikonal/Absorption follows as a special case from Gribov's Reggeon diagram technique where Gribov hypothesis is realised, when one replaces the coupling constants by two body form factors which allow for the breakup of the colliding particle. This produces a cut off preventing large momenta from travelling across the dia
dratic limit eikonal and Reggeon calculus correspond to the same physical picture of colliding hadrons which scatter each other's internal structure. This amounts formally to Glauber scattering of nuclei (24). This formal approach suggests an extension of the eikonal model: here the nth order terms of a Glauber expansion of the scattering is taken between nuclei (composite hadrons) in powers of the nucleon-nucleon (quark-quark) scattering amplitude factors are characterized by internal wave functions and correspond to a kernel introduced into the nth order. The nucleon-nucleon (quark-quark) scattering amplitude corresponds to the Reggeon propagators. Formally, physics one could regard such an extended eikonal as the multiple scattering expansion of an optical potential of a nuclear correlation function. (25) Whereas in first order the target remains in its ground state directly nuclear density next to the elementary nucleon-nucleon scattering amplitude, in higher order it experiences r excitations. When in higher order the target has been lifted out of its ground state after the first encounter, establishes the connection with the final scattering in that particular higher order and allows for the nucleus state. The closure as the sum over all excited nuclear states is related to the many body correlation function form factor.

The correspondence between Gribov and q-number Glauber scattering has been stressed by Lovelace (26) and future phenomenology. We have listed the ingredients of Gribov's multiple scattering expansion and related different approaches and corresponding terms from nuclei scattering.

<table>
<thead>
<tr>
<th>Particles</th>
<th>Nuclei</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Hadron</td>
<td>Nucleus</td>
</tr>
<tr>
<td>(2) Constituent e.g. quark, parton</td>
<td>Nucleon</td>
</tr>
<tr>
<td>(3) Gribov vertex i.e. impact factor (27), fixed pole residue, Veneziano fixed pole residue (26), Dispersion (superconvergent) sum rule of absorptive part of Regge-particle scattering (28), in connection with Gribov-Migdal reggeon unitarity condition (29), Feynman integral over Mandelstam cross (14)</td>
<td>Generalized form factor</td>
</tr>
</tbody>
</table>
|                       | Fourier transform of int dispos
| (4) Reggeon propagator | Hadron viewed as non-re bound state and due to co
|                       | sum over the infinite num st
|                       | partition functions of two dimensional Coulomb gases (30) | Nucleon-nucleon scatteri

- 9 -
Our phenomenological purposes are to take into account the effective contribution of the inelastic intermediate states, the wrong phase of the cut predicted by the absorption model. We therefore include the mutual orientation of the reggeons by relating the Gribov vertex to the hadronic quark-antiquark oscillator model of Pagnamenta (30). We do this by expressing the Gribov vertex product as a correlation kernel parameter representation. The correlation parameter - the "Gribov c" - corresponds, if interpreted in the qQ model or fluctuation length, which gives the average separation of the quarks. We have, however, found that phase suitably and produce the polarization of \( \Pi^{-}p \rightarrow \Pi^{0}n \) the Gribov c becomes persistently a negative number which obscures the relation to the oscillator length. This implies that when Reggeon and Pomeron poles are being cut their range is shrinking by comparison to the single exchanged poles. The shrinking of range is then the phase of the helicity nonflip isovector amplitude: it reverses the sense of rotation around the origin of the diagram. There are, on the other hand, two essentially different models which actually produce an excess of such a pole at the crossover position. The other chooses a large range of c. For both models the c has, to a greater or lesser extent, only a cosmetic effect in the sense that it forces agreement but it is not responsible for the change of the phase. For those models c is positive, however: by the oscillator length. For this reason we do not interpret c. We merely state that it is the parameter effective contribution from inelastic intermediate states. Any analogy to nuclear physics can only help parameterization for the Gribov vertices. It will, therefore, come as no surprise that the totally different world leads to parameter values which are in contradiction to their non-relativistic counterparts.
PART ONE

THEORETICAL DISCUSSION
II - THE REGGEON AND POMERON CONTRIBUTION TO THE S-CHANNEL HELICITY AMPLITUDE
OF THE PROCESS $A + C \rightarrow B + D$

When studying hadronic interactions at high energies with hadrons made up of constituents one chooses approximate momentum frame IMF (31). In such a frame of reference the hadron moves with nearly the speed of light and effect freezes the internal motion of the constituents. We therefore decompose (28) the four momenta of the process $A + C \rightarrow B + D$ into longitudinal and transverse components

$$\rho_{\lambda} = \rho_{\lambda}^{\parallel} + \rho_{\lambda}^{\perp}$$

where

$$\rho_{\lambda}^{\parallel} = \left( \frac{E_{A}}{t}, 0, 0, \rho_{\lambda} \right)$$

and

$$\rho_{\lambda}^{\perp} = \left( 0, \rho_{\lambda}, \rho_{\lambda}, 0 \right)$$

The longitudinal components are large

$$\rho_{\lambda}^{\parallel} \approx \frac{1}{2} \sqrt{s} \rightarrow \infty$$

and the transverse components are of the order of

The four momentum transfer is

$$t = \sqrt{s} \left( p_{A} - p_{B} \right)^{2} = q^{2}$$

In the IMF one has

$$q^{2} = t = -k^{2}$$

with the Reggeon momenta $k$

in the exchange diagram of fig. 1
is two dimensional and is given by the sum of the transverse components of the initial and final three momenta

\[ p_n^T = C_p_n^T (p_{r1}) \]

This situation is demonstrated in fig. 2

\[ p \rightarrow A, B, C, D \]

The relation between the relativistic invariant scattering amplitude and the centre of mass scattering amplitude is given by:

\[ M(c.m.) = \frac{\gamma}{\epsilon} \mathcal{M}(\xi, \phi) \]

\( a \) Scattering of \( A + C \rightarrow B + D \) in the infinite momentum frame.

\( b \) One can always put the azimuthal angle \( \phi \) equal to zero such that the scattering takes place in the plane.

The relation between the relativistic invariant scattering amplitude and the centre of mass scattering amplitude is given by:

\[ M(c.m.) = \frac{\gamma}{\epsilon} \mathcal{M}(\xi, \phi) \]
with $p_i$ and $p_f^\perp$ the three-dimensional initial and final momentum, $E$ the total energy and $\Theta$ the solid angle in the c.m. system, $s_1$ and $s_2$ are the spins of the colliding particles.

The differential cross section in the c.m. system reads then

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2s_1 + 1)(2s_2 + 1)} \frac{1}{p_i^2} |M(c, \Theta)|^2$$

with $\Omega$ the solid angle in the c.m. system, $s_1$ and $s_2$ are the spins of the colliding particles.

Replacing the c.m. scattering amplitude by the invariant amplitude we obtain the invariant differential cross section

$$\frac{d\sigma}{d^2t} = \frac{s}{p_i^2} \frac{d\sigma}{d\Omega} = \frac{1}{(2s_1 + 1)(2s_2 + 1)} |M(c, t)|^2$$

Our normalization factor $N$ is consequently $N = 1$ whereupon the optical theorem

$$\sigma_{tot} = C(\alpha \mu V)^{\frac{1}{2}} J_m^0 M(c, t = 0)$$

reads

$$\sigma_{tot} = \frac{4\pi}{\gamma} J_m^0 M(c, t = 0) \frac{\sqrt{m_b}}{\text{GeV}/c}$$
We have used the isospin decomposition for $\pi N$ scattering

\[ H\left(\pi \rightarrow \pi^+ \rightarrow \pi^0\right) = \begin{pmatrix} 0 \text{M} \end{pmatrix} + \begin{pmatrix} \text{M} \end{pmatrix} \]

\[ H\left(\pi \rightarrow \pi^+ \rightarrow \pi^0\right) = \sqrt{2} \begin{pmatrix} \text{M} \end{pmatrix} \]

with the index 0 denoting isoscalar and 1 isovector exchange.

The formalism will enable us to construct for the $\pi N$ system in principal the isoscalar amplitude by

\[ 0H = P + P' + P\sigma P + P'\sigma P + P\sigma P' + P'\sigma P + P\sigma P + P'\sigma P \ldots \]

and the isovector amplitude by

\[ 1H = \sigma + \sigma P + \sigma P' + \sigma P \sigma P + \sigma P \sigma P \ldots \]

where $P$, $P'$, and $P$ denote Pomeron, P' and P Regge poles respectively. $\otimes$ symbolises the cut theory Regge-Regge cuts should in case of $\pi \rightarrow \pi^0$ be cancelled due to Finkelstein's selection rules.
We denote a general isospin s-channel helicity amplitude by

\[ m(p^j) \]

with the helicities \( \lambda^i \) of the external particles \( j = A, B, C, D \). \( \Delta \) is the total momentum transfer \( \Delta = \Delta' \).

Further we have

\[ \Delta^2 = \sum_{j=1}^{\text{#}} k_j^2 \gamma_j = -t = |t| \]

and

\[ k_j^2 = -t_j = |t_j| \]

\( m(p^j) \) indicates the overall net helicity. It is a function of the net helicities from the contribution of the individual exchanges \( \mu^j \) with \( \mu = |\lambda^i - \lambda^j| \) with \( \lambda^i = \lambda_A - \lambda_B \) and \( \lambda^j = \lambda_C - \lambda_D \).

\( \mu_j = 0 \) indicates that the \( j \)th Reggeon has net helicity nonflip

\( \mu_j = 1 \) net helicity flip

The sum overall net helicities \( \rho = \sum_j \mu_j \) defines the overall net helicity in \( m(p^j) \) such that an even number of poles change the projection of the net helicity nonflip total amplitude is obtained when an even number of poles change the projection of the net helicity nonflip total amplitude.

Thus \( m(p = \text{even}) = 0 \) and \( m(p = \text{odd}) = 1 \).
We obtain the meson-nucleon helicity amplitude for the single scattering of the $p$ pole and an arbitrary num
by the Pomeron. There is growing confirmation (5) that the isoscalar amplitude is not diagonal in helicity s
not conserve helicity. It is for this reason that we allow the vacuum pole to change the projection of the nuc

We decompose any amplitude in a helicity nonflip and helicity flip contribution indicated by 0 and 1 respectively

$$\mathcal{M} = \mathcal{M}_0 + i \mathcal{M}_2 \cdot \frac{\mathbf{q}_2}{|\mathbf{q}_2|} \cdot \mathbf{H}_n \cdot \mathbf{H}_j = \mathcal{M}_0 + i \mathcal{M}_2 \cdot \mathbf{H}_n \cdot \mathbf{H}_j,$$

where

$$\mathbf{\Omega}_j = \begin{pmatrix} 0 & -\hat{j} \\ \hat{j} & 0 \end{pmatrix}$$

and the polarization vector is perpendicular to the scattering plane fig. 2b and points along the \(\hat{j}\) axis.

$$\frac{\hat{j}_\rho}{|\hat{j}_\rho|} = \frac{\frac{\mathbf{q}_2}{|\mathbf{q}_2|} \cdot \mathbf{H}_n}{|\mathbf{H}_n|} = \hat{\gamma}.$$

We write down the propagator of the \(\mathcal{P}\) regge pole and the Pomeron \(\mathcal{P}'\). They differ in so far as we ass
fixed pole in its residue sufficiently strong to cancel the present nonsense wrong signature zero (NWSZ) wher
such a fixed pole. We accomplish the reggeization of the Feynman propagator for the elementary particle ex
the replacement:

$$\frac{1}{t - m^2} \rightarrow \frac{d\hat{x}}{t} \frac{\hat{x}}{\sin \hat{\pi} (\hat{x} - \hat{J})} \left( \frac{s + \frac{1}{2} t - \frac{1}{2} \sum_{\ell} m_{\ell}^2}{s_0} \right).$$

where \(\hat{x} = \hat{J}\) at the pole \(t = m^2\) and \(\hat{\pi}\) is the signature. \(m_{\ell}\) are the masses of the external particle and n
exchanged particle, \(s\) is the total c.m. energy squared and \(s_0\) the energy scale factor traditionally chosen
since any other choice is equivalent to the introduction of an exponential factor into the residue. \(\hat{x}\) is the
signature factor \[ \frac{1 + \tau \frac{e^{-i\pi\alpha}}{2}}{\sin \pi \alpha} \] and the energy factor \[ \left( \frac{\frac{1}{2} - \frac{1}{2} \frac{\alpha}{\pi} m^2}{\frac{1}{2}} \right)^{\alpha - \frac{1}{2}} \]

Thus \[ \sin \pi (\alpha - \frac{1}{2}) \approx \sin \pi (\alpha - \frac{1}{2}) \left|_{\tau = \frac{1}{2}} \right. + \frac{\partial}{\partial \tau} \sin \pi (\alpha - \frac{1}{2}) \left|_{\tau = \frac{1}{2}} \right. \]

noting that the signature and \( \alpha^2 - \frac{1}{2} \) factor are unit at \( \tau = \frac{1}{2} \) (for high energy the masses of the four particles are irrelevant) we see that this relation is exact at the pole \( \tau = m^2 \). The Gell Mann ghost eliminating mechanism is given by making the replacement

\[ \frac{\bar{u}}{\sin \pi \alpha} = \Gamma(\alpha) \Gamma(1 - \alpha) \]

and dividing by \( \Gamma(\alpha) \) for natural parity exchange \( (-1)^{\tau - \eta} \eta \) where \( \eta \) is the intrinsic parity.

For natural parity exchange \( (-1)^{\tau - \eta} \eta \) we make the replacement \( \frac{\bar{u}}{\sin \pi \alpha} = -\Gamma(-\alpha) \Gamma(1 + \alpha) \) and divide through \( \Gamma(1 + \alpha) \).

In the case of \( \alpha \) exchange we obtain

\[ \frac{1}{\tau - m^2} \rightarrow -\alpha^\prime \Gamma(1 - \alpha) \frac{1 - e^{-2\pi \alpha}}{2} \left( \frac{\pi}{\alpha^\prime} \right) \alpha - 1 \]
This agrees with the Regge limit of the Veneziano formula if the couplings are constant.

The poles of the Gamma function are at 0 and negative integer values, hence there are only poles in the resonant, not in the scattering region. This resembles the propagator of the one particle exchange model.

The first pole at $\alpha = 1$. In the scattering region the Gamma function can be approximated by, let us say, for simplicity, however, we use only one.

The signature factor gives a zero at integer values of $\alpha$, which correspond to wrong signature points (valid when $C = 1$). This means that in the resonance region poles occur on a trajectory in steps where $\alpha = \frac{\nu}{J}$ that is to say for the $P$ pole with $\nu = 1, 3$ etc. The wrong signature zeros occur again for $\nu = 1$, i.e. the wrong signature $-\nu = -J$. These zeros are, since they occur in the resonance region, responsible for the dip structure of the differential cross section - a triumph for the NWSZ Regge pole.

We separate the $t$-dependence in the Regge energy factor

$$ (S_{\alpha, J})^{\alpha} e^{\nu} = (S_{\alpha, J})^{\alpha - 1} e^{\nu} \ln(S_{\alpha, J}) |t| $$

since we are in the scattering region we have $t = -/t$ always.

The Gamma function we parameterize for convenience just by one exponential (there are no poles nor zeros i.e. where $\nu = 1, 3$ etc. The wrong signature zeros occur again for $\nu = 1$, i.e. the wrong signature $-\nu = -J$. These zeros are, since they occur in the resonance region, responsible for the dip structure of the differential cross section - a triumph for the NWSZ Regge pole.

We split, for calculational reasons (performing the cut) the negative signature factor

$$ \frac{\nu}{J} C_{\nu} \left( \frac{\nu}{J} \right) = A_p \rho - B_p |t| $$

Thus we write the propagator as

$$ G_p \left( S_{\alpha, J} \right) = (S_{\alpha, J})^{\alpha - 1} \sum_{J=1}^{2} \frac{g_{J/2}^{(2)}(\omega)}{\nu} e^{\nu} \ln(S_{\alpha, J}) |t| $$

where

$$ g_{J/2}^{(2)}(\omega) = \frac{\alpha_j}{\nu} A_p / \frac{1}{2} \quad g_{J/2}^{(2)}(\omega) = - \frac{\nu}{\alpha_j} $$
with the Regge interaction region of the nonrotating part

$$B_1 = B_0 + \alpha f \ln (s_{AB})$$

and the Regge interaction region of the rotating part

$$B_2 = B_0 + \alpha f \left( \ln (s_{AB}) - \frac{1}{\nu} \right)$$

When we just consider the pole alone then it is more convenient to use the half angle form of the signature and write then for the propagator

$$G_p^{-1} (s, t, u) = \sum_{\alpha} \alpha f \left( \ln (s_{AB}) \right)^{-1} A_0 \ln \frac{t}{u} \left( \Delta_{\alpha} \right)^{-1}$$

We replace the coupling constant (Veneziano limit) with form factors and write for the exponentially parameterized Regge residue i.e. for Particle-Regge-Particle vertex where $\alpha$ indicates the Regge term

$$P^\mu (s, t, u) = \sum_{\alpha} \sum_{\nu} \alpha f \left( \ln \frac{s_{AB}}{\nu} \right) C_{\alpha \nu} \left( \Delta_{\alpha} \right)^{\nu}$$

$$- \frac{1}{2} \left( R_{CD}^2 + \gamma \right) t$$
The once fitted values of the residues are expected to be not too far away from the values of the coupling constant $\gamma$ extrapolated to the resonance region $t = \gamma^2$.

Conservation of angular momentum and parity requires the introduction of the factors $\hat{A}^i \hat{A}^j$:

$$\hat{A}^i \hat{A}^j = \left( \frac{|L|}{\alpha m_\nu} \right)^C |\lambda_1 \lambda_2 \lambda \lambda^*| \xi \phi (\lambda^* \lambda)$$

with $m_\nu$ the nucleon mass introduced for dimensional reasons. $\phi$ is measured against the $\hat{x}$ axis, the phase angle of the Reggeon momentum $\hat{k}$, it is measured in addition to the conventional Regge phase which is $\hat{x}$ axis. The phase had to be introduced since the scattering plane $p_\sigma \cdot k_2$ is not fixed due to the integration over convolution.

We express the full Regge pole amplitude by

$$M^A_{\sigma} (s, k) = \left( \frac{|h_1|}{\alpha m_\nu} \right)^A \exp \left( \sum_{x=1, 2} \beta^A (s) \xi \phi \right)^A$$

with $\beta^A (s) = \beta^A (s) \xi \phi (\xi + 1) - \beta^A (s) \xi \phi \beta^A (\xi) \xi \phi (\xi - 1) = \beta^A (s)

$$\beta^B (s) = \beta^B (s) \xi \phi (\xi - 1) - \beta^B (s) \xi \phi \beta^B (\xi + 1) = \beta^B (s)$$

$$\gamma^A_{\sigma 1} (s) = \gamma^A_{\sigma 1} (s) \xi \phi (\xi) + \gamma^A_{\sigma 1} (s) \xi \phi (\xi + 1) = \gamma^A_{\sigma 1} (s)$$

$$\gamma^A_{\sigma 2} (s) = \gamma^A_{\sigma 2} (s) \xi \phi (\xi - 1) + \gamma^A_{\sigma 2} (s) \xi \phi (\xi + 1) = \gamma^A_{\sigma 2} (s)$$

$$\gamma^B_{\sigma 1} (s) = \gamma^B_{\sigma 1} (s) \xi \phi (\xi) + \gamma^B_{\sigma 1} (s) \xi \phi (\xi + 1) = \gamma^B_{\sigma 1} (s)$$

$$\gamma^B_{\sigma 2} (s) = \gamma^B_{\sigma 2} (s) \xi \phi (\xi - 1) + \gamma^B_{\sigma 2} (s) \xi \phi (\xi + 1) = \gamma^B_{\sigma 2} (s)$$
For the Pomeron propagator on the other hand we write
\[ G_P(s, k^2) = \frac{1 + \alpha P(k)}{\sin \pi \alpha P(k)} \]
and since
\[ 1 + \alpha P(k) = \frac{i \pi (\frac{\tau + i}{2} - \alpha P(k))}{\cos \left( \frac{\tau + i}{2} - \alpha P(k) \right)} \]
we obtain for the propagator \( T^{+1} \)
\[ G_P(s, k^2) = \frac{2 \cos \left( \frac{\tau}{2} (1 - \alpha P(s)) \right) + \frac{\pi \alpha P(s)}{2}}{k^2} \]
and the full expression for the Pomeron pole amplitude reads now (with the slow varying \( \cos (c_1 - \alpha P(s)) \) into the residue)
\[ M_P(s, k^2) = \hat{M}^{\alpha}(s, k^2) \]
with
\[ \hat{M}^{\alpha}(s, k^2) = \frac{\alpha P(s)}{\alpha P^{(0)}} (\frac{\alpha P(s)}{\alpha P^{(0)}})^{-1} \]
and
\[ \alpha P^{(0)} = \alpha P^{(0)} \ln (\frac{s}{\alpha P^{(0)}}) - i \frac{\pi}{2} \]
III - GRIBOV'S REGGEON DIAGRAM TECHNIQUE

III.1 - Gribov's evaluation of the Mandelstam diagram - the basis of the Reggeon diagram technique

In Gribov's determination of the asymptotic form of Mandelstam's diagram the use of Sudakov variables has proven in order to separate between negligible and important invariants, e.g. in separating the effect for large energy and from the transverse momentum space. This is mainly achieved by decomposing the internal momentum into a vector plane of large vectors and into a vector perpendicular to this plane. Equally one can achieve the same by working in a momentum frame, this is because the hadrons are decaying while they are moving with high velocity and the Gribov the probability amplitudes for decay and recombination which are in the Glauber picture analogous to the wave function amplitude for decay and recombination which are in the Glauber picture analogous to the wave function amplitude.

In order to succeed in picking out the essential regions of integration, Gribov applied the Sudakov variables to the left and right cross of the Mandelstam diagram. For large s, the two body amplitudes comparable with the Glauber scattering amplitude will be large but will fall off considerably fast when t is beyond the exchanged square mass. The Regge feature, therefore Gribov factorised the two body amplitude. These restrictions put heavy constraints on integration. Further avoiding that the asymptotic contribution of the Mandelstam cut disappears, one has to restrict 0 and 1 so the Sudakov contours cannot be distorted to infinity because they are pinched between the singularities on both sides of the contour. One can work this out by representing the propagator of the left or right hand cross by

In applying the Mellin transforms to the Green's function of the Regge poles, one obtains a partial wave decomposition function a la Sommerfeld-Watson, whereas the partial wave amplitude is expressed as the Mellin pole. Now one obtains for the asymptotic behaviour of the Mandelstam cut built up from two Gribov vertices and two Mellin poles. The Gribov vertices incorporate our ignorance of decay and recombination of hadrons.

Let us now look closely at how the dominant region of integration is picked out. The main result will be that the final Mandelstam cut will be a two dimensional integral of the Regge poles over a plane perpendicular to the incident momentum supported by our intuition, namely by imagining two interacting hadrons at high energy as two flat absorbing discs, a two-dimensional transverse world. We consider the two Gribov vertices which represent the left and the right end points of the propagation. The lines themselves are complicated Feynman diagrams. We already mentioned the condition for the internal propagators t to be infinitely away from their mass shell. So one obtains an integral over \( \Lambda^* \) and over the four dimensional momentum.
1 stands for the left and 2 for the right hand cross  

This 4-dimensional momentum is decomposed into $\chi = 1$ (left) and $\chi = 2$ (right) and into the two-dimensional transverse momenta $k_1$ and $k_2$ are the Sudakov variables when the momenta are expressed by

$$k_i = \alpha_i P_i + \beta_i P_2 + k_1$$

The propagator expressed in these variables gives the mass conditions namely the 4 masses depend only on $\alpha_1$, $\beta_1$, $k_{1L}$, $q_L$, $k_{2L}$ and $q_L$. These are the same variables on which the integrant over $\alpha_1$, $\beta_1$, and $k_L$ depends, thus making a further integral left with a function which depends only on $q$ and $k$. Since the left and the right hand cross are symmetrical in $P_1$ and $P_2$, these conclusions also apply to the right hand cross, so that one is actually left with a function which depends only on $q$ and $k$. How can we now understand that the exchange of the incoming momenta $p_1$ and $p_2$ moves in a two-dimensional world of timelike dimension, namely rapidity, from a source provided by the annihilation of 2 incoming hadrons to the sink again outgoing hadrons?

One can find the single partial wave explicitly by applying the Mellin transformation to the absorptive part of the Green's function conserving angular momentum and "energy" namely the angular momentum and energy of the produced particles. So Gribov showed that the discontinuity across the Mandelstam cut looks very similar to the exchange of two quasi-particles with conserved momentum and "energy" in its intermediate states and on its mass shell. With this, Gribov can now express the pole of a Reggeon with a non-relativistic quasi-particle one can talk about its "mass" which is inverse twice its slope and also the velocity and further on the Green's function for it either in energy or transfer momentum or in its conjugate virtual parameter and rapidity ("time"). This gives us almost a picture for Reggeons which diffuse along in space and time.
We have to notice that only pairs of hadrons serve as a source and as a sink of produced and absorbed quasi particles. The Reggeon propagators represent the interaction, the integration is over the first angular momentum and the transverse momentum and energy and angular momentum are conserved. We can further say that the Gribov vertices do not depend on energy if we are in the high energy region; essential singularities appear in Sudakov variables which are of the order of inverse s so that the integral works like a power series. The vertex function is inversely proportional to s. Further, the vertices are real because of the space-like Reggeon propagator. Alternatively to the Gribov perturbation approach to the Mandelstam diagram, White derived the genuine rigorous footing. He started from t-channel unitarity and projected out the partial wave of the 4 particle integrals into the complex s plane by a helicity contour integral. The contour got pinched in the helicity plane between the Regge poles "nonsense wrong signature inverse square root branch points" of the product of the scattering amplitudes which t-channel integral which in turn generates a cut at the moment it hits the end point of the integral. However, this analysis was that this cut has a negative sign, which was always intuitively felt from the phenomenological point of view.

A brief outline of Gribov's method (14)

fig. 1. The Mandelstam diagram

Gribov found asymptotic value for s → ∞ by applying Sudakov variables to loop momenta:
decompose inner momenta in plane of light like fourvector and space like two vector perpendicular to this plane:

\[ k_2^* = \alpha \xi p_2^* + p_\xi p_1^* + k_{2*} \]
\[ p_1^* = p_1 - \frac{m^2}{2} p_2 \]
\[ p_2^* = p_2 - \frac{m^2}{2} \]

volume of integration:

\[ \text{d}^4 k = \frac{1}{2} |s| \text{d} \alpha \text{d} \beta \text{d}^2 k_4 \]

For left hand cut:

\[ \alpha_1 = \alpha_1' \beta s + k_{4*}^2 - m^2 + i\epsilon \]
\[ \alpha_2 = (p_1' - \frac{m^2}{2}) s + k_{4*}^2 - m^2 + i\epsilon \]
\[ \alpha_3 = (\alpha_4 - \alpha_2') (k_{1*} - k_{2*}) s + (k_{1*} - k_{2*})^2 + i\epsilon \]
\[ \alpha_4 = - (\alpha_4 - \alpha_2') s - (p_1' - p_2') m^2 \]
\[ + (\alpha_4 - \alpha_2') (p_1' - p_2') s + \]
\[ + (k_{1*} - k_{2*})^2 - q_{2*}^2 (-p_1' + p_2') \]

put close to mass shell

with similar expression for the right hand cut.

After integration over \( \alpha \), the integrand still has singularities in \( \alpha \) on both sides of Re \( \alpha \) due to the interference from the third order \((s-u)\) double spectral function i.e. \((\alpha_2 - \alpha_4)\) and \((\alpha_1 - \alpha)\) consequently the integration contour (Pinch singularity)
The assumptions made are -

(1) \( f_{ij} \) large when energy large: \( \otimes f_{ij} \sim O(e) \)

\[ S_1 = C (k_1 + k_2)^2 \sim 2k_1k_2 \]

\[ S_2 = C (p_1 + p_2 - k_1 - k_2)^2 \sim 2(Cp_1 - k_2)(Cp_2 - k_2) \]

(2) Momentum transfer and masses

\[ k^2, (Cq - k)^2, (Cq + k)^2 \]

If these variables become of order \( e \) amplitude decreases sharply and region is unimportant.

Regions of interest:

| \( k_1^2 \sim k_2^2 \sim m^2 \) |
| \( \alpha_1 \sim \pi_2 \sim m^2 \) |
| \( \pi \sim \beta_1 \sim 1 \) |
| \( \alpha \sim \alpha_1 \sim \alpha_2 \sim 1 \) |

From there it follows

\[ S_1 \sim \pi_1 \alpha_2 s \]

\[ S_2 \sim (1 - \pi_1)(1 - \alpha_2)s \]

\[ k^2 = \alpha \beta_1 + k_2^2 \sim k_1^2 \]
Factorization of Regge amplitudes $s \to \infty$

\[ f_1 = g_1 \left( k_1^2 (k_0 - k_1)^2, k_0^2 \right) g_2 \left( k_2^2 (k_0 + k_2)^2, k_0^2 \right) G(k_1^2, 2k_0 k_2^2) \]

\[ f_2 = g_1' \left( (p_1 - k_1)^2, k_0^2 \right) G' \left( (p_2 - k_2)^2, (p_1 - k_1 + q - k_0)^2 \right) G'' \left( (p_2 - k_2 + q - k_0)^2, q^2 \right) \]

Sommerfeld-Watson G's: (Green's function)

\[ G = - \sum \frac{\text{d}^{2l_1}}{k_i} \frac{\text{d}^{2l_2}}{k_i} G_{k_1} (k_2) \left( \alpha_2 \phi_1, s \right)^{l_1} \]

\[ G' = - \sum \frac{\text{d}^{2l_1}}{k_i} \frac{\text{d}^{2l_2}}{k_i} G'_{k_1} (k_2) \left( \alpha_2 \phi_1, s \right)^{l_1} \]

\[ G'' = - \sum \frac{\text{d}^{2l_1}}{k_i} \frac{\text{d}^{2l_2}}{k_i} G''_{k_1} (k_2) \left( \alpha_2 \phi_1, s \right)^{l_1} \]

where $g_i$ are the signature factors.
Insert the $G$'s into the Feynman integral and use $k^2 = \alpha + \lambda_2 \nu \lambda_2^2 (C q - 1)^2 \nu (C q - \lambda_2)^2$

The result of all integrations:

$$\Gamma (s, q^2) = \frac{i \pi}{24} \int \frac{d k_1}{2 \pi} \int \frac{d k_2}{2 \pi} \int \frac{d \ell}{2 \pi} \int \frac{d \ell}{2 \pi} \frac{N_{k_1}^2 \nu (C q - \lambda_2) G_{k_2} G_{k_1} G_{A_{12}} G_{A_{23}}}{(2 \pi)^4}$$

signature \[ \frac{d}{\ell_{k_1}} = - \frac{1 + \nu}{\lambda_1 - \lambda_2}, \quad G_{k_2} G_{k_1} (s_{13}) = \frac{1}{\ell_1 - \lambda_2 (s_{13})} \]

Mellin pole

Reggeon production amplitude = Gribov vertex
\[ \text{Im} F(s, q^2) = \frac{i}{2} \int \frac{d \ell_1}{\omega_1} \int \frac{d \ell_2}{\omega_2} \gamma_2, \gamma_3 \int \frac{d \ell_4}{\omega_4} N_{\ell_1, \ell_2} \chi_{\ell_3} \chi_{\ell_4} \left( C(q^2) \right)^{j-1} \text{Im} F(s', q^2) \]

with \( \text{Re} \frac{\gamma_2}{\omega_2} = \frac{\gamma_1}{\omega_1} \)

**t-channel partial wave amplitude**

\[ \frac{f_j(s)}{f_j(s')} = \frac{2}{\nu} \int \frac{d \ell_4}{\omega_4} \left( \frac{s}{s'} \right)^{-j-1} \text{Im} F(s', q^2) \]

**partial wave amplitude** = Mellin projection x Absorptive part of the scattering amplitude

\[ \frac{f_j(s)}{f_j(s')} = \frac{\int \frac{d \ell_1}{\omega_1} \int \frac{d \ell_2}{\omega_2} \int \frac{d \ell_4}{\omega_4} \gamma_2, \gamma_3 \int \frac{d \ell_4}{\omega_4} N_{\ell_1, \ell_2} \chi_{\ell_3} \chi_{\ell_4} \left( C(q^2) \right)^{j-1} \text{Im} F(s', q^2)}{j + 1 - \ell_1 - \ell_2} \]
The partial wave amplitude is well defined and analytic in the region to the right of its singularities

\[ \text{integrations run to the right of the singularities of } \mathcal{G}_R \mathcal{G}_L \text{ and } \lambda_j \text{ is analytic right of } \mathcal{C}_j \]

Integration over \( \mathcal{C}_2 \) and evaluated pole at \( \lambda_2 = j + 1 - \ell \)

\( \Re j > \Re (\ell_1 \mathcal{C}_2 + \ell_2 \mathcal{C}_1) - 1 \) (domain of validity \( j + 1 - \ell, \mathcal{C}_1 \rightarrow \ell_2 \mathcal{C}_2 \)).

Fixed \( j \) : pole in \( \mathcal{C}_2 \) \( \ell_2 = j + 1 - \ell, \mathcal{C}_1 \) lies to the right of \( \mathcal{C}_2 \).

Similarly for \( \ell_1 \).
now deform $C_2$ around the pole in $l_2$

\[
\left( \oint_{C_2} \frac{C(q^2)}{C_l} \right) = \oint_{C_1} \frac{\alpha l_1}{\alpha^{2l_1}} \oint \frac{\alpha^2 l_2}{C^{2l_2}} \, G_{\beta_1} \left( k_1 \right) G_{\beta_2} \left( k_2 \right) \left( C_1 - k_1 \right)^2 \left( C_1 - k_2 \right)^2 \, N^2 \left( \alpha \right) \left( \alpha + 1 \right) \left( \alpha + 2 \right) \left( \alpha + 3 \right) \left( \alpha + 4 \right)
\]

the singularity from $G$ is at

\[
\left( \alpha \right) \left( \alpha + 1 \right) \left( \alpha + 2 \right) \left( \alpha + 3 \right) \left( \alpha + 4 \right)
\]

the singularity from $G'$ is at

\[
\left( \alpha \right) \left( \alpha + 1 \right) \left( \alpha + 2 \right) \left( \alpha + 3 \right) \left( \alpha + 4 \right)
\]

Momentum conservation: \( q = k_1 + k_2 \) in the vertices

Energy: \( j + 1 \)

Energy conservation: \( j - 1 = \beta_1 - 1 + \beta_2 - 1 \)

On the other hand, evaluating the pole contribution of $G$, $G'$

\[
\left( \oint_{C_2} \frac{C(q^2)}{C_l} \right) = \oint \frac{\alpha^2 l_2}{C^{2l_2}} \, \frac{\partial \sigma_{\alpha_1 \alpha_2}}{\partial \alpha_{j+1}} \, N^2 \left( \alpha \right) \left( \alpha + 1 \right) \left( \alpha + 2 \right) \left( \alpha + 3 \right) \left( \alpha + 4 \right)
\]

fig. 9

b) C_1 runs between the two poles of $G_1$.
III.11. THE REGGE PARTICLE SCATTERING AMPLITUDE

Gribov's theory indeed concentrates the unknown into one function - its Gribov vertex $N$. The Regge-particle amplitude factorizes due to the factorization property of the Regge residues. Thus, let us consider the nucleon vertex $\alpha p \alpha p \rightarrow \bar{p}^\ast$ of

![Diagram]

Fig. 10: Gribov two-Reggeon cut

The three momenta $p$, $\tilde{p}$ and $p^\ast$ define the scattering in the $\hat{s}$, $\hat{t}$ plane in fig. 11.
Fig. 11 demonstrates that the Regge particle amplitude in Fig. 12 is a function of the angle $\phi$ between the two dimensional Reggeon momenta.
The Regge-particle scattering amplitude has the same analytic properties in the subenergy plane as the ordinary amplitude. (29) That is to say it has poles and cuts due to the presence of physical intermediate states.

The third order double spectral function is symbolized by the Mandelstam cross -

\[ N \alpha \rho \times \rho \]

\[ \Rightarrow \]

\[ S_i \]

Fig. 15

Singualrities in the sub-energy plane due to third order
double spectral function $s - u$
The convergence for large internal masses \( s_i = k_i^2 \) is rapid enough that one can rotate the contour and one - the contributions from integrands over large semi-circles, hence:

\[
N^{(2)} \frac{\partial \rho}{\partial \rho} \sim \int \frac{d \omega}{\omega} \Im \left[ \int \frac{d^d k}{(2\pi)^d} \tilde{N}(\omega) \right]
\]

Now let us consider the AFS cut

\[
N_{AFS} = \int \frac{d^d k}{(2\pi)^d} \tilde{N}(\omega) \]

The dashed line denotes that the discontinuity is taken through the pole (elastic intermediate state) of the propagator. AFS unitarity integral then leads to a spurious cut with positive sign:

\[
q_i^2 = s_i > m_i^2 \] is the total sub energy squared flowing through the blob.
$\tilde{N}$ falls off more rapidly than $\frac{1}{s_1}$ when $s_1 \to 2$

$$\tilde{N}(s) = \frac{\beta C(s_1, k_2^2)}{q_1^2 - m^2 + i\epsilon}$$

The Regge residues possess poles and branch points in the sub energy plane $s_1$. The vertices depend on their $s$ when $s_1$ increases. The vertices are cut analytic, i.e. they only have a right hand cut. Thus, drawing the situation in the sub-energy plane $s_1 = q_1^2$ we see that at fixed $t$

Which demonstrates that the cut singularity (if integrated up to $+\infty$) exactly cancels the pole from the propagator. See this even more clearly by wrapping the contour individually around pole and cut. This example emphasizes the nature of Gribov's theory in comparison to $\beta^2$. The AFS model is sufficiently convergent so as to rotate in the sub energy plane. However, the integrand does not fall off rapidly enough, i.e. it is still sizeable for $\beta^2$ or even $\beta^2 = s$. Thus values $\beta^2 \sim m^2$ do not give the dominant contribution. This in turn demonstrates why
produce a genuine cut. AFS performs all unitarity integrals represented by fig.16 by taking the discontinuity through the pole of the propagator. The discontinuity across the cut exactly cancels is in fact the discontinuity which cuts through the vertex function due to which this cancellation occurs.

Since a reggeon can be represented by a multiperipheral ladder, it needs a large number of rungs to cut through to cancel the positive contribution obtained by AFS. Then, however, the threshold of the discontinuity is large, where \( n' \) is the number of ladders. Such a large mass is possible for AFS but not for Gribov. In Gribov's theory, the assumption is the sufficiently rapid fall off in the virtual masses such as \( s_t \). This rapid fall off dampens the discontinuity of the cuts with a high number of rungs cut through.

An underlying field theory with convergence properties such as the ones assumed by Gribov - the damping of \( \nu \) and momentum transfers when they exceed a particle mass \( m^2 \) - has not yet been found. But it is assumed that gauge theories could provide such a theory. (15)
III. III - GRIBOV RULES FOR THE "RALEIGH-SCHRÖDINGER" PERTURBATION THEORY

Gribov's analysis of the asymptotic behaviour of Feynman diagrams produced a two dimensional field theory. The fluctuations described by this field take place in impact parameter space and a variable (rapidity) which formally corresponds to time and which is conjugate to a quantity which formally corresponds to the energy.

The Laplace-Mellin transform plays in this a central role (15).

\[ \Phi(x, j, l^2) = \int_0^\infty \frac{d^2 \log s}{s} \frac{\log (C_{j-1})}{C_{j-1}} M(x, l^2) \]

Gribov's interpretation of fig. 18 - the one Reggeon exchange graphs -

![Diagram of Reggeon exchange graphs]

vanishes if \( \frac{Q}{Q_0} > \frac{1}{2} \)

fig. 18
A source C of energy $1-j$ and nonrelativistic momentum $k$ creates a quasi particle, a Reggeon of momentum $k_1$ and energy $1-\alpha(k^2)$. It will be annihilated at the sink A where it transfers its momentum.

Note

$$1-\alpha(Ck^2) - (1-j^2) = j - \alpha(Ck^2)$$

$$\frac{q_0}{q} < \frac{2}{2}$$

minimum

Gribov rules for the one-reggeon exchange graph -

(1) The creation vertex (CP in our diagram) is given by

$$P_{CD} C(k^2) \rho \frac{q_0}{q} C(j - \alpha(Ck^2))$$

(2) Annihilation vertex (AB) -

$$P_{AB} C(k^2)$$

(3) Into the propagator enters the difference between the energy of the source C and the energy of the Reggeon $\frac{1}{1-\alpha(Ck^2) - (1-j)}$

\[ \frac{1}{\text{Energy of Reggeon}} \quad \frac{1}{\text{Energy of source}} \]  

\[ \frac{1}{j-1 - \alpha'k^2 + C(1-\alpha(Ck^2))} \]  

for linear trajectory

\[ \frac{1}{E - \frac{k^2}{2m} + (1-\alpha(Ck^2))} \]  

Non-relativistic quasi particle

Energy gap: $1-\alpha(Ck^2)$
E is the conjugate variable to $T = i \log s$ i.e. rapidity in "time". The mass is $\frac{1}{2} \alpha' c = m$. The speed in $(k^2, \tau)$ space

$$\nu = \frac{m}{c} \left( \frac{4 \alpha' \alpha E}{1 + \alpha' \alpha} \right)^{1/2}$$

The Green's function for the diffusion of a Reggeon, see Abarbanel (34)

$$G(k, \tau) = \frac{\Theta(\tau) \tau^{(1 + \alpha')}}{4 \pi \alpha' \tau^{1 + \alpha'}} e^{-\frac{\tau^2}{4 \alpha' \tau}}$$

and is shown in fig. 19

a) Space-time picture of a Reggeon (34)  
b) for Mandelstam cut
The damping factor \( e^{-2C(\phi^2)} \) in the Green's function implies that for \( g \rightarrow \rho \) only Reggeons with \( C(\phi^2) \rightarrow 0 \) Pomerons survive in the case of diffractive scattering. At attainable energies, however, simple picture does not hold as there are many complicated interactions involved.

**Gribov rules for the two-Reggeon exchange graphs**

(e.g. The leading cut contribution to \( \gamma^p \rightarrow \gamma^A \))

![Diagram of Gribov rules for the two-Reggeon exchange graphs](image)

1) The vertex for the production of a pair of Reggeons with momenta \( k_1, k_2 \) during the scattering of a pion is given by

\[
\mathcal{A} = \int \frac{d\tilde{C}}{2\pi} \left\{ 1 - \frac{\tilde{C}}{2} N(C_{(1)}^{(2)}, k_{(1)}, k_{(2)}) \right\}
\]

with the product of the signature - \( \text{Im sign} \: \alpha_1 \: \text{sign} \: \alpha_2 \).
2) The annihilation vertex
\[ = N C_{k_2, k_3} \]

3) A Reggeon decay amplitude
\[ = \gamma \alpha_1 \alpha_2 \xi_{\alpha_1 \alpha_2}^{-\frac{N}{2} C_{k_1, k_2}} \]

4) Annihilation amplitude of two Reggeons into one
\[ = \gamma \alpha_1 \alpha_2 C_{k_2, k_3} \]

5) One Reggeon propagator
\[ \frac{1}{\xi - \alpha (C k_2^2)} \]

6) Two Reggeon propagator
\[ \frac{1}{1 - \alpha_1 + 1 - \alpha_2 - (\xi - \alpha)} \]

7) Momentum transfer integration
Partial wave of the unenhanced Reggeon cut and absorptive part

\[ - \frac{C_0}{2} \left( j + 1 - \alpha_1 - \alpha_2 \right) \]

Laplace-Mellin transform:

\[ \text{Im} \, M \left( \frac{q^2}{s}, \frac{b^2}{s} \right) = \int \frac{d\alpha}{C_1} \left( \frac{b_2}{s} \right)^{j-1} \phi(c_i, 4) \]

\[ = \int \frac{d\alpha b_1}{C_1} \frac{d\alpha b_2}{C_2} N_1 \left( b_1, b_2, b_3 \right) N_2 \left( c_1, c_2 \right) \]

where the N's are taken at \( j - \alpha_1 + \alpha_2 - 1 \)
Gribov rules indicate that:

1) the unenhanced graph vanishes unless $\frac{g}{g_0} < \frac{\alpha}{\alpha_0}$

2) the semi-enhanced graph vanishes unless $\frac{g}{g_0} > 2 \frac{\alpha}{\alpha_0}$

3) the full enhanced graph vanishes unless $\frac{g}{g_0} > 3 \frac{\alpha}{\alpha_0}$
IV - A CORRELATION MODIFIED EIKONAL MODEL

With our basic ingredients explicitly defined we enter the main part of the thesis. The derivation and justification of our modified eikonal model where one Reggeon and \( n - 1 \) Pomeron can be exchanged and where non vacuum amplitudes are allowed to change the projection of the nucleon spin.

We symbolize such a model by the typical Gribov diagram

\[
\text{fig. 21}
\]

We understand the scattering amplitude \( M(s, t, \theta) \) as expanded into a series consisting of \( n \)th order cut term helicity contribution. Thus we put

\[
\mathcal{M}^{(n)}(s, t) = \sum_{\rho} \mathcal{M}^{(\rho)}(s, t, \theta)
\]

and write down the \( s \)-channel helicity contribution to the \( n \)th order term in Gribov's 'multiple expansion' of the reaction \( A + C \rightarrow B + D \)

\[
\mathcal{M}^{(n)}(s, \theta) = \sum_{\rho} \oint \mathcal{M}^{(\rho)}(s, t, \theta) \prod_{\lambda=1}^{m} \mathcal{C}(b_{\lambda}, s) \, d\omega^{(\rho)}
\]
The reggeon 4-momenta are \( q = (0, k, 0) \) since, for high energy small angle scattering the reggeon momenta are approximated by the transverse component of the final three momentum of the colliding particle projected onto a perpendicular to the scattering plane. The longitudinal momentum transfer of the projectile has been neglected. Scattering (24) the two dimensional integration approximates the integration over the sphere by an integration which is tangent to the sphere at forward direction.

Thus we use \( Z_i^2 = -1 \) and \( \Delta^2 = -t \) for the total momentum transfer \( \Delta = \sum_{i=1}^{n} k_i^2 \).

The helicity sum splits into net helicity nonflip and net helicity flip indicates by \( m(p) \) which is a function of the helicities \( u_i \) carried by the exchanged reggeons, counted by the index \( i \) from 1 up to \( n \) the order of exchange. The \( u_i = 1 \) and denotes reggeon helicity nonflip and the \( u_i = 1 \) reggeon helicity flip. \( p = u_i \) defines the net helicity such that even \( p \) in net helicity nonflip and odd \( p \) in net helicity flip. Gribov's two body amplitude has been written as correspond multiple scattering expansion in powers of the basic nucleon-nucleon (quark-quark) scattering amplitude - resembling nuclei scattering - and in increasing orders of a many-body transition form factor taking into account the contribution of inelastically excited intermediate states between the internal structure of the colliding hadrons. In addition, the amplitude includes factors for shower formation which are reminiscent of the Michigan formula.

The momentum conserving delta function enters the reggeon phase space as

\[
\delta^{(n)} \left( \Delta - \sum_{i=1}^{n} k_i^2 \right) = \int \delta^2 (\Delta - \sum_{i=1}^{n} k_i^2) \prod_{i=1}^{n} d^2 k_i^2 \]
We consider the $n$th order term of the correlation modified eikonal and symbolize the dependence of the Gribov vertices on the angle of the exchanged Reggeon and Pomerons by the $n$th order extension of the Mandelstam diagram.

The Gribov vertex factorizes in the case of $\pi N$ scattering into a nucleon (AB) and pion vertex (CD). Only nucleon vertex can helicity change take place. We give $N$ the following functional expression:

$$m_{(p)} N^{\mu_1, \mu_2, \ldots, \mu_n} (b_1, \ldots, b_n, \delta) = m_{(p)} \tilde{N}^{(\mu_1, \ldots, \mu_n)} (b_1, \ldots, b_n, \delta) \tilde{N}_{\mu_1} (b_1) \ldots \tilde{N}_{\mu_n} (b_n) \prod_{i=1}^{n} H^{\mu_i} (|b_i|)$$

The $H$'s and $\tilde{b}$'s are angular momentum factors and Regge residues respectively as defined in II (page 19).
runs from 1 to \( n \) where \( i = 1 \) indicates the Reggeon and \( i = 2, \ldots, n \) the Pomeron-exchange. The interesting to any deviation from 1 reflects the effective contribution of inelastic intermediate states besides the elastic process. The shower factor \( \lambda \) measures the normalization of the contribution of such inelastic intermediate states and their dependence on momentum transfer.

In a modification of the eikonal model, Ter-Martirosyan (21) has proposed the quasi-eikonal model by introducing a deviation from the eikonal model due to the ratio of the cross sections for diffractive dissociation to the cross section for elastic scattering. They can be \( s \)-dependent, see Kaidalov (36) and for applications see Boreiskov et al (20). The way to take into account shower formations in intermediate states in the framework of the quasi-eikonal model is the factorized Gribov vertices as expanded in a series in the complete system of physical intermediate states in the language of nuclear physics to the expansion in an increasing order of the nuclear correlation function.
is not a Feynman diagram nor an AFS diagram since the intermediate particles are taken on mass shell.

\[ \frac{1}{q_f^2 - m_f^2 + i\epsilon} \rightarrow -\frac{i}{\pi} \int \frac{d^2 q}{q^2 - m^2} \quad \text{see Ter-Martirosyan (16)} \]

This procedure leads to the "optical" contribution which leads to the eikonal model. The optical contribution agrees with the Absorption model when initial and final state interactions are the same. The singularities are on the left and right hand side of the complex plane. In the absence of enhanced branch points, i.e., small masses \( s_f \), the absorptive part of the Regge-scattering amplitude falls off more rapidly than \( \frac{1}{s_f} \). One can then rotate both contours such that

\[ \text{left} \rightarrow \text{right} \]

and obtain a superconvergent relation for the Regge-particle scattering amplitude which leads in the case of the dominance to the equality of initial and final state rescattering, i.e., the absorption model see Kaidalov (28).
The contribution of each link in fig. 24 b) increases due to the shower formation in comparison with the elastic states in fig. 24 a). The Reggeon-induced production of a cascade of particles will be different from the one by the Pomeron. Also helicity flip might contribute differently from nonflip. In practice we only fit one number per helicity.

\[ \sum l l_n^m \sum l l_n^m \lambda_{l l_n} \lambda_{l l_n} = \lambda_{l l_n} \]

Note that a determination by Ravenhall and Wyld (38) led to the result that from four diffusively produced states, only two, namely \( A_1 + N \) and \( T + N^* C / 670 \), made a significant contribution leading to \( \lambda \sim 1.2 \) to be generally assumed that only pomeron-induced production is significantly large. In contrast, those induced by \( T \) and helicity flips are small (21).

What remains, and promises to be of considerable effect by comparison with the absorption/elkonal, is the dependence on the momentum transfer, not only on the overall one \( \Delta \) but also on \( |l_1| \Delta \) and \( |l_2| \Delta \) which leads to a similar dependence on \( \Delta \) and \( |l_1| \) and \( |l_2| \) i.e. a renormalization of the shape of the cut and of the poles to higher order exchanges. Thus the resulting elkonal will be modified in its overall \( \Delta \) dependence and in addition phase undergoes a modification. This \( |l_1| \sim |l_2| \Delta \) and possible \( s \)-dependence is represented by \( \Delta \). In the case of second order exchange one could parameterize \( \tilde{N} \) algebraically by

\[ \tilde{N} = \frac{1}{1 + C |l_1| + |l_2|} \] or exponentially

\[ \tilde{N} = e^{-c_1 + c_1 |l_1| + c_2 |l_2|} \]
These form factors reflect the composite structure of the colliding particles as they do in the quark model calculation of Benofy, Shrauner and Cho (39) and Harrington and Pagnamenta (30). We choose the exponential representation as to obtain simple Gaussian integrals. In the case of the nth order contribution we parameterize such that \( \tilde{N} \) factorize. This is accomplished by the following parameterization:

\[
\tilde{N}(\mathbf{q}) \tilde{N}(\mathbf{k}_1,\ldots,\mathbf{k}_n) = \exp \left[ -\frac{C}{2} \sum \right] \left\{ \left( k_1 - k_2 \right)^2 + \left( k_2 - k_3 \right)^2 + \ldots + \left( k_{n-1} - k_n \right)^2 \right\}
\]

We write an analogous expression for

\[
\tilde{N}(\mathbf{k}_1,\ldots,\mathbf{k}_n, \mathbf{q}) = \exp \left[ -\frac{C}{2} \sum \right] \left\{ \left( k_1 - k_2 \right)^2 + \ldots + \left( k_{n-1} - k_n \right)^2 \right\}
\]
In a further simplification we set \( C_N = C \) in this case

\[
\begin{align*}
\sum_{\pi} \langle \mu_1 \ldots \mu_n \rangle N_{\pi}(k_1, \ldots, k_n, \Delta) N_{\bar{\pi}}(l_1, \ldots, l_n, \Delta) = \langle \mu_1 \mu_2 \ldots \mu_n \rangle K_{\ell_1 \ldots \ell_n, \ell} = \frac{C A^2 - N C}{\Lambda^2 - A^2}.
\end{align*}
\]

due to conservation momentum transfer \( \Delta = \sqrt{k_1^2} \) we call \( K \) the correlation kernel and the correlation potential the "Grilov c". The \( n \) in the exponent is the order of exchange. The introduction of a correlation kernel in the cut integral has been strongly suggested by Høgaa sen and Krzywicki (40). There the forms

\[
K = \exp \left[ 2D k_1 k_2 \right] \quad \text{and} \quad K = \exp \left[ C (k_1 - k_2)^2 \right]
\]

were put forward. Another form is the one by Lovelace (26). \( K = \sqrt{k_1 k_2} \).
We write the nth order contribution to the s-channel helicity scattering amplitude as

\[
\mathcal{M}^{(n)}(s, t, \gamma) = \mathcal{C}^{(n)} \int \mathcal{M}^{(n)}(s', b^2) \mathcal{K}^{(n)}(b_1, \ldots, b_n, \gamma) \delta^2(A_{\gamma} - \frac{b_1}{2}) \cdots \delta^2(b_n) \nu_{\gamma} \cdot \nu_{\pi} \cdot \nu_{\rho} \cdot \nu_{\delta} \cdot \nu_{\alpha} \cdot \nu_{\beta}
\]

with

\[
M^{(n)}(s', b^2) = \left( \frac{|k_1|^2}{2m_\gamma} \right)^{n/2} \mathcal{P} \left( \mathcal{M}(s') \right) \mathcal{E} \sum_{\sigma=1}^{n/2} C_{\sigma} b^2
\]

and

\[
M^{(n)}(s, b^2) = \left( \frac{|k_2|^2}{2m_\gamma} \right)^{n/2} \mathcal{P} \left( \mathcal{M}(s) \right) \mathcal{E} \sum_{\sigma=1}^{n/2} C_{\sigma} b^2
\]

as defined in II pages 20 and 21. Further, we put the correlation kernel as on page 52 of this chapter

\[
K^{(n)}(b_1, \ldots, b_n, \gamma) = \mathcal{C}^{(n)} \sum_{\sigma=1}^{n/2} b^2
\]

Before we discuss the normalization \( \nu_{\gamma}^{(n)} \) we remark on the crossing symmetry (up to \( \frac{1}{\nu} \)) of the Gribov cut. The symmetry of the cut under crossing \( s \rightarrow -s \) is the product of the symmetries of the Reggeons, (Pomeron) \( \mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \ldots \mathcal{A}_n \).

The signature of the cut is the product of the signatures of the poles.
In this respect it is interesting to remark that the absorption model as mentioned in the introduction (page 2) traditionally calculated with the help of the Sopkovich prescription which lacks a-4 crossing symmetry, i.e. it is not line reversal. We mentioned that the Gribov cut corresponds to the absorption model, see page 49, if initial and final rescatterings are equal. This is however, generally not necessarily the case. How one can actually restore crossing symmetry in the absorption model has been nicely demonstrated by Quigg (41). It consists of adding the crossed graphs to the conventional graphs of the absorption model, i.e. to add elastic scattering of an initial state particle with a final state particle, as shown in fig. 27.

![Crossed Graphs](image)

*fig. 27*

Quigg's crossing symmetric absorption model

Quigg demonstrated further how the crossed terms are already included in the Gribov cut. The Regge-particle scattering amplitude contains all possibilities - thus also the production of crossed reggeons.
Quigg's argument

The Gribov cut includes crossed and uncrossed graphs. The absorption model can be made crossing symmetric by averaging the crossed graphs with the uncrossed graphs. The crossed graphs can be understood since the Regge-particle scattering amplitude contains all possible orderings of the constituents namely a and b interchanged and c and d interchanged give crossed graph as in fig. 29
We can now state more precisely the limit cases of the Gribov cut in the case of second order exchange.

We write

\[ M^{(2)} = \int N G_1 G_2 \, dQ \]

which reduces in the case of \( N = \text{const.} \) to the absorption model with coupling constants - see Adjei et al (4) initial and final rescatterings are equal. The contour integral in the sub energy plane includes only the real part of the intermediate elastic pole. The residue is a constant. The \( N \)'s can contain form factors instead of coupling constants. Any deviation, however, of \( N \) from the product of the pole residues involved in the ex is a way to pick up contributions of other singularities in the sub energy plane. By writing \( M^{(2)} \) as

\[ M^{(1)} = \int M_1 M_2 K \, dQ \]

one can say that any deviation of \( K \) from 1 takes effectively account of the contribution of inelastic intermediate states and \( K = \text{const.} \) corresponds to the crossing symmetric i.e. line reversal version of the absorption model such as the one developed by Quigg. However \( K = \text{const} \neq 1 \) only determines the normalization of the Gribov N, i.e. the contribution of the inelastic intermediate states to the cut in forward direction. \( K \neq \text{const.} \) on the other hand measures the effect these intermediate states have at \( /t/ < 0 \). This has a considerable effect on the position of the cut. Note, however that \( K \) has to be real so as not to destroy the crossing symmetry. In practice a small part might be feasible as long as it does not break the symmetry too drastically.
Now we are going to fix the normalization $\nu^{(n)}$ in

$$M^{(n)} = \nu^{(n)} \int^\infty \cdots \int^\infty \prod_{\mu} dk_\mu k^2 \prod_{\mu} \phi_k d^3 k$$

First let us consider spinless scattering. $M^{(n)}$ is the $n$th order term in an eikonal expansion if $K$ factorizes. For this reason we have chosen it as on page 51.

Other forms, such as

$$m^p \bar{\eta} \eta_{\bar{n}} = \mu^p \bar{c} \eta_{\bar{n}} \left( \frac{1}{\lambda} \dot{c}_\mu - \frac{1}{\lambda} k_\mu \right)^2$$

leading to

$$m^p \bar{\eta} \eta_{\bar{n}} = \mu^p \bar{c} \eta_{\bar{n}} \left( \frac{1}{\lambda} \dot{c}_\mu - \frac{1}{\lambda} k_\mu \right)^2$$

and

$$\bar{\eta} \eta_{\bar{n}} = \mu^p \bar{c} \eta_{\bar{n}} \left( \frac{1}{\lambda} \dot{c}_\mu - \frac{1}{\lambda} k_\mu \right)^2$$

do not factorize at once. However, the quadratic forms with crossed terms can be diagonalized by carrying out similarity transformations.
We now write down the s-channel partial wave series for an elastic scattering amplitude. With our normal see I, page 12, and Appendix page 207 we find

\[ M(s,A) = \frac{-i\pi}{q^2} \sum_{j=1}^{\infty} C_j M(s_j) \cdot \bar{\rho}_j (\cot \theta_j) \]

We are going over to the impact parameter representation, see Appendix page 211, and make the replacement

\[ \sum_{j=1}^{\infty} \longrightarrow \frac{2q^2}{r_0} \int_0^{r_0} \rho \, dr \quad \text{and} \quad \bar{\rho}_j (\cot \theta_j) \longrightarrow \bar{\rho}_0 (\frac{|\rho|}{r_0}) \text{and} \]

thus

\[ M(s,A) = \frac{-i\pi}{q^2} \int_0^{r_0} \rho \, M(C,\rho) \cdot \bar{\rho}_0 (\frac{|\rho|}{r_0}) \, dr \]

\[ M(C,\rho) \] is the sum over all nth order products of the phase shifts, i.e., the Regge pole amplitudes which Fourier transformed from momentum transfer space into impact parameter space. If we call these phase \( \theta_i \), then the s-channel partial wave reads

\[ M(C,\rho) = \sum_{m=1}^{\infty} \left( \frac{2i}{M} \delta_{(b,i)} \right)^m = - \epsilon^{2i \delta_{(b,i)}} - 1 \]

\( M(C,\rho) \) is the partial wave amplitude for the angular momentum \( J = q_s b^{-\frac{1}{2}} \) where \( q_s \) is the magnitude of the s-channel centre of mass three momentum \( |p_C| = |p_D| \) for the scattering \( A + C \rightarrow B + D \) and \( b \) the impact parameter.

\[ \begin{array}{c}
\text{impact parameter} \\
\text{beam} q_s \\
\text{fig. 30}
\end{array} \]
Fourier transform back into impact parameter space: $M(s, b)$

$$H(s, b) = \frac{\omega}{8\pi} \int \frac{d^2 b}{x} M(s, b) \cos \theta$$

The connection with $H(s, b) = \frac{\omega}{8\pi} \int d^2 b M(s, b) \int_0^\infty C(|b|) d\theta$ is found by making use of the identity

$$J_m(x) = (-i)^m \frac{\pi}{2^m \Gamma(m + \frac{1}{2})} \int_0^{\pi} e^{i x \cos \phi} \cos^{m - \frac{1}{2}} \phi d\phi$$

thus

$$\int e^{i \theta \cdot \ell} d^2 b = \frac{\omega}{8\pi} \int d^2 b \int_0^\infty C(|b|) d\theta$$

then

$$H(s, b) = \frac{\omega}{8\pi} \int \frac{d^2 b}{x} M(s, b) \cos \theta$$

the partial wave amplitude $M(s, b)$ can be obtained by inverting $H(s, b) = \frac{\omega}{8\pi} \int d^2 b H(s, b) \int_0^\infty C(|b|)$
by using the Fourier-Bessel integral

$$f(x) = \int_0^\infty J_n(\alpha x) \alpha \left\{ \int_0^\infty f(x') J_n(\alpha x') \alpha dx' \right\} d\alpha$$

Or

$$M(\alpha, \beta) = \frac{i}{\pi} \int_0^\infty M(\alpha, \beta) J_0(\alpha \phi) \alpha d\alpha$$

and since

$$M(\alpha, \beta) = \sum_{n=1}^\infty \left( \frac{\alpha n \phi(\alpha, \beta)}{\pi} \right)^n \delta(\alpha, \beta) = 2i \delta(\alpha, \beta)$$

and we define the eikonal phase in terms of the phase shift:

$$\chi(\alpha, \beta) = 2 \delta(\alpha, \beta)$$

then we have

$$\int_0^\infty \delta(\alpha, \beta) - 2i \delta(\alpha, \beta) = i \chi(\alpha, \beta)$$
and  
\[ \hat{\lambda}(x(s, \beta)) = \frac{\lambda}{i} \int_0^\infty \frac{dz}{2\pi} \frac{\zeta^{(1)}}{\lambda_0^{(1)} \lambda_0^{(2)}} \frac{\partial \zeta^{(1)}}{\partial \beta_0} \]  
where \( M^{(1)}(s, \beta) \) is the Regge pole.

now  
\[ M^{(n)}(s, \beta) = \sum_{j=1}^{\infty} M^{(n)}(s, \beta) \]  
and  
\[ M^{(n)}(s, \beta) = \frac{\sqrt{\pi}}{2} \int_0^\infty \frac{dz}{2\pi} \frac{\zeta^{(n)}}{\lambda_0^{(1)} \lambda_0^{(2)}} \frac{\partial \zeta^{(n)}}{\partial \beta_0} \]  
but  
\[ \delta^{(n)}(s, \beta) = \left( \frac{i \lambda(s, \beta)}{m!} \right)^n \]  
thus  
\[ M^{(n)}(s, \beta) = \frac{\sqrt{\pi}}{2} \int_0^\infty \frac{dz}{2\pi} \frac{\zeta^{(n)}}{\lambda_0^{(1)} \lambda_0^{(2)}} \frac{\partial \zeta^{(n)}}{\partial \beta_0} \chi(s, \beta) \chi(s, \beta) \cdots \chi(s, \beta) \, d\beta \]  
now we compare this with our nth order Gribov integral:  
\[ M^{(n)}(s, \beta) = \frac{\sqrt{\pi}}{2} \int_0^\infty \frac{dz}{2\pi} \frac{\zeta^{(n)}}{\lambda_0^{(1)} \lambda_0^{(2)}} \frac{\partial \zeta^{(n)}}{\partial \beta_0} \chi(s, \beta) \chi(s, \beta) \cdots \chi(s, \beta) \, d\beta \]
in order to be able to work in the impact parameter representation we choose the Fourier integral representation of the delta function:

$$\delta^2(|a - a'|) = \frac{1}{(2\pi)^2} \int \frac{1}{2\pi} e^{i(\alpha - \alpha') \cdot \mathbf{b}} d\mathbf{b}$$

then the Gribov integral becomes

$$M_{\chi_{\alpha}^{\pi_i}}(s, t) = V_{\pi_i} \int e^{i\mathbf{b} \cdot \mathbf{b}} \left[ \int e^{-i\mathbf{b} \cdot \mathbf{b}} M_1(s, b) d\mathbf{b} \right] \left[ \int e^{-i\mathbf{b} \cdot \mathbf{b}} M_2(s, b) d\mathbf{b} \right] \cdots \left[ \int e^{-i\mathbf{b} \cdot \mathbf{b}} M_{\pi_i}(s, b) d\mathbf{b} \right]$$

by inspection

$$V_{\pi_i} \left[ \int e^{-i\mathbf{b} \cdot \mathbf{b}} \chi_1(s, b) \chi_2(s, b) \cdots \chi_{\pi_i}(s, b) d\mathbf{b} \right]$$

and cut

$$\int_{m_1}^{\chi_{\alpha}^{\pi_i}} \frac{\chi_{\alpha}^{\pi_i} \chi_{\alpha}^{\pi_i} \chi_{\alpha}^{\pi_i}}{(m-1)! (\mathbf{b}^2)^{m-1}}$$
Thus we write for the nth order contribution to the s-channel helicity scattering amplitude for the exchange of Reggeon (the $\rho \rightarrow \pi^- \pi^+$) and n-1 Pomerons which are allowed to change the projection of the nucleon spin is not helicity conserving.

$$\mathcal{M}^{(m)}(s, t, u) = \frac{\lambda_{C+}^{(m-1)}}{(m-1)!} \frac{1}{(2\pi)^n} \int \ldots \int \mathcal{K}_{\pi, \ldots, \pi}^{(n)}(l_{\pi, \ldots, \pi}) \delta^2(q - q')$$

with Reggeon, Pomeron and the correlation kernel as defined on page 53 and in addition the Fourier integral of the delta function we obtain:

$$\mathcal{M}^{(m)}(s, t, u) = \sum_{n=0}^{\infty} \mathcal{M}_{n}^{(m)}(s, t, u)$$

$$= \sum_{n=0}^{\infty} \mathcal{M}_{n}^{(m)}(s, t, u)$$
The conventional Regge phase has been defined along the $\hat{z}$ axis, see also part II, page 12, fig. 2. The plane of the Reggeon momenta $k_1$, $k_2$ and the total momentum transfer $\Delta$ is then as shown in fig. 31.

The mutually orientated transverse components of the Reggeon momenta.
We then write

\[ \sum_{n} \int_{-\infty}^{\infty} \frac{d^2 h_n}{(2\pi)^2} \langle \hat{\gamma}_p h_n \rangle \langle \hat{\gamma}_p h_j \rangle \langle \hat{\gamma}_p h_j \rangle \]

and consequently the Regge eikonal reads

\[ \chi_{\text{Regge}}(s,k) = \sum_{n} \frac{\beta_{\text{P}}(c)(C^{-1})^n C^{-1}}{(\alpha_{\text{Regge}})^n} \int_{-\infty}^{\infty} \frac{d^2 h_n}{(2\pi)^2} \langle \hat{\gamma}_p h_n \rangle \langle \hat{\gamma}_p h_j \rangle \langle \hat{\gamma}_p h_j \rangle \]

and the Pomeron eikonal

\[ \chi_{\text{Pomeron}}(s,k) = \frac{(-1)^{n-1} \beta_{\text{P}}(c)(C^{-1})^n C^{-1}}{(\alpha_{\text{Pomeron}})^n} \int_{-\infty}^{\infty} \frac{d^2 h_n}{(2\pi)^2} \langle \hat{\gamma}_p h_n \rangle \langle \hat{\gamma}_p h_j \rangle \langle \hat{\gamma}_p h_j \rangle \]
We now carry out the integration over momentum-transfer $k_i$ and $k_j$, i.e. the Bessel integrals:

in order to evaluate the integrals we make use of the Fourier-Bessel transforms:

$$
\int_0^{2\pi} \int_0^\infty j_\nu (k r) e^{-\alpha x^2} dr dx = \frac{\Gamma (\frac{\nu}{2} + 1)}{(2\alpha)^{\frac{\nu}{2} + 1}} e^{-\frac{x^2}{4\alpha}}
$$

Thus we obtain

$$
\sum_j \int_0^{2\pi} \int_0^\infty j_\nu (k_j r) e^{-\alpha x^2} dr dx = \frac{(-i)^n |b|^n}{(2^{\frac{n}{2} + 1})^n} \frac{\Gamma (\frac{n}{2} + 1)}{\prod (2^{\frac{j-1}{2} + 1})^n}
$$

thus

$$
\sum_{j=1}^{n} \alpha_j^2 b_j^2 \cdots \alpha_j^n b_j^n = \frac{(-i)^n |b|^n}{\prod (2^{\frac{j-1}{2} + 1})^n}
$$

and

$$
\text{and} \quad \prod_{j=1}^{n} \alpha_j \cdots \alpha_j^n b_j^n = \frac{\Gamma (\frac{n}{2} + 1)}{\prod (2^{\frac{j-1}{2} + 1})^n}
$$
Using the identity

\[ \mathcal{F}(x) = (-i)^n \int \frac{dx}{2\pi} e^{i\cos x} \cos nx \]

we obtain

\[ \mathcal{F}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-x^2} \int \frac{dx}{\sqrt{2\pi}} e^{i\cos x} \cos nx \]

and

\[ M_{\mu \nu}(x, A) = e^{-i\phi A_\mu} \int \frac{dx}{\sqrt{2\pi}} e^{-x^2} \int \frac{dx}{\sqrt{2\pi}} e^{i\cos x} \cos nx \]

\[ -\frac{i}{\sqrt{2\pi}} \int \frac{dx}{\sqrt{2\pi}} e^{i\cos x} \cos nx \]
The nth order formula for the \( g\gamma P^{n-1} \) helicity cut contribution to the \( s \)-channel helicity scattering amplitude is given by:

\[
M^{m(p)}(s, t) = \hat{V}^{(n)}(\Delta^2, q^2) \int_0^\infty \frac{d\xi}{\xi^{n+1}} \frac{p - m(p)}{2} \frac{1}{\Delta^2 - (2\Delta^{n+1} + m(p))^2}
\]

with

\[
\hat{V}^{(n)} = \sum_{\{g\}} \frac{(-1)^{n-2} \Gamma(n) \Gamma(n+1) \Lambda^{2n-2}}{(n-1)! (\Lambda^{n+1} \Lambda^{2n+1} \Lambda^{2n})^2} \left( \sum_{i=1}^\infty \sum_{j=1}^\infty \sum_{k=1}^\infty \frac{P_{(i,j,k)}}{P^2 + \Lambda^{2n+1}} \frac{P_{(i,j,k)}}{P^2 + \Lambda^{2n+1}} \right)
\]

and

\[
\frac{\Lambda^{n+1}}{\Lambda^{n+1} + m(p)^2} \frac{1}{\Lambda^{2n+1}} \left( \frac{\Lambda^{n+1}}{\Lambda^{n+1} + m(p)^2} \frac{1}{\Lambda^{2n+1}} \right)^{n+1}
\]

where \( m(p) \) is the momentum of the pion.
In II page 16, we have decomposed an amplitude by

\[ M = M^0 \delta_0 + i \alpha (\hat{p}_a \times \hat{q}) M' \]

with \( \delta_0 \) the unit matrix.

Thus we obtain for the second order helicity cut -

\[ \left( M^m \right)_{\mu \nu} = \left( M^0 \right)_{\mu \nu} \left( M^0 \right)_{\rho \sigma} - \left( M^1 \right)_{\mu \nu} \left( M^1 \right)_{\rho \sigma} \]

and for the third order contribution

\[ \left( M^m \right)_{\mu \nu} = \left( M^0 \right)_{\mu \nu} \left( M^0 \right)_{\rho \sigma} - \left( M^1 \right)_{\mu \nu} \left( M^1 \right)_{\rho \sigma} - \left( M^0 \right)_{\mu \nu} \left( M^1 \right)_{\rho \sigma} - \left( M^1 \right)_{\mu \nu} \left( M^0 \right)_{\rho \sigma} \]
Our correlation modified eikonal model then reads -

**Pomeron exchange only**

\[
M^0(c,s,a,c) = \frac{\bar{\nu}_M}{2\lambda^0} \int \frac{d^2\delta_p}{2\pi} \chi \frac{2 \lambda^0_p}{2 \lambda^0} \left( e^{-\beta \lambda^0_s \delta_p^0} - 1 \right) e^{i \Phi_p} \text{ d}^2\delta_p
\]

\[
M^1(c,s,a,c) = \frac{\bar{\nu}_M}{2\lambda^1} \int \frac{d^2\delta_p}{2\pi} \chi \frac{2 \lambda^1_p}{2 \lambda^1} \left( e^{-\beta \lambda^1_s \delta_p^1} - 1 \right) e^{i \Phi_p} \text{ d}^2\delta_p
\]

**Pomeron exchange + one Reggeon**

\[
M^0(c,s,a,c) = \frac{\bar{\nu}_M}{2\lambda^0} \int \frac{d^2\delta_p}{2\pi} \chi \frac{2 \lambda^0_p}{2 \lambda^0} \left( e^{-\beta \lambda^0_s \delta_p^0} - 1 \right) e^{i \Phi_p} \text{ d}^2\delta_p
\]

\[
M^1(c,s,a,c) = \frac{\bar{\nu}_M}{2\lambda^1} \int \frac{d^2\delta_p}{2\pi} \chi \frac{2 \lambda^1_p}{2 \lambda^1} \left( e^{-\beta \lambda^1_s \delta_p^1} - 1 \right) e^{i \Phi_p} \text{ d}^2\delta_p
\]
The partial wave amplitude for the contribution of the Pomeron, which can change the projection of the nucleon spin

\[
\delta_{\lambda_i}^{\lambda_i}(s, t) = \frac{(-1)^{m_i}}{\sqrt{t} (2m_V)^{\lambda_i}} \sum_{\lambda'_1, \lambda'_2} \frac{\delta_{\lambda'_1}^{\lambda'_1}(s, t)}{(2m_V)^{\lambda'_1}} \phi_{\lambda'_2}^{\lambda'_2}(s, t) \frac{E_{\lambda'_1}^{\lambda'_1}}{4 \pi} \frac{b^2}{\eta^2} \eta^{\lambda_i}_{\lambda'_1}
\]

and for the Reggeon:

\[
\delta_{\lambda_i}^{\lambda_i}(s, t) = \frac{(-1)^{m_i}}{\sqrt{t} (2m_V)^{\lambda_i}} \sum_{\lambda'_1, \lambda'_2} \frac{\delta_{\lambda'_1}^{\lambda'_1}(s, t)}{(2m_V)^{\lambda'_1}} \phi_{\lambda'_2}^{\lambda'_2}(s, t) \frac{E_{\lambda'_1}^{\lambda'_1}}{4 \pi} \frac{b^2}{\eta^2} \eta^{\lambda_i}_{\lambda'_1}
\]

Eikonal \( \rightarrow \chi = 2\delta \) -- partial wave
And finally we state as the second order example, i.e. the correlation modified Gribov-Absorption cut:
(it is this formula which we use in our phenomenological investigation)

**Helicity nonflip**

\[-(2) \lambda^0_{\mu_1} \sqrt{\frac{m^2}{m_{\mu_1}^2}} \left( \frac{p^0_{\mu_1}}{\omega} \right) \frac{p_{\mu_1}}{P_{\mu_1}^0} (\omega) \frac{p_{\mu_1}}{P_{\mu_1}^0} (\omega) \left( \frac{1}{1 + \frac{m_{\mu_1}^2}{m_{\mu_1}^2}} \right) \leq \frac{m_{\mu_1}^0 + C \left( m_{\mu_1}^0 + m_{\mu_1}^0 \right)}{m_{\mu_1}^0 + k_c + m_{\mu_1}^0} \text{[Eq.]}\]

**Helicity flip**

\[-(2) \lambda^1_{\mu_1} \sqrt{\frac{m^2}{m_{\mu_1}^2}} \left( \frac{p^1_{\mu_1}}{\omega} \right) \frac{p_{\mu_1}}{P_{\mu_1}^1} (\omega) \frac{p_{\mu_1}}{P_{\mu_1}^1} (\omega) \left( \frac{1}{1 + \frac{m_{\mu_1}^2}{m_{\mu_1}^2}} \right) \leq \frac{m_{\mu_1}^1 + C \left( m_{\mu_1}^1 + m_{\mu_1}^1 \right)}{m_{\mu_1}^1 + k_c + m_{\mu_1}^1} \text{[Eq.]}\]
V. THE "DERIVATIVE RULE" APPLIED TO THE CORRELATION MODIFIED WEAK ABSORPTION MODEL WITH (A) TRADITIONAL POMERON INPUT AND (B) HARTLEY-KANE POMERON INPUT

(A) At a preliminary stage of our investigation we applied the "derivative rule" in order to obtain the helicity flip amplitude. We change our notation such that:

\[ \lambda_p = \frac{1}{2} \bar{g}_{p_{(2)}}(c^2_{(1)}) A \quad \lambda_p = B \quad \lambda_p = \frac{1}{2} \bar{g}_{p_{(2)}}(c^2_{(1)}) \quad \lambda_p = \frac{C_{p_{(1)}}}{c^2_{(1)}} \quad \lambda_p = \frac{1}{2} \bar{g}_{p_{(2)}}(c^2_{(1)}) \]

and write for the Regge pole:

\[ M_{1, C_{S_{(2)}}} = \frac{1}{2} A_1 \bar{g}_{C_{S_{(2)}}} \rho - \frac{1}{2} B_2 \]

and for the Pomeron:

\[ M_{2, C_{S_{(2)}}} = \frac{1}{2} A_1 \bar{g}_{C_{S_{(2)}}} \rho - \frac{1}{2} B_2 \]

where:

\[ A_1 = A_2 \quad A_2 = - A_2 \quad B_1 = B_1 \quad B_2 = B_1 \]

Inserting the pole, the pomeron and the correlation kernel into the Gribov expression we obtain:

Helicity nonflip cut amplitude:

\[ M_{(S_{(2)}, A)} = - \frac{1}{2} C_{p_{(2)}} \left( \frac{R_2}{\tau} \right) \rho - \frac{1}{2} \sum_{i, j} A_i \frac{A_j^{R_2}}{R_2 + B_1 + B_2} \]

We now apply the derivative rule:

\[ \phi_{S_{(2)}, A} = \phi_{S_{(2)}}(c^2_{(1)}) \]

where \( c_o \) is an arbitrary parameter.
Helicity flip cut amplitude of correlation modified weak absorption by means of the "derivative rule" with traditional Pomeron input.

we obtain:

\[ M(\sigma, b) = c_0 (2 \sqrt{s}) \left\{ -\frac{1}{2} C_{\text{cor}} \beta \left[ \frac{R^2}{s} - \left(\frac{R^2}{s} + c_0 \right) \right] \right\} \]

\[ \times \sum_{\ell^+ \ell^-} \frac{A_i}{\frac{R^2}{s} + B_i + c_0} \left[ \frac{\left(\frac{R^2}{s} + c_0\right)^2}{\frac{R^2}{s} + B_i + c_0} - \left(\frac{R^2}{s} + c_0\right) \right] J_{0} \left( R_0 \sqrt{p^2 - 2\pi k^2} \right) \]

(B) We now state our correlation modified version of a HARTLEY-KANE POMERON INPUT (10) although the phenomenological investigation has not yet been carried out.

\[ M_{\text{HK}}(\sigma, b) = -\lambda \sum a \beta \left( \frac{k_2^2}{2} + a \right) e^{-\frac{k_2^2}{2}} J_0 \left( R_0 \sqrt{p^2 - 2\pi k^2} \right) \]
Correlation modified weak absorption with Pomeron input à la Hartley-Kane.

\[
M(s_\perp, t) = -\frac{\lambda_s}{4\sqrt{\eta}} \left[ \sum_{\alpha+i} \frac{A_i}{(b+Q_i+t\epsilon)} \right] \left[ \frac{(b+2\epsilon)^2}{(b+Q_i+t\epsilon)} \right]^{1/2} \\
+ \frac{\lambda_s}{4\sqrt{\eta}} \left[ \sum_{\alpha+i} \frac{A_i}{(b+Q_i+t\epsilon)} \right]^{1/2} \left[ \frac{(b+2\epsilon)^2}{(b+Q_i+t\epsilon)} \right]^{1/2}
\]

We now use the "derivative rule"

\[
\phi_{\perp} (s_\perp, t) = \frac{C_0}{\alpha_Y} \phi_{\perp} (s_\perp, t)
\]
And find the following result: Correlation modified weak absorption with Pomeron input a la Hartl helicity flip cut amplitude by means of the "derivative rule".

\[
M(\xi, \xi') = c_0 \left( -\frac{\lambda S^{\xi(\xi')}_{\xi}}{4\sqrt{\pi}} \right) \left[ 2a\lambda^{-\frac{(b+c)}{4}} \right] \sum_{i=1}^{n} \frac{A_i}{(b+3i+4c)} + \left( \frac{b+3c}{b+3i+4c} \right)^2 \left( \frac{R_0^2}{b+3i+4c} \right) \left( \bar{\lambda} - \frac{\lambda}{\sqrt{\pi}} \right) + \left( \frac{b+3c}{b+3i+4c} \right) \left( \frac{R_0}{\sqrt{b+3i+4c}} \right) \left( \bar{\lambda} - \frac{\lambda}{\sqrt{\pi}} \right) - \frac{(b+3c)}{b+3i+4c} \left( \frac{R_0}{\sqrt{b+3i+4c}} \right) \left( \bar{\lambda} - \frac{\lambda}{\sqrt{\pi}} \right)
\]
PART TWO

PHENOMENOLOGICAL INVESTIGATION*

* Figure numbers start again from 1.
A discussion of the mechanism by which the correlation modified absorption model rectifies the incorrect behaviour predicted by traditional reggeized absorption.

The pion-nucleon system is completely determined by 4 complex amplitudes, namely both isoscalar amplitudes in their helicity non-flip and helicity flip states. These amplitudes can, in turn, be extracted experiment if there exists a complete set of measurements of the observables, i.e. the differential the polarization and the spin-rotation parameters R and A. Such a complete set of data exists only laboratory momentum of the incident pion at 6 GeV/c and small momentum transfer. Beyond \(|t| = \frac{1}{4}\) GeV^2, the lack of spin-rotation data prevents us from having a non-ambiguous view of the amplitudes' structure. However, in a case such an amplitude analysis \((5)\) is model-independent only up to an overall phase unless the method analyticity \((5)\) is used. The overall phase is associated with the dominating amplitude, namely the isoscalar amplitude. The phase away from forward direction of this amplitude, as found by Pietarinen, to considerably exceed the value as predicted by Barger and Phillips \((12)\). This is consistent with the assumption of Ambats et al. \((5)\). By assuming that the helicity flip isovector amplitude is strongly regge-dominated, one concludes from the constant phase difference which they have found to exist between the helicity flip helicity non-flip isoscalar amplitude, that the effective trajectories of the two amplitudes have equal \(|t| = \frac{1}{4}\) GeV^2. Pietarinen \((5)\) found a ratio of real to imaginary part of the isoscalar helicity non-flip isoscalar amplitude of -50% at 6 GeV/c for \(|t| = \frac{1}{3}\) GeV^2. We have incorporated this piece of information into our effective pomeron.
We now discuss how the gradual alteration of the effective Pomeron phase in connection with the introduction of correlation between the simultaneously exchanged pomeron and reggeon in an absorption model can produce structure of the helicity amplitudes as found by the amplitude analysis at least at small momentum transfers fixed laboratory momentum of 6 GeV/c.

We represent the amplitudes in the complex plane such that they are characterized by the following quantities:

1. Their strength $f$ in forward direction
2. The slope $Q^2$ of the assumed exponential fall-off with $|t|$ of this strength
3. Their initial phase $\theta$
4. The rotation velocity $\phi$ of this phase in dependence on $|t|$

Thus we write any amplitude as

$$\mathcal{A}_{\mu_1, \mu_2}^{\alpha, \beta} = \frac{1}{\sqrt{\pi}} \exp \left\{ -\frac{Q^2}{2} t + \lambda (\phi + \frac{\theta}{2}) \right\}$$

Its particular type is specified by its indices. These indices indicate:

1. The upper left indices $\alpha, \beta$ correspond to the $t$-channel isospin state
2. The right indices denote individual $s$-channel helicity states. The net helicity state $m$ is the individual helicity states $n_j$ where $n_j = 0$ is helicity nonflip and $n_j = 1$ helicity flip if $n_j + n_2 = 1$ helicity nonflip if $n_j + n_2 = 1$. $n = 1$ helicity flip.
3. $P, C, \text{th}, \exp$ stand for the pole, the cut, their sum which makes up our theoretical total helicity
exp denotes the amplitude as extracted by amplitude analysis from experiment.
4. The lower left indices $1, 2$ refer to the non-rotating and rotating parts respectively into which the theoretical amplitudes due to their regge signature.
VI.1 - The Crossing Symmetric Weak Cut Reggeized Absorption Model as a special case of the Gribov cut.

By setting in the Gribov cut integral the correlation kernel equal to 1 we recover the absorption model in its original form. We use the weak-cut version of this model as a starting point for the discussion about the possible modifications to traditional absorption. Gradually we then introduce the correlation in several model variants and observe the deviation of these model variants with respect to the reference model. We mean, by weak-cut, that the fixed parameter present at wrong signature points will not contribute multiplicatively to the regge residues. Thus they are not nonsense wrong signature zeros (NWSZ). Since the pomeron stays positive throughout the cross-section and the pole changes sign only in the convolution integral and the resulting cut turns out to be small. This is not the case in the standard model which was the alternative version originally adopted by the Michigan group. In this view, the third order correlation function is assumed to have a strong influence on the regge-residue in the form of strong fixed poles which cancel the help of an enhancement factor \( \lambda^2 \) pole and cut become comparable and generate the dip by interference. The weak-cut version, in contrast, just fills in the zeros already predicted by the pole. ( Unfortunatel the dip to smaller values in \( \lambda/\tau \).) The weak-cut model does not add any new parameter beyond those already present i.e. \( \lambda = 1 \). There is still some considerable amount of flexibility within the frame of the traditional weak-cut reggeization allows a choice of ghost-eliminating mechanisms, exponential factors in the residues and the trajectories. We adopt the exponential factors in the residue and choose an energy scale factor \( \epsilon = 1 \text{ (GeV)} \) choice is equivalent to the introduction of an exponential factor in the residue. Thus we absorb any deviation from our trajectory chooses nonsense. Thus both helicity poles vanish at \( \lambda/\tau = 0.647 \) for our particular choice of initial rho-trajectory. We employ the Gell-Mann ghost-killing mechanism and the Gamma function left over by this exponential parameterization of the residue function.

The parameters in Table I provide a reasonable choice for treating pion-nucleon charge exchange at 6 GeV/c with our reference model. We state explicitly the dependence of the cut characterizing quantities on all parameters down initial cut strength, shrinking velocity of this strength, initial phase angle and rotation velocity of it. This on the level of amplitudes, what is needed to improve absorption and trace the effect the correlation has on the We use six Regge-parameters which are:

1. the two helicity dependent residues \( \hat{D}_k \) at \( \lambda/\tau = 0 \)
2. the two helicity-dependent residue slopes \( \hat{J}_k \) of their exponential fall-off with increasing \( \lambda/\tau \).
3. the two helicity-independent parameters which determine the linear regge-trajectory i.e. the intercept and slope \( \beta \). We relate intercept and slope via the following expression \( \beta = 1 + \epsilon \beta \) with mass of the rho meson \( m_\rho = 773 \text{ GeV/c}^2 \).
There are four parameters for the effective pomeron. The traditional absorption model is characterized by its use helicity conserving purely imaginary and stationary pomeron as the absorptive factor. In this way we have effectively only for the pomeron, the residue $\delta^N_0$ at $/t/ = 0$ and the slope $\lambda^N_0$ of the exponential fall-off with increasing $/t/$.

paraneters, by fitting the helicity nonflip isoscalar amplitude as taken from the amplitude analysis by Ambats et al. the opacity is fixed at $C_{Op} = 0.79$. Then the model will involve altogether five free parameters in order to describe cross-section and predict the non-vanishing polarization of the recoil nucleon. The parameter values in Table I do a "best fit" to the differential cross-section. They rather provide a good set of initial values liable to systematic im;
clearly demonstrate success and failure of the traditional weak-cut reggeized absorption model. For this reason we cut, the theoretical total amplitude obtained as sum of pole and cut, and the total amplitude as found in the amplitude et al. One comment to the amplitude analysis is in order. Because of the arbitrary nature of the overall phase, At all the phases relative to the helicity nonflip isoscalar amplitude. They denote a parallel component pointing in the reference amplitude and a perpendicular one orthogonal to this direction. At $/t/ = 0$ their reference amplitude is purely imaginary and corresponds to $10\degree$. Moreover by assuming a regge-behaved helicity flip isovector amplitud phase difference observed between the isovector helicity flip and the isoscalar helicity nonflip amplitude of about 60 drawn that both amplitudes rotate counterclockwise with increasing $/t/$ and with the same velocity. This phase beh;
by transferring from "parallel-perpendicular" plane to the complex plane. However, beyond $/t/ = 0.4$, the difference diminishes increasingly fast. We present in Table IIa and IIb the numerical values of the amplitudes at , by Ambats et al in their analysis. These values are: $/t/ = 0.00, 0.05, 0.15, 0.25, 0.35, 0.45, 0.55$. For the r we can only compare with Ambats total amplitude at $/t/$ values up to $/t/ = 0.35$. For the values $/t/ = 0.45$ and $0.55$ w the magnitude but not with the phase, since this phase depends on a model for the isoscalar reference amplitude. (In Table IIC we arrange real and imaginary total theoretical amplitudes as calculated by the traditional absorption r the polarization according to the formula -

\[ \text{Polarization} = -2 \text{Im} \Theta^{0T}_{1T}/\text{differential cross section} \]

The observables as given in Table IIC are given by Ambats for the additional $/t/$ values of $/t/ = 0.65$ and $0.8$. These exhibit clearly the structure of the observables. We see in fig. 1 the differential cross section measured in mb$^{-1}$ contributions to the momentum transfer distributions from the pole, the cut, and the pole + cut. Compare Table IIr results, the theoretical differential cross section in column 3 and the experimental values in column 4. Immediate
theoretical curve deviates from an excellent description of the experimental differential cross section up to \( t/ = \). This value. Although the dip up to \( t/ = .025 \) is at the correct position, it is vastly underestimated. Furthermore, at \( t/ = .8 \) cannot be reached at the heights it should be. Nevertheless, beyond \( t/ = .4 \) the theoretical curve still qualitative feature of the data.

In Fig. 2 we see that the predicted polarization is disastrously wrong. This is particular to the traditional absorptive strong or weak nature, and demonstrates its most serious failure. Since the rate of change of the differential cross to the scattering angle is proportional to the polarization, we observe that the 90% minimum of the polarization is position as the dip in the differential cross section. This dip in turn originates in NWSZ of both helicity poles at \( t/ \) fills in the zero insufficiently, yet moves it to the desired position. A different choice of the parameters for the tr exert a strong influence on the position of the dip, and the minimum of the polarization. They could both be moved i for \( \mathcal{A} \) \( (t/ = .5-1.00 /t/ \) for example. We see in fig. 1 that the single rho pole fits the differential cross sector the zero which has to be filled in. In particular the pole alone exhibits the forward turn over around \( t/ = .05 \), giving the dominating presence of the helicity flip amplitude which was only suppressed by the angular momentum factor at to Table II(a).

Of greater importance to our discussion, however, is the specific way in which the traditional absorption happens to polarization. The three parts of Table II contain all the information on amplitude level necessary to understand the The helicity regge poles do not differ in their phase. They start (in our particular case) at 43, 20° and rotate antici- 72, 36° per units in \( t/ \) around the origin of the Argand diagram. With a confidence up to \( t/ = .35 \) the amplitude anal- behaviour only for the helicity flip amplitude. The helicity nonflip amplitude, on the contrary, rotates in a clockwise polarisation is generated by the relative phase difference between the helicity flip and the helicity nonflip amplitude, should be positive. Traditional absorption treats the helicity nonflip pole as relatively strong by comparison with the strong absorption in the helicity nonflip case is accompanied by a phase of the cut relative to the one of the pole which let us say, typically 5°. Such a cut makes the pole lose about one degree at \( t/ = 0 \). By comparison, although the helicity nonflip residue. (See Table I). This increases both the relative phase of flip cut to flip pole and the flip cut roughly in comparison to equal residue slopes. Due to this the positive start of the polarization is lost. This different
with respect to the major alterations which traditional absorption has to undergo. The reason that traditional absorption polarization only in a very small /t/ region 0 ≤ /t/ ≤ 0.075, is due to the different strengths by which the helicity pole helicity flip pole loses slightly on strength after it has been absorbed, the effect on its phase, especially for small /t/ behaviour is sharply in contrast to that of the helicity nonflip pole. See Tables IIb and IIa. Away from /t/ = 0 the pole by the relative rotation velocity per /t/ of helicity nonflip cut and pole. In the traditional absorption the rotation velocity with the pole is almost negligible about 4-5 degree per /t/ compared with the 72° of the pole. Thus, although the cut the pole is already at very small /t/ out of phase with the cut by 180°. From then onward, the absorbed pole starts to rotation away from the weakly absorbed helicity flip pole whose phase remains virtually unaffected. Thus, in principle between helicity nonflip cut and pole passes through 180° the phase difference between the two helicity amplitudes is positive to negative.

We have, in fig. 3-9 displayed the structure of the helicity amplitudes for the isovector exchange as obtained with the traditional absorption. Fig 3 shows the moduli of the helicity nonflip amplitude. Its structure reflects the two absorption - the NWSZ has been shifted from /t/ = 0.65 to a smaller value in /t/. At the same time the zeros of the r part become separated. This separation converts the zero of the regge-pole in the differential cross section into a dip in the differential cross section. Thus the separation is actually needed, but what the differential cross section cannot to separation has unfortunately been arranged the wrong way round by traditional absorption. This could only be revealed data now available for larger values in /t/. The Argand diagram in fig. 8a clearly demonstrates the zero structure of amplitude where we have given the pole and show that the cut places the zero of the real part at /t/ = 0.25 and the zero part at /t/ = 0.325 (GeV/c)^2. For comparison we have drawn, in the same figure, the amplitude as obtained in the am Ambats et al. which places the zero of the real part at about /t/ = 0.25 (GeV/c)^2 and the zero of the imaginary part at Note, however, that those values are found in the parallel perpendicular plane in reference to the isoscalar helicity no has been taken as the parallel component. These values are determined by Ambats et al. with a precision which is not this plane requires a model for the phase of the reference amplitude. One assumption about this phase is the crossover is known in this plane with an accuracy up to 0.025 and placed at /t/ = 1.5 (GeV/c)^2 into the complex plane requires a model for the phase of the reference amplitude. One assumption about this phase is et al have taken it, that the phase of the isoscalar nonflip amplitude behaves in a very similar fashion to the helicity flip: it rotates anticlockwise with the velocity of a regge pole starting by being out of phase of about 60° which amounts to
This initial value and its sense of rotation agrees with dispersion relations. This assumption is only reliable for a value of about \( t = 0.35 \) (GeV/c). For larger values in \( t \), however, a rotation in the opposite direction occurs such that the amplitude crosses the positive imaginary axis at about \( 0.4 \) to \( 0.6 \) into the first quadrant again involving elastic scattering as an absorptive factor, which takes in form of an appropriate parameterization of the isoscalar amplitude into account can satisfy within the frame of strong absorption a positive polarization.

Our final aim, however, is to construct both isospin amplitudes out of poles with the correct absorption properties, if applied to a complicated effective Pomeron, seems to lead occasionally to correct results in absorption, if applied to a complicated effective Pomeron, seems to lead occasionally to correct results in the range in \( t \) up to \( t = 2.00 \) (GeV/c)^2 and for different energies as accomplished within the frame of Gribo Sh. H. Eremyan (23). This is the best description ever achieved. The number of parameters is still great but would result in less parameters or, if not, then at least those which are less obscure. We are going to gradually build up such a model. At this stage we have discussed weak cut absorption by assuming a simple form of the elastic scattering amplitude - namely the Pomeron - purely imaginary and fixed pole and the effective amplitude namely the total isoscalar amplitude itself. The isovector amplitude through the process of absorption.

The incorrect relative phase between the two helicity amplitudes as seen in fig. 5 causes the polarization to become wrong. Note the proportionality between fig. 2 and fig. 5. Fig. 7 also shows the modulus of the helicity flip amplitude which is the cause of the wrong polarization. The phase of the helicity flip amplitude is governed by the elastic polarization unambiguously known up to \( t = 2.00 \). Elastic polarization rise and fall of models in strong interactions.

\* Ross \*\* Anderson et al \*\*\* Hartley and Kane
Traditional reggeized Absorption model at fixed energy; (fixed energy is indicated through hats on the relevant parameters).

The Pomeron is s-channel helicity conserving, purely imaginary and stationary.

Helicity non-flip - (Non-rotating part of the cut)

\[ \xi_1^{\infty} = \frac{\lambda_{R1} \hat{\rho}_1 \hat{\rho}_2}{2 \hat{\pi} \left( \text{Re} \lambda_P^0 + \hat{\lambda}_P^0 \right)} \]

\[ \phi_1^{\infty} = 0 \]

\[ \psi_1^{\infty} = \frac{\text{Re} \hat{\lambda}_P^0 \hat{\lambda}_P^0}{\text{Re} \hat{\lambda}_P^0 + \hat{\lambda}_P^0} \]

\[ \phi_1^{\infty} = 0 \]

Rotating part of the cut

\[ \xi_2^{\infty} = \frac{\lambda_{R2} \hat{\rho}_2 \hat{\rho}_2}{2 \hat{\pi} \left( \text{Re} \lambda_P^0 + \hat{\lambda}_P^0 \right)} \]

\[ \phi_2^{\infty} = -i \text{Im} \left( \phi \right) - \text{re} \text{Im} \frac{1}{\text{Re} \lambda_P^0} \]

\[ \psi_2^{\infty} = \frac{\text{Re} \hat{\lambda}_P^0 \left( \lambda_P^0, \text{Re} \hat{\lambda}_P^0 + \hat{\lambda}_P^0 \right)^2 + \text{Re} \hat{\lambda}_P^0}{\left( \text{Re} \hat{\lambda}_P^0 + \hat{\lambda}_P^0 \right)^2} \]

\[ \phi_2^{\infty} = 0 \]
Traditional reggeized Absorption model at fixed energy
(fixed energy is indicated through hats on the relevant parameters, see Table I for parameter values).

The Pomeron is s-channel helicity conserving, purely imaginary and stationary

Helicity flip - (Non-rotating part of the cut)                             (Rotating part of the cut)

\[ \phi_{1}^{01} = 0 \quad \phi_{2}^{01} = - \frac{\lambda_{1}^{\alpha} \lambda_{2}^{\beta} \lambda_{2}^{\gamma} \text{Re} \lambda_{p}^{\delta}}{2 \pi \left( \text{Re} \lambda_{p}^{\delta} + \lambda_{2}^{\gamma} \right)} \]

\[ \psi_{1}^{01} = \frac{\text{Re} \lambda_{p}^{\delta} \lambda_{2}^{\gamma}}{\text{Re} \lambda_{p}^{\delta} + \lambda_{2}^{\gamma}} \quad \psi_{2}^{01} = - \frac{\text{Re} \lambda_{p}^{\delta} \lambda_{2}^{\gamma} \lambda_{2}^{\gamma} \text{Re} \lambda_{p}^{\delta}}{\left( \text{Re} \lambda_{p}^{\delta} + \lambda_{2}^{\gamma} \right)^{2}} \]

\[ \phi_{1}^{01} = 0 \quad \phi_{2}^{01} = - \frac{\text{Re} \lambda_{p}^{\delta}}{\left( \text{Re} \lambda_{p}^{\delta} + \lambda_{2}^{\gamma} \right)^{2}} \]
We introduce in several model variants a correlation between the exchanged Regge Pomeron and measure this correlation with respect to this fixed set of parameters of the traditional absorption model. The given values of the parameter are not determined by a minimization procedure. They nevertheless provide a reasonable choice to see the discussion about the mechanism of the correlation as set against the independent typical for the traditional absorption. The indices $\rho$ for Rho Regge pole and

<table>
<thead>
<tr>
<th>Parameter type</th>
<th>Value</th>
<th>Unit</th>
<th>Parameter type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inc. lab momentum $P_{\text{lab}}$</td>
<td>6.00</td>
<td>GeV/c</td>
<td>Residue slope</td>
<td>$\lambda^0_{\text{R}}$</td>
</tr>
<tr>
<td>Tot. c.m. energy $S$</td>
<td>12.163</td>
<td>(GeV/c)$^2$</td>
<td>Initial Pomeron phase</td>
<td>$\delta_0^0$</td>
</tr>
<tr>
<td>Energy scale $S_0$</td>
<td>1.00</td>
<td>(GeV/c)$^4$</td>
<td>Correlation real part</td>
<td>$\Re c^0$</td>
</tr>
<tr>
<td>Intersect of trajectory $\alpha_{\text{L}}^{\infty}$</td>
<td>.52</td>
<td></td>
<td>Correlation imaginary part</td>
<td>$\Im c^0$</td>
</tr>
<tr>
<td>Slope of trajectory $\delta^0_{\text{R}}$</td>
<td>.804</td>
<td>(GeV/c)$^2$</td>
<td>Cut enhancement $\lambda_{\text{R}}^0$</td>
<td>1.00</td>
</tr>
<tr>
<td>Residue const. $\beta_{\text{R}}^0$</td>
<td>.431</td>
<td>(GeV/c)$^3$</td>
<td>Helicity flip $\delta_{\text{R}}^0$</td>
<td>2.00</td>
</tr>
<tr>
<td>Residue slope $\lambda_{\text{R}}^0$</td>
<td>8.00</td>
<td>(GeV/c)$^3$</td>
<td>Helicity flip $\delta_{\text{R}}^1$</td>
<td>4.00</td>
</tr>
<tr>
<td>Intercept of trajectory $\alpha_{\text{L}}^{\mu}$</td>
<td>1.00</td>
<td></td>
<td>Helicity flip $\delta_{\text{R}}^{\mu}$</td>
<td>0.00</td>
</tr>
<tr>
<td>Slope of trajectory $\delta^0_{\text{L}}$</td>
<td>0.00</td>
<td>(GeV/c)$^2$</td>
<td>Helicity flip $\delta_{\text{L}}^0$</td>
<td>0.00</td>
</tr>
<tr>
<td>Residue const. $\beta_{\text{L}}^0$</td>
<td>6.16</td>
<td>(GeV/c)$^2$</td>
<td>Helicity flip $\delta_{\text{L}}^1$</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Traditional Absorption model (parameters from Table I)

\[ T^0_\varphi = 0.958 \sin \frac{\pi}{2} (0.52 - 0.804 |t|) e^{-j(43.20^\circ + 72.36^\circ |t|)} \]

\[ T^0_c = -0.071 \frac{1}{2} e^{-j(0.00^\circ + 0.00^\circ |t|)} \]

\[ + 0.07 \frac{1}{2} e^{-j(0.81^\circ + 13.8^\circ |t|)} \]

\[ T^1_\varphi = 4.75 |t|^{1/2} \sin \frac{\pi}{2} (0.52 - 0.804 |t|) e^{-j(43.20^\circ + 72.36^\circ |t|)} \]

\[ T^0_\varphi = -0.25 |t|^{1/2} \sin \frac{\pi}{2} (0.804 - 0.52 |t|) e^{-j(0.00^\circ + 0.00^\circ |t|)} \]

\[ + 0.233 |t|^{1/2} e^{-j(-57.36^\circ + 36.4^\circ |t|)} \]
TABLE IIa  Traditional Absorption model (helicity nonflip amplitudes given in $(mb)^2/GeV/c$)
| \( t' \) | Traditional Absorption model (helicity nonflip amplitudes given in \((\text{mb})^2/\text{GeV}/c)\) |
|---|---|---|
| \( t' = .55 \) | \( T_P = .001 e^{i 83.00^\circ} \) | \( t' = .65 \) | \( T_P = .00 \) | \( t' = .8 \) | \( T_P = .00 \) |
| \( T_c \) | \( .016 e^{i 230.91^\circ} \) | \( T_c \) | \( .014 e^{i 230.15^\circ} \) | \( .0072 \) |
| \( T_{t4} \) | \( .017 e^{i 29.09^\circ} \) | \( T_{t4} \) | \( .014 e^{i 230.15^\circ} \) | \( .0072 \) |
| \( T_{pp} \) | \( .055 e^{i ?} \) | \( T_{pp} \) | \( ? \) | \( ? \) |
### Table II b: Traditional Absorption Model (helicity flip amplitudes given in (mb)^2/GeV/c)

<table>
<thead>
<tr>
<th>t/ = 0.05</th>
<th>t/ = 0.15</th>
<th>t/ = 0.25</th>
<th>t/ = 0.35</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_\Phi^-</td>
<td>0.626 i 41.8°</td>
<td>0.623 i 54.0°</td>
<td>0.441 i 61.2°</td>
</tr>
<tr>
<td>T_\Phi^0</td>
<td>0.57 i 23.8°</td>
<td>0.66 i 27.5°</td>
<td>0.66 i 23.5°</td>
</tr>
<tr>
<td>T_\Phi^1</td>
<td>0.47 i 9.0°</td>
<td>0.55 i 53.8°</td>
<td>0.75 i 2.16°</td>
</tr>
<tr>
<td>T_\Phi\bar{\chi}^-</td>
<td>0.76 i 51.1°</td>
<td>0.6 i 56.6°</td>
<td>0.316 i 6.9°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t/ = 0.45</th>
<th>t/ = 0.55</th>
<th>t/ = 0.65</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_\Phi^-</td>
<td>-0.136 i 75.7°</td>
<td>-0.15 i 63.0°</td>
</tr>
<tr>
<td>T_\Phi^0</td>
<td>0.053 i 23.4°</td>
<td>0.06 i 25.2°</td>
</tr>
<tr>
<td>T_\Phi^1</td>
<td>0.81 i 8.0°</td>
<td>0.23 i 19.7°</td>
</tr>
<tr>
<td>T_\Phi\bar{\chi}^-</td>
<td>0.09 i 2°</td>
<td>0.076 i 2°</td>
</tr>
</tbody>
</table>
Polarization Chart (traditional reggeized Absorption model with parameters from Table I). Real and imaginary part of the helicity amplitudes are arranged according to
\[ \text{Polarization} = -2 \text{Im} T^0 T^1 / \text{differential cross section} \]

<table>
<thead>
<tr>
<th>( h )</th>
<th>( \text{Re} T^0 )</th>
<th>( \text{Im} T^1 )</th>
<th>( \text{Re} T^0 \text{Im} T^1 )</th>
<th>( \alpha / \alpha )</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.36</td>
<td>0.00</td>
<td>-0.36</td>
<td>3.67</td>
<td>3.62 ± 0.07</td>
</tr>
<tr>
<td>0.5</td>
<td>0.23</td>
<td>0.34</td>
<td>-0.23</td>
<td>3.73</td>
<td>3.73 ± 0.03</td>
</tr>
<tr>
<td>0.15</td>
<td>0.05</td>
<td>0.19</td>
<td>-0.08</td>
<td>0.30</td>
<td>0.27 ± 0.02</td>
</tr>
<tr>
<td>0.15</td>
<td>0.01</td>
<td>0.30</td>
<td>-0.01</td>
<td>1.63</td>
<td>1.01 ± 0.01</td>
</tr>
<tr>
<td>0.35</td>
<td>-0.01</td>
<td>0.18</td>
<td>-0.001</td>
<td>0.06</td>
<td>0.38 ± 0.04</td>
</tr>
<tr>
<td>0.45</td>
<td>0.01</td>
<td>0.08</td>
<td>-0.01</td>
<td>0.08</td>
<td>0.01 ± 0.01</td>
</tr>
<tr>
<td>0.55</td>
<td>-0.002</td>
<td>0.02</td>
<td>-0.001</td>
<td>0.01</td>
<td>0.59 ± 0.01</td>
</tr>
<tr>
<td>0.65</td>
<td>-0.009</td>
<td>0.03</td>
<td>-0.011</td>
<td>0.017</td>
<td>0.59 ± 0.01</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.006</td>
<td>(-0.057)</td>
<td>-0.008</td>
<td>0.035</td>
<td>0.04 ± 0.01</td>
</tr>
</tbody>
</table>
TABLE IIId  Polarization chart analogous to Table IIc with one difference -  λ cut = 2.00  
The boost factor improves the differential cross section - but not the polarization

<table>
<thead>
<tr>
<th>δ</th>
<th>( \frac{Q_{e}^{T}}{Q_{e}^{T}} \cdot \frac{T_{A}^{T}}{T_{A}^{T}} - \frac{T_{A}^{'T}}{T_{A}^{'T}} \cdot \frac{Q_{e}^{T}}{Q_{e}^{T}} )</th>
<th>( d^2\sigma / d\Omega )</th>
<th>( \Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00</td>
<td>.055 * .00 - .357 * .00</td>
<td>.371</td>
<td>.362 ± .07</td>
</tr>
<tr>
<td>.05</td>
<td>.206 * .00 - .21 * .00</td>
<td>.308</td>
<td>.624 ± .03 - .010</td>
</tr>
<tr>
<td>.15</td>
<td>.02 * .245 - .02 * .328</td>
<td>.397</td>
<td>.26 ± .06 - .010</td>
</tr>
<tr>
<td>.25</td>
<td>- .032 * .314 - (- .0179) * 1.372</td>
<td>.112</td>
<td>.10 ± .12 - .24</td>
</tr>
<tr>
<td>.35</td>
<td>- .042 * .158 - (- .036) * .0315</td>
<td>.089</td>
<td>.03 ± .04 - .06</td>
</tr>
<tr>
<td>.55</td>
<td>- .044 * .075 - (- .036) * .039</td>
<td>.067</td>
<td>.01 ± .01 - .96</td>
</tr>
<tr>
<td>.65</td>
<td>- .027 (- .026) - (- .015) (- .060)</td>
<td>.005</td>
<td>.005 ± .001 - .96</td>
</tr>
<tr>
<td>.65</td>
<td>- .0126 (- .063) (- .046) (- .013)</td>
<td>.007</td>
<td>.005 ± .001 - .606</td>
</tr>
<tr>
<td>.8</td>
<td>- .0126 (- .063) (- .046) (- .013)</td>
<td>.007</td>
<td>.005 ± .001 - .606</td>
</tr>
</tbody>
</table>
TABLE IIe  
In order to show that the failure of traditional absorption is mainly due to the amplitude as far as the polarization is concerned (this applies in particular if we combine the theoretical helicity flip values with the helicity nonflip value) et al (Model for isoscalar had to be used however)

<table>
<thead>
<tr>
<th>t[1]</th>
<th>$Q_e T^0 \phi T^1$ - $\bar{J}^0 \theta R^1$</th>
<th>$\delta^e/\lambda t$</th>
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</thead>
<tbody>
<tr>
<td>.00</td>
<td>-.078 x .00 - .548 x .00</td>
<td>.367</td>
</tr>
<tr>
<td>.05</td>
<td>.235 x .387 - .190 x .875</td>
<td>.350</td>
</tr>
<tr>
<td>.15</td>
<td>.048 x .677 - .093 x .306</td>
<td>.279</td>
</tr>
<tr>
<td>.25</td>
<td>.084 x .389 - (.073) x .163</td>
<td>.127</td>
</tr>
<tr>
<td>.35</td>
<td>-.009 x .853 - (.066) x .057</td>
<td>.045</td>
</tr>
<tr>
<td>.55</td>
<td>.016 x .853 - (-.001) x .067</td>
<td>.011</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.008</td>
</tr>
<tr>
<td>$T_0$</td>
<td>$T_0$</td>
<td>$\Theta_{T_0}$</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>-----------------</td>
</tr>
<tr>
<td>0.10</td>
<td>6.16</td>
<td>23.21</td>
</tr>
<tr>
<td>0.05</td>
<td>5.42</td>
<td>21.96</td>
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<tr>
<td>0.15</td>
<td>3.53</td>
<td>12.24</td>
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<tr>
<td>0.25</td>
<td>2.63</td>
<td>6.01</td>
</tr>
<tr>
<td>0.35</td>
<td>1.65</td>
<td>2.85</td>
</tr>
<tr>
<td>0.45</td>
<td>1.17</td>
<td>1.34</td>
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<tr>
<td>0.55</td>
<td>.801</td>
<td>.635</td>
</tr>
<tr>
<td>0.65</td>
<td>.556</td>
<td>.39</td>
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<tr>
<td>.8</td>
<td>.319</td>
<td>.105</td>
</tr>
<tr>
<td>1.0</td>
<td>.157</td>
<td>.024</td>
</tr>
<tr>
<td>1.2</td>
<td>.073</td>
<td>.006</td>
</tr>
<tr>
<td>1.5</td>
<td>.024</td>
<td>.001</td>
</tr>
<tr>
<td>2.0</td>
<td>.004</td>
<td>.0001</td>
</tr>
</tbody>
</table>

**MODEL la Elastic chart**
Traditional reggeized absorption model with $\lambda_{cut} = 1.00$

Differential cross-section measured in $\text{mb/}(\text{GeV/c})^2$
Polarization of pion-nucleon charge exchange at 6 GeV/c by the traditional Reggeized absorption model (purely imaginary and pomeron) with \( \Lambda \) cut = 1.0.
Traditional absorption model with $\lambda_{\text{cut}}$ =
Phase of helicity nonflip isovector amplitude after traditional absorption

Phase of isovector, traditional
Relative phase between helicity flip isovector amplitude and helicity nonflip isovector amplitude as obtained from the traditional absorption model with $\lambda_{\text{cut}} = 1.70$. 

![Graph showing the relative phase between helicity flip and nonflip isovector amplitudes.](image)
Traditional absorption model (purely imaginary and stationary Pomeron and \( \Lambda \) cut = 1.00)
See Table IIa for numerical values

---

Helicity nonflip pole
\[ \rightarrow \]
Absorbed amplitude
\[ \rightarrow \]

Numbers indicate \( |t| \) values

Scale:
1 cm = 0.019 (mb)²/GeV/c

---

Argand diagram for the helicity nonflip amplitude

fig 6
Traditional absorption model (purely imaginary and stationary Pomeron and cut = 1.00)
See Table IIb for numerical values.

Helicity flip pole
Absorbed amplitude

Numbers indicate /t/ values
Scale
1 cm = 0.038(mb)^1/2/GeV/c

Argand diagram for t isovector amplitude
fig 7
Argand diagram for the helicity nonflip isovector amplitude obtained with the help of a traditional absorption model (purely imaginary and stationary Pomeron and \( \Lambda \) cut = \( \cdots \)).
Previous diagram for the helicity nonflip isovector amplitude enlarged for large $|t|$ region.

![Graph showing $\sqrt{mb/(GeV^2)}$]
Argand diagram for the helicity flip isovector amplitude for small $|t|$ of traditional absorption model (purely imaginary and stationary Pomeron).

The numbers on the curve indicate values in $|t|$.

---

Fig. 9a
Previous diagram for helicity flip isovector amplitude enlarged for large \( \ell \ell \) region.
VI. II - The Gribov Cut represented as vector in the Argand Diagram

The parameterization of the cut allows for a moving helicity conserving Pomeron with real part in forward direction. The correlation parameter \( c \) can be helicity dependent and has a slow energy dependence in order to stabilize a possible dependence of the phase. The actual values of the parameters give rise to a set of models, out of which we choose those that satisfy the optimum between theoretical constraints and phenomenological necessity.

Helicity non-flip cut

\[
\mathcal{T}_{\mu}^{\nu}(s, s', \lambda^2, \mu^2, \sigma^2, \sigma^0, \lambda^2, \mu^2, \sigma^2, \sigma^0) = -\frac{1}{2} \epsilon \langle \Phi^{0} - \Phi^{0} | t \rangle \langle \Phi^{0} - \Phi^{0} | t \rangle + \frac{1}{2} \epsilon \langle \Phi^{0} - \Phi^{0} | t \rangle \langle \Phi^{0} - \Phi^{0} | t \rangle
\]

Helicity flip cut

\[
\mathcal{T}_{\mu}^{\nu}(s, s', \lambda^2, \mu^2, \sigma^2, \sigma^0, \lambda^2, \mu^2, \sigma^2, \sigma^0) = -\frac{1}{2} \epsilon \langle |t|^{-1} \rangle \langle \Phi^{0} - \Phi^{0} | t \rangle + \frac{1}{2} \epsilon \langle |t|^{-1} \rangle \langle \Phi^{0} - \Phi^{0} | t \rangle
\]
We have split the cut into "non rotating" and "rotating" vectors in the complex plane indicated by 1 and 2 respectively.

Their explicit dependence on all parameters involved is as follows:

\[ J_1^\infty = \frac{(2 \sqrt{\pi})^{-1} \chi_0^0 \beta_0^0 \beta_0^0 (\gamma_0)}{\left\{ (\lambda_0^0 + \alpha_{\lambda}^0 \mu (\gamma_0) + \lambda_0^0 + \alpha_{\lambda}^0 \mu (\gamma_0) + i \Re (c_i^0 + c_2^0 \mu (\gamma_0)))^2 + \left( 4 \left( c_1^0 + c_2^0 \mu (\gamma_0) \right) \right)^{-1} \right\}} \]

\[ J_2^\infty = \frac{(2 \sqrt{\pi})^{-1} \chi_0^0 \beta_0^0 \beta_0^0 (\gamma_0)}{\left\{ (\lambda_0^0 + \alpha_{\lambda}^0 \mu (\gamma_0) + \lambda_0^0 + \alpha_{\lambda}^0 \mu (\gamma_0) + i \Re (c_i^0 + c_2^0 \mu (\gamma_0)))^2 + \left( 4 \left( c_1^0 + c_2^0 \mu (\gamma_0) \right) \right)^{-1} \right\}} \]

\[ \frac{1}{2} \]
Slope of the exponential fall off with $/t/$ of the strength of the non rotating part of the helicity non flip cut

\[ a_{1,1} = \left\{ \left( \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) \right) \left( \lambda_\omega + \alpha_\omega \ln(\gamma_\omega) \right) + \Re \left( c_i \cdot c_i \ln(\gamma_i) \right) \left( \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) \right) + \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) \right\} \left\{ \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) + \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) + 4 \Re \left( c_i \cdot c_i \right) \right\} \\
+ \left\{ \Im \left( c_i \cdot c_i \ln(\gamma_i) \right) \left( \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) + \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) \right) \right\} \left\{ 4 \Im \left( c_i \cdot c_i \ln(\gamma_i) \right) \right\}^{\frac{1}{2}} + \left\{ 4 \Im \left( c_i \cdot c_i \ln(\gamma_i) \right) \right\}^{\frac{1}{2}} \left\{ \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) \right\} \\
+ \left\{ \Re \left( c_i \cdot c_i \ln(\gamma_i) \right) \left( \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) + \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) \right) \right\} \left\{ 4 \Re \left( c_i \cdot c_i \ln(\gamma_i) \right) \right\}^{\frac{1}{2}} + \left\{ 4 \Im \left( c_i \cdot c_i \ln(\gamma_i) \right) \right\}^{\frac{1}{2}} \left\{ \lambda_\rho + \alpha_\rho \ln(\gamma_\rho) \right\} \right\}^{\frac{1}{2}} \]
Slope of the exponential fall-off with /t/ of the strength of the rotating part of the helicity non flip cut

\[ \psi_{\zeta} = \left\{ \left( \lambda_0^\phi + \lambda_0^\varphi \ln(C_{\zeta}) \right) \left( \lambda_0^\phi + \lambda_0^\varphi \ln(C_{\zeta}) \right) + Re \left( c_0^i + c_0^2 \ln(C_{\zeta}) \right) \left( \lambda_0^\phi + \lambda_0^\varphi \ln(C_{\zeta}) \right) \right\} - \frac{\pi \alpha^\phi}{2} \left( \frac{\ln \lambda_0^\phi}{\lambda_0^\phi} + \frac{\ln \lambda_0^\varphi}{\lambda_0^\varphi} \right) \]
Initial angle of the "non rotating part" of the helicity non flip cut

\[ \phi_{1}^{0} = \delta_{p} - \frac{\pi}{2} - \omega \chi_{j} - \frac{4}{ \lambda_{p} + \alpha_{p} \lambda_{i} (\lambda_{e}) + \lambda_{g} + \alpha_{g} \lambda_{i} (\lambda_{e}) + 4 \Re (c_{i} + c_{l})} \]

Initial angle of the "rotating part" of the helicity non flip cut

\[ \phi_{2}^{0} = -\omega \chi_{i} (0) + \delta_{p} - \frac{\pi}{2} - \omega \chi_{j} - \frac{4}{ \lambda_{p} + \alpha_{p} \lambda_{i} (\lambda_{e}) + \lambda_{g} + \alpha_{g} \lambda_{i} (\lambda_{e}) + 4 \Re (c_{i} + c_{l})} \]
Rotation velocity per /t/ of the "non rotating part" of the helicity non flip cut

\[ \Phi^\circ_{\perp} \approx \left[ \left\{ \lambda_p + \alpha_L \lambda(c_{5/0}) \right\} C_{\lambda} + \alpha_L \lambda(c_{5/0}) + \lambda_p + \alpha_L \lambda(c_{5/0}) - \frac{\alpha_L \lambda}{2} \right] \left[ \left\{ \lambda_p + \alpha_L \lambda(c_{5/0}) \right\} C_{\lambda} + \alpha_L \lambda(c_{5/0}) + \lambda_p + \alpha_L \lambda(c_{5/0}) + \frac{\alpha_L \lambda}{2} \right]^{-1} \]
Rotation velocity per \( t \) of the "rotating part" of the helicity non flip cut

\[
\Phi_{\infty}^{(2)} = \left[ \begin{array}{c}
- \frac{1}{2} \alpha_{\phi} \left( \lambda_{\theta} + \alpha_{\phi} \nu_{\theta} (L_{\theta}) + \beta_{\eta} \rho_{\eta} (L_{\eta}) \right) + \gamma_{\eta} \rho_{\eta} (L_{\eta}) \\
\end{array} \right] + \left[ \begin{array}{c}
- \frac{1}{2} \alpha_{\phi} \left( \lambda_{\phi} + \alpha_{\phi} \nu_{\phi} (L_{\phi}) + \beta_{\phi} \rho_{\phi} (L_{\phi}) \right) - \frac{1}{2} \alpha_{\phi} \left( \lambda_{\phi} + \alpha_{\phi} \nu_{\phi} (L_{\phi}) + \beta_{\phi} \rho_{\phi} (L_{\phi}) \right) + \left( \lambda_{\phi} + \alpha_{\phi} \nu_{\phi} (L_{\phi}) + \beta_{\phi} \rho_{\phi} (L_{\phi}) \right) \end{array} \right] \]
Strength of the "non rotating part" of the helicity flip cut in forward direction

\[
\frac{\left< T_{L}^{-1} \right>}{J_{p}} \frac{1}{\lambda_{p}} \left| \beta_{p} \right|^{2} \left\{ \left( \lambda_{p}^{2} + \alpha_{p} \beta_{p} (\gamma_{p}) + 2 \Re (c_{1} + c_{2} \beta_{p} (\gamma_{p})) \right) + 2 \Re (c_{1}^{*} + c_{2}^{*} \beta_{p} (\gamma_{p})) - \frac{T_{L}}{2} \right\}
\]

Strength of the "rotating part" of the helicity flip cut in forward direction

\[
\frac{\left< T_{L}^{-1} \right>}{J_{p}} \frac{1}{\lambda_{p}} \left| \beta_{p} \right|^{2} \left\{ \left( \lambda_{p}^{2} + \alpha_{p} \beta_{p} (\gamma_{p}) + 2 \Re (c_{1} + c_{2} \beta_{p} (\gamma_{p})) \right) + 2 \Re (c_{1}^{*} + c_{2}^{*} \beta_{p} (\gamma_{p})) \right\}^{2} + \frac{1}{2} \frac{T_{L}}{J_{p}} (c_{1} + c_{2} \beta_{p} (\gamma_{p})).
\]
Slope of the exponential fall-off with \( t \) of the strength of the non rotating part of the helicity flip cut

\[
\frac{\mu}{2} = \left[ \left\{ (\lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2})) (\lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2})) + \text{Re} \left( c_1 + c_2 \mu (\frac{\gamma}{2}) \right) \right\} \right]^{-1} + \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \text{Re} \left( c_1 + c_2 \mu (\frac{\gamma}{2}) \right) \frac{\alpha \epsilon \rho}{\mathbf{2}} \left\{ \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \text{Re} \left( c_1 + c_2 \mu (\frac{\gamma}{2}) \right) \frac{\alpha \epsilon \rho}{\mathbf{2}} \left\{ \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \text{Re} \left( c_1 + c_2 \mu (\frac{\gamma}{2}) \right) \frac{\alpha \epsilon \rho}{\mathbf{2}} \left\{ \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \text{Re} \left( c_1 + c_2 \mu (\frac{\gamma}{2}) \right) \frac{\alpha \epsilon \rho}{\mathbf{2}} \left\{ \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \text{Re} \left( c_1 + c_2 \mu (\frac{\gamma}{2}) \right) \frac{\alpha \epsilon \rho}{\mathbf{2}} \left\{ \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \lambda \rho + \alpha \epsilon \mu (\frac{\gamma}{2}) + \text{Re} \left( c_1 + c_2 \mu (\frac{\gamma}{2}) \right) \right\} \right\} \right\} \right\} \right\}
Slope of the exponential fall-off with $t_f$ of the strength of the "rotating part" of the helicity flip cut

$$
\eta_{01} \frac{d}{d t_f} = \left[ \left( \lambda_{0} + \alpha_{0} \, h_n \left( \gamma_{0} \right) \right) \left( \lambda_{1} + \alpha_{1} \, h_n \left( \gamma_{1} \right) \right) + \text{Re} \left( c_{1} + c_{2} \, h_n \left( \gamma_{2} \right) \right) \left( \lambda_{0} + \alpha_{0} \, h_n \left( \gamma_{0} \right) \right) + \lambda_{1} + \alpha_{1} \, h_n \left( \gamma_{1} \right) + \text{Im} \left( c_{1} + c_{2} \, h_n \left( \gamma_{2} \right) \right) \right] \left( \eta_{0} \frac{d}{d t_f} + \frac{\text{Re} \left( c_{1} + c_{2} \, h_n \left( \gamma_{2} \right) \right)}{\lambda_{1}} \right) - \frac{\text{Re} \left( c_{1} + c_{2} \, h_n \left( \gamma_{2} \right) \right)}{\lambda_{1}} \\
\times \left\{ \lambda_{0} + \alpha_{0} \, h_n \left( \gamma_{0} \right) + \lambda_{1} + \alpha_{1} \, h_n \left( \gamma_{1} \right) + 4 \text{Re} \left( c_{1} + c_{2} \, h_n \left( \gamma_{2} \right) \right) \right\} + \left\{ \lambda_{0} + \alpha_{0} \, h_n \left( \gamma_{0} \right) + \lambda_{1} + \alpha_{1} \, h_n \left( \gamma_{1} \right) + 4 \text{Re} \left( c_{1} + c_{2} \, h_n \left( \gamma_{2} \right) \right) \right\} \right\}
$$
Initial angle of the "non rotating part" of the helicity flip cut

\[ \phi_1^{0} = \delta_0 - \frac{\pi}{2} - 2 \mu \cot \gamma - \frac{4 J_m(c_1 + c_2 \lambda c(\frac{\pi}{2}))}{\lambda_0 + \alpha_0 \lambda c(\frac{\pi}{2}) + \lambda_0^* + \alpha_0^* \lambda c^*(\frac{\pi}{2}) + 2 \Re \overline{c} c} \]

Initial angle of the "rotating part" of the helicity flip cut

\[ \phi_2^{0} = -\frac{\pi}{2} \delta_0 + \delta_0 - \frac{\pi}{2} - 2 \mu \cot \gamma - \frac{4 J_m(c_1 + c_2 \lambda c(\frac{\pi}{2}))}{\lambda_0 + \alpha_0 \lambda c(\frac{\pi}{2}) + \lambda_0^* + \alpha_0^* \lambda c^*(\frac{\pi}{2}) + 2 \Re \overline{c} c} \]

\[ + \mu \cot \gamma \frac{2 J_m(c_1 + c_2 \lambda c(\frac{\pi}{2}))}{\lambda_0 + \alpha_0 \lambda c(\frac{\pi}{2}) + \lambda_0^* + \alpha_0^* \lambda c^*(\frac{\pi}{2}) + 2 \Re \overline{c} c} \]
Rotation velocity per \( /t/ \) of the "non rotating part" of the helicity flip cut:

\[
\Phi_{01}^{01} = \left[ \frac{1}{2} \right] \left\{ \Im \left( c_1^* + c_2^* \mu_n(c_{1\alpha}) \right) \left( \lambda_\rho + \alpha_\rho \mu_n(c_{1\alpha}) + \lambda_\rho + \alpha_\rho \mu_n(c_{1\alpha}) - \frac{\omega_\rho}{2} \right) + \alpha_\rho \mu_n(c_{1\alpha}) + \Im \left( c_1^* + c_2^* \mu_n(c_{1\alpha}) \right) \right\} - \left\{ \lambda_\rho + \alpha_\rho \mu_n(c_{1\alpha}) \right\} \left( \lambda_\rho + \alpha_\rho \mu_n(c_{1\alpha}) \right) + \Re \left( c_1^* + c_2^* \mu_n(c_{1\alpha}) \right) \left( \lambda_\rho + \alpha_\rho \mu_n(c_{1\alpha}) \right) \left[ \frac{1}{2} \right] \left\{ \Im \left( c_1^* + c_2^* \mu_n(c_{1\alpha}) \right) - \frac{\omega_\rho}{2} \right\}^{-1}
\]

+ \left\{ \Im \left( c_1^* + c_2^* \mu_n(c_{1\alpha}) \right) - \frac{\omega_\rho}{2} \right\}^{-1}
Rotation velocity per $\ell/2$ of the "rotating part" of the helicity flip cut

$$\mathcal{H}^{01}_{2} = \left\{ \begin{array}{l} -\tau \alpha \left( \lambda_0 + \alpha \lambda_2 \nu \left( e_{12} \right) \right) + \Re \left( C_1 + C_2 \nu \left( e_{12} \right) \right) + \bar{\eta} \left( C_1 + C_2 \right) \right. \\
\left. \left( \lambda_0 + \alpha \lambda_2 \nu \left( e_{12} \right) \right) + \lambda_0 + \alpha \lambda_2 \nu \left( e_{12} \right) \right\} - \frac{5 \alpha_0}{2} \left( \lambda_0 + \alpha \lambda_2 \nu \left( e_{12} \right) \right) + \Re \left( C_1 + C_2 \nu \left( e_{12} \right) \right)
\end{array} \right.$$
Correlation modified Absorption model - Gribov cut

The explicit dependence of the helicity nonflip cut characterizing quantities at \(t/ = 0\) and fixed energy

Initial cut strength and initial phase angle of non-rotating and rotating part

\[
\phi_{1}^{0} = \delta_{P} - \frac{\xi}{2} - \cot \frac{\delta}{4} \left( \frac{4}{\Im \phi} - \frac{\xi}{2} \right) \frac{1}{\Re \phi + \Re \phi + 4 \Re \phi}
\]

\[
\phi_{2}^{0} = \delta_{P} - \frac{\xi}{2} - \cot \phi_{0} \left( 4 \frac{4}{\Im \phi} - \frac{\xi}{2} \right) \frac{1}{\Re \phi + \Re \phi + 4 \Re \phi}
\]
Correlation modified Absorption model - Gribov cut

The explicit dependence of the helicity nonflip cut characterizing quantities at $/t/ \neq 0$ and fixed energy

Shrinking velocity with $/t/$ of cut strength of non-rotating and rotating part

\[ \Psi_1^{oo} = \frac{(\text{Re} \hat{\lambda}_p^0) \hat{\lambda}_p^0 + (\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0)^2}{(\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0 + 4 \text{Re}^0)^2 + (4 \text{Im} e - \frac{\text{Im} e}{2})^2} \]

\[ + (\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0 + 4 \text{Re}^0)^2 + (4 \text{Im} e - \frac{\text{Im} e}{2})^2 \]

\[ (\text{Re} e)^2 \left\{ 4 (\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0) \right\} - \text{Im} e \left\{ 4 \hat{\lambda}_p^0 \frac{\text{Im} e}{2} + (\text{Im} e)^2 \right\} \]

\[ + (\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0 + 4 \text{Re}^0)^2 + (4 \text{Im} e - \frac{\text{Im} e}{2})^2 \]

\[ \Psi_2^{oo} = \frac{(\text{Re} \hat{\lambda}_p^0) \hat{\lambda}_p^0 + (\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0)^2}{(\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0 + 4 \text{Re}^0)^2 + (4 \text{Im} e - \frac{\text{Im} e}{2})^2} \]

\[ + (\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0 + 4 \text{Re}^0)^2 + (4 \text{Im} e - \frac{\text{Im} e}{2})^2 \]

\[ (\text{Re} e)^2 \left\{ 4 (\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0) \right\} - \text{Im} e \left\{ 4 \hat{\lambda}_p^0 \frac{\text{Im} e}{2} + (\text{Im} e)^2 \right\} \]

\[ + (\text{Re} \hat{\lambda}_p^0 + \hat{\lambda}_p^0 + 4 \text{Re}^0)^2 + (4 \text{Im} e - \frac{\text{Im} e}{2})^2 \]
Correlation modified Absorption model - Gribov cut

The explicit dependence of the helicity nonflip cut characterizing quantities at $|t| \neq 0$ and fixed energy on

Phase angle rotation velocity with $|t|$ of non-rotating and rotating part

\[
\begin{align*}
\Phi^0_\perp & = - \left( \frac{\tau_\perp}{2} \right) \left( \lambda_\perp \right)^2 + \frac{\text{Re} \phi \{ 4 \lambda_\perp \} + 4 \left( \text{Re} \phi \right)^2 + 4 \left( \text{Im} \phi \right)^2}{(\text{Re} \lambda_\perp + \lambda_\perp + 4 \text{Re} \phi)^2 + (4 \text{Im} \phi - \frac{\pi}{2})^2} \\
\Phi^\perp_\perp & = - \left( \frac{\tau_\perp}{2} \right) \left( \lambda_\perp \right)^2 + \frac{\text{Re} \phi \{ 4 \lambda_\perp \} + 4 \left( \text{Re} \phi \right)^2 + 4 \left( \text{Im} \phi \right)^2 + (\text{Td} \phi)^2}{(\text{Re} \lambda_\perp + \lambda_\perp + 4 \text{Re} \phi)^2 + (4 \text{Im} \phi - \frac{\pi}{2} + \text{Td} \phi)^2} \\
\end{align*}
\]
Correlation modified absorption model - Gribov cut (isovector amplitude)

The explicit dependence of the helicity flip cut characterizing quantities at $|t| = 0$ and fixed energy on all parameters

Initial cut strength and initial phase angle of non-rotating and rotating part

\[ \phi_1^- = \frac{\lambda^\perp \phi_1 \phi_1}{2} \left\{ (\Re \lambda_\perp + t \Re \lambda')^2 + \frac{\xi \xi'}{2} \right\} \]

\[ \phi_2^- = \frac{\lambda^\perp \phi_2 \phi_2}{2} \left\{ (\Re \lambda_\perp + t \Re \lambda')^2 + \frac{\xi \xi'}{2} \right\} \]
Correlation modified absorption model - Gribov cut (isovector amplitude)

The explicit dependence of the helicity flip cut characterizing quantities at \( |t| \neq 0 \) and fixed energy on all parameters involves shrinking velocity with \( |t| \) of cut strength of non-rotating and rotating part.

\[
\psi_{01}^{\omega} = \frac{\{ \text{Re} \hat{\lambda}_{P}^{\omega} \} \hat{\lambda}_{p}^{\omega} \{ \text{Re} \hat{\lambda}_{P}^{\omega} + \hat{\lambda}_{p}^{\omega} \} + \left( \frac{\Delta x}{\omega} \right)^2 \hat{\lambda}_{p}^{\omega} + \text{Re}' \{ \text{Re} \hat{\lambda}_{P}^{\omega} \} \hat{\lambda}_{p}^{\omega} + (\omega)}
\]

\[
+ \frac{\{ \text{Re} \hat{\lambda}_{P}^{\omega} + \hat{\lambda}_{p}^{\omega} \} + \{ \text{Re} \hat{\lambda}_{P}^{\omega} - \hat{\lambda}_{p}^{\omega} \} + \{ \text{Re} \hat{\lambda}_{P}^{\omega} + \hat{\lambda}_{p}^{\omega} \}}{\text{Re}' \{ \text{Re} \hat{\lambda}_{P}^{\omega} + \hat{\lambda}_{p}^{\omega} \} + (\omega) - (\omega)}
\]

\[
\psi_{02}^{\omega} = \frac{\{ \text{Re} \hat{\lambda}_{P}^{\omega} \} \hat{\lambda}_{p}^{\omega} \{ \text{Re} \hat{\lambda}_{P}^{\omega} + \hat{\lambda}_{p}^{\omega} \} + \left( \frac{\Delta x}{\omega} \right)^2 \hat{\lambda}_{p}^{\omega} + \{ \text{Re} \hat{\lambda}_{P}^{\omega} \} \hat{\lambda}_{p}^{\omega} + (\omega)}
\]

\[
+ \frac{\{ \text{Re} \hat{\lambda}_{P}^{\omega} + \hat{\lambda}_{p}^{\omega} \} + \{ \text{Re} \hat{\lambda}_{P}^{\omega} - \hat{\lambda}_{p}^{\omega} \} + \{ \text{Re} \hat{\lambda}_{P}^{\omega} + \hat{\lambda}_{p}^{\omega} \}}{\text{Re}' \{ \text{Re} \hat{\lambda}_{P}^{\omega} + \hat{\lambda}_{p}^{\omega} \} + (\omega) - (\omega)}
\]
Correlation modified absorption model - Gribov cut (isovector amplitude)

The explicit dependence of the helicity flip cut characterizing quantities at $t/\not= 0$ and fixed energy on all
Phase angle rotation velocity with $t/\not= 0$ of non-rotating and rotating part.

\[
\begin{align*}
\tilde{\Phi}_1^a &= - \left( \frac{\xi_q^2}{2} \right) \left( \hat{\lambda}_p^a \right)^2 + \text{Rec}' \left\{ \frac{\hat{\lambda}_p^a}{\lambda_q^a} \right\} + \frac{1}{2} (\text{Rec}')^2 + \frac{1}{2} (t J^c')^2 \\
&= \left( \text{Re} \hat{\lambda}_p^a + \hat{\lambda}_p^a + \text{Re} C^a \right)^2 + \left( \frac{t J^c}{2} - \frac{\xi_q^2}{2} \right) \\
&+ \frac{1}{J^c} \left\{ \text{Re} \hat{\lambda}_p^a + \hat{\lambda}_p^a + \text{Re} C^a \right\}^2 + \left( \frac{t J^c}{2} - \frac{\xi_q^2}{2} \right)
\end{align*}
\]

\[
\begin{align*}
\tilde{\Phi}_2^a &= - \left( \frac{\xi_q^2}{2} \right) \left( \hat{\lambda}_p^a \right)^2 + \text{Rec}' \left\{ \frac{\hat{\lambda}_p^a}{\lambda_q^a} \right\} + \frac{1}{2} (\text{Rec}')^2 + \frac{1}{2} (t J^c')^2 + \frac{1}{2} (\xi_q^2)^2 \\
&= \left( \text{Re} \hat{\lambda}_p^a + \hat{\lambda}_p^a + \text{Re} C^a \right)^2 + \left( \frac{t J^c}{2} - \frac{\xi_q^2}{2} - \frac{\xi_q^2}{2} \right) \\
&+ \frac{1}{J^c} \left\{ \text{Re} \hat{\lambda}_p^a + \hat{\lambda}_p^a + \text{Re} C^a \right\}^2 + \left( \frac{t J^c}{2} - \frac{\xi_q^2}{2} - \frac{\xi_q^2}{2} \right)
\end{align*}
\]
VI. III. - Several Model Variants towards the solution of the phase problem of the helicity nonflip isovector amplitude

VI. III. 1 - Model Variant Ia - Purely Real Correlation Model

- the s-channel helicity conserving effective pomeron is purely imaginary and stationary.

\( |t| = 0 \)

Purely real correlation parameter \( c \) given in units of \( (\text{GeV}/c)^2 \)

It is known that by comparison to the helicity flip amplitude the helicity non-flip amplitude needs to be strengthened or weakened the cut in forward direction at will, depending on whether one chooses a negative or positive correlation parameter \( c \). However, the ratio of non-rotating to rotating strength will quickly deviate from 1 towards larger values, because, if the model has any chance to rectify the situation, one would expect a cut which is stronger in its real part. Traditionally, both parts are approximately equally strongly absorbed. The increase of the correlation parameter even worse. In fact, although the initial phase of the rotating cut is strongly rotated in anticlockwise direction, there is no way to rotate the non-rotating part of the cut, since we have used a stationary phase. The ratio prevents any substantial net gain in phase for the total cut from being more than a few degrees at a time to a Michigan enhancement factor of about \( \lambda = 2 \). Any further attempt to increase the strength with the help of the correlation parameter would result in a severe loss in phase. For example, using the parameters specified in table I, we obtain a triangle of 228.51° with a strength of about \( \sqrt{10} \) of the pole in the complex plane, which leaves a relative angle of 185.31 degrees. This is much too close to the critical value of 180° where the polarization changes its sign. The correlation parameter corresponding to \( \lambda = 2 \) has the value \( c = -1.8 \) keeping everything else fixed, has reached the optimal net gain in phase namely 3.18°. With \( c = -2.2 \) corresponding to a \( \lambda = 2.52 \), the strength is larger than the traditional absorption would give, namely 227.77°. The reason for this behaviour is that for a negative correlation parameter the non-rotating part of the cut possesses a singularity due to the introduction of \( c \). For the particular parameter values this singularity occurs at \( c = -2.925 \). The rotating part of the cut, by contrast, cannot possess such a singularity as it is the Regge slope in the denominator.
Model Variant Ia

\[ 0 \leq |t| \leq 35 \text{ (GeV/c)}^2 \]

Purely real correlation parameter \( c \)

Although the introduction of a purely real \( c \) namely \( \text{Re} \, c = -1.8 \text{ (GeV/c)}^2 \) resulted only in a negligible net gain in a few degrees, there is still hope that the situation might improve once \( |t| \) starts to increase.

The nature of the failure of the traditional absorption model is the unfortunate effect of the combination of features -

1. The relative cut pole phase is too close to 180° at \( |t| = 0 \text{ (GeV/c)}^2 \).
2. Although the cut rotates with increasing \( |t| \) away from the pole, it does so too slowly by comparison with the fast following pole, thus the pole already catches up with the cut at the critical phase difference of 180° at very small \( |t| \) values and there causes the polarization to change its sign from positive to negative.

By switching on the negative and purely real correlation one not only increases the cut strength at forward \( |t| \) but also weakens the exponential fall-off of the cut strength with increasing \( |t| \); e.g., for \( \text{Re} = -1.8 \text{ (GeV/c)}^2 \) the slope is by a quarter of its former value. Also, nonrotating and rotating slope fall-off in the case of the traditional model are slightly different (the rotating part a bit faster than the non-rotating part). At \( \text{Re} = -1.8 \text{ (GeV/c)}^2 \) the all slopes are exactly equal. A further increase in \( \text{Re} \) lowers both slopes considerably quickly, but does so for the non-rotating part. Again this is a consequence of the singularity at \( \text{Re} = -1.92 \text{ (GeV/c)}^2 \).

The hope that the disappointing situation met in forward direction might improve away from \( |t| = 0 \text{ (GeV/c)}^2 \) is unsubstantiated because we observe that \( \text{Re} \) not only cannot initiate a rotation of the non-rotating part but also slows down the already existing rotation velocity of the rotating part of the traditional absorption model which causes a complete standstill.
VI. III. 2 - Model Variant Ib - complex correlation model as a crossing symmetry violating solution to the phase problem of the helicity nonflip isovector amplitude

\[ |t| = 0 \ (\text{GeV}/c)^2 \]

Complex correlation parameter $c$

Due to the unwanted increase in strength of the non-rotating part of the cut relative to the rotating part, the real correlation parameter has so far been disappointing. By allowing for an imaginary part of the correlation, one can increase and keep the ratio at 1 if one accompanies the $\text{Re} c = -1.8 \ (\text{GeV}/c)^2$ with an $\text{Im} c = +0.63 \ (\text{GeV}/c)^2$. The presence of the imaginary part of the correlation causes the non-rotating part, which is traditionally real, to acquire an imaginary part. Unfortunately, the ratio stabilizing positive $\text{Im} c$ rotates both non-rotating parts clockwise such that the net effect for the total cut phase at $|t| = 0 \ (\text{GeV}/c)^2$ is much worse than it would be with traditional absorption.

A negative imaginary part of $c$ can, however, account for the strong absorption part of the pole partially or even totally at the expense of the real part. Absorption modifying models that produce such a behaviour of their cut in an ad hoc fashion and have produced several basic features of the observable correctly. By introducing a complex correlation, we produce this behaviour naturally. If we choose for example $\text{Re} c = -1.35 \ (\text{GeV}/c)^2$ and $\text{Im} c = -1.35 \ (\text{GeV}/c)^2$ we obtain a purely imaginary cut at forward direction whose strength correspon
Model variant Ib

\[ 0 < |t| < 0.35 \ (GeV/c)^2 \]

Complex correlation parameter \( c \)

We have remarked that the reason for the persistent failure of the traditional absorption cut is due to its too early diagram at \( |t| = 0 \ (GeV/c)^2 \) together with its too slow rotation with increasing \( |t| \). This causes the total amplitude to rotate clockwise which speeds up when the 180° relative cut-pole phase border has been crossed. This is in complete contrast with the actual behaviour of the amplitude, as has been revealed by model independent determinations of the amplitude. The feature of the amplitude in the \( |t| \) region under consideration is its zero structure. Especially where the "crossover" to the zero of the parallel part is concerned (parallel with respect to the dominating helicity non-flip isoscalar \( s \) analysis by Ambats et al. has a definite advantage over others. The more precise determination of this zero at \( |t| \) due to their particle-anti-particle relative normalization uncertainty of \( \pm 1.5\% \) which leads to an uncertainty of only \( \pm 0.025 \ (GeV/c)^2 \) whereas a normalization uncertainty of \( \pm 5\% \) as in previous determinations led to an uncertainty of the reference amplitude which is model independent unless it is determined by the method of fixed-\( t \) analyticity.

Whereas the zero of the parallel part seems to be a fairly reliable constraint for model building, the knowledge of the perpendicular part suffers from an uncertainty due to the inconsistency present in the polarization data measured by the CERN data. The ARGONNE data are persistently 20% lower than the CERN data which affects mostly the perpendicular part: the ARGONNE data are taken, the perpendicular part has a zero at \( |t| = 0.25 \ (GeV/c)^2 \) whereas in the case of the CERN data it is zero at \( |t| = 0.2 \ (GeV/c)^2 \) not too far away from the actual "crossover". The ARGONNE data are therefore 20% lower than the CERN data. As in the case of the RRRT-Phase angle, our model absorbs the imaginary part at the expense of the real part. RRRT provide the non-rotating part of the cut with a positive ad hoc phase angle of 90° which amo...
of the real part of the cut. This model is the most extreme of its kind. Intermediate versions are for example the model by J. Anderson et al (10) which correct only the rotating part of the pole, and the model by Sadoulet (10) which corrects non-rotating and rotating parts of the cut differently and employs a non-flat pomeron. The newest determination of the crossover zero position by Ambats et al puts a severe constraint on model building. Although all three models above reproduce the observables and several features of the amplitude, the difficulties which these models encounter in trying to move the imaginary zero in the vicinity of the actual position of the crossover at $t/\lambda = 0.15$ demonstrate the necessity of cut strength in the imaginary part. (That this is so is clearly seen in the fact that RRRT can move its zero from $t/\lambda = 0.2$ to the crossover position, and this is so only because its additional resource of strength comes from the non-rotating part. The non-rotating part is, due to the phase angle, fully transformed into the imaginary part. This effect doubles the cut strength.) No other model has such a resource in cut strength. The introduction of a complex parameter can somehow simulate the various versions of these phase-modifying models. A purely imaginary $c$ can change the cut phase. However, both parts of the cut obtain an equal amount of change. A purely imaginary $c$ doubles the approximate equality of the size of the two cut parts. Thus one can arrange for a $45^\circ$ positive phase angle with $\text{Im}c = -2.6 \text{(GeV/c)}^2$ and obtain a purely imaginary cut at least at $t/\lambda = 0$. However, although we obtain the correct phase rotation we lose in comparison with RRRT on strength. This happens firstly because of the $45^\circ$ additional phase which applies to both parts of the cut and reduces in principle the effective strength with respect to RRRT from $\lambda = 2$ to $\lambda = 1$. Secondly, a purely imaginary $c$ weakens the cut and the net effect is such that for the optimal phase angle there will be enhancement (nor loss) in strength of the imaginary part of the cut and there will be total cancellation of the real part. If our phase modification is not $t/\lambda$ independent. Both phases have a small rotation velocity. Such a purely imaginary $c$ weakens the model by J. Anderson apart from the weak $t/\lambda$ dependence. Both models fail at the crossover because of their lack in cut strength. For both models the simulating $\lambda$ is $\lambda = 1$. Our zero, as well as that of Anderson, is better than $t/\lambda = 0.4$. By switching on the real part of the correlation we encounter a new source of strength and also a phase in phase. In order to avoid the additional unwanted contribution in phase we have to turn down the size of $\text{Im}c$. This provides in turn a further boost in strength. The values for $c$ which move the crossover to $t/\lambda = 0.15$ are $\text{Re}c = -0.36$ and $\text{Im}c = -1.025 \text{(GeV/c)}^2$. The effective cut strength is then $\lambda = 2.14$ and we see that in the real part of $c$ we have a large gain of strength which a purely ad hoc phase modification cannot provide.
Helicity nonflip isovector amplitude (parameters used from Table I)

Model variant Ia - purely real correlation parameter, purely imaginary and stationary pomeron

Non-rotating part

\[
\begin{align*}
\mathcal{A}_{0,0}^\chi &= 1.749 / 11.7 + 4e^\circ \\
\not\Phi_{1,0}^\chi &= 0 \\
\not\Phi_{1,0}^\chi &= 36.5 + 37.29e^\circ + 10.08e^\circ / (11.7 + 6e^\circ) \\
\not\Phi_{1,0}^\chi &= 0
\end{align*}
\]

Rotating part

\[
\begin{align*}
\not\Phi_{0,0}^\chi &= 1.749 \\
\not\Phi_{0,0}^\chi &= -93.6^\circ = 2.52 / 11.7 + 6e^\circ \\
\not\Phi_{1,0}^\chi &= 36.5 + 37.29e^\circ + 10.08e^\circ / (11.7 + 6e^\circ)^2 + 6.35
\end{align*}
\]
Model variant Ib

(purely imaginary and stationary pomeron)

Helicity nonflip isovector amplitude (parameters used from Table 1)

Dependence of the cut characterizing quantities on a complex correlation parameter

Non-rotating part

\[
\xi^0_1(\alpha^0) = \frac{\alpha^0}{\left(\xi_1 + \xi_2 \text{Re}^0\right)^2 + C \left(\xi_1 \text{Im}^0\right)^2}
\]

\[
\Phi^0_1(\alpha^0) = -\frac{\alpha^0}{\left(\xi_1 + \xi_2 \text{Re}^0\right)^2 + C \left(\xi_1 \text{Im}^0\right)^2}
\]

\[
\Phi^\infty_1(\alpha^0) = \frac{18.39 \text{ Im}^0}{\left(\xi_1 + \xi_2 \text{Re}^0\right)^2 + C \left(\xi_1 \text{Im}^0\right)^2}
\]
Model variant Ib
(purely imaginary and stationary pomeron)
Helicity nonflip isovector amplitude (parameters used from Table 1)
Dependence of the cut characterizing quantities on a complex correlation parameter

Rotating part

\[
\begin{align*}
\psi_2^{00} &= \frac{7n}{\left( C_{21} + \Re \cos \theta \right)^2 + C \left( \Re \cos \theta - 2.5 \right)^2} \\
\phi_2^{00} &= -93.6^\circ - 9.0^\circ \frac{\Re \cos \theta - 2.5^\circ}{n \Re \cos \theta} \\
\psi_2^{01} &= \frac{361.8^\circ + 27.8^\circ \Re \cos \theta - 37.8^\circ \Im \cos \theta + 46.1^\circ \left\{ \left( \Re \cos \theta \right)^2 + \left( \Im \cos \theta \right)^2 \right\}}{n \Re \cos \theta} \\
\phi_2^{01} &= -37.5^\circ + 27.8^\circ \Re \cos \theta - 26.8^\circ \Im \cos \theta + 10.5^\circ \left\{ \left( \Re \cos \theta \right)^2 + \left( \Im \cos \theta \right)^2 \right\}
\end{align*}
\]
Model variant Ib - parameters as in Table I. Complex correlation parameter and purely imaginary and s

in addition \( \text{Re} C^0 = -2.2 \ (\text{GeV/c})^{-2} \) \( \text{Im} C^0 = -1.025 \ (\text{GeV/c})^{-2} \)

\[
T_\Phi = 0.82 \sin \frac{\pi}{2} (0.52 - 0.861 |t|) e^{8.00 |t|} \{ 3.50^\circ + 72.36^\circ |t| \}
\]

\[
T_C^0 = -0.169 |t| e^{-2.35 |t|} \{ 57.72^\circ + 13.056^\circ |t| \}
+ 0.103 |t| e^{-2.06 |t|} \{ -87.26^\circ + 60.9^\circ |t| \}
\]

For Helicity flip (keep traditional absorption: \( \text{Re} C' = \text{Im} C' = 0 \))

\[
T_\Phi' = 0.656 (|t|)^{1/2} \sin \frac{\pi}{2} (0.52 - 0.861 |t|) e^{-8.00 |t|} \{ 3.50^\circ + 72.36^\circ |t| \}
\]

\[
T_C' = -0.253 (|t|)^{1/2} \{ 1.92 |t| \} \{ 0.00^\circ + 0.00^\circ |t| \}
+ 0.253 (|t|)^{1/2} \{ -2.05 |t| \} \{ -57.26^\circ + 30.10^\circ |t| \}
\]
| $|\tau| = 0$ | $|\tau| = 0.05$ | $|\tau| = 0.15$ |
|---|---|---|
| $T_p$ | $0.621 \pm 1.3 \times 10^{-2}$ | $0.89 \pm 1.45 \times 10^{-2}$ | $1.64 \pm 1.05 \times 10^{-2}$ |
| $T_c$ | $0.17 \pm 1.71 \times 10^{-2}$ | $1.52 \pm 2.4 \times 10^{-2}$ | $1.21 \pm 2.63 \times 10^{-2}$ |
| $T_{\theta}$ | $0.52 \pm 2.9 \times 10^{-2}$ | $3.13 \pm 2.9 \times 10^{-2}$ | $1.9 \pm 2.9 \times 10^{-2}$ |
| $T_{\theta \gamma}$ | $0.607 \pm 3.5 \times 10^{-2}$ | $3.02 \pm 3.5 \times 10^{-2}$ | $1.0 \pm 2.6 \times 10^{-2}$ |

| $|\tau| = 0.25$ | $|\tau| = 0.35$ | $|\tau| = 0.45$ |
|---|---|---|
| $T_p$ | $0.056 \pm 1.2 \times 10^{-2}$ | $0.026 \pm 1.2 \times 10^{-2}$ | $0.007 \pm 1.5 \times 10^{-2}$ |
| $T_c$ | $0.056 \pm 1.7 \times 10^{-2}$ | $0.077 \pm 1.4 \times 10^{-2}$ | $0.061 \pm 1.6 \times 10^{-2}$ |
| $T_{\theta}$ | $0.06 \pm 5.0 \times 10^{-2}$ | $0.089 \pm 5.3 \times 10^{-2}$ | $0.057 \pm 6.1 \times 10^{-2}$ |
| $T_{\theta \gamma}$ | $0.023 \pm 5.0 \times 10^{-2}$ | $0.04 \pm 5.0 \times 10^{-2}$ | $0.06 \pm 2 \times 10^{-2}$ |
TABLE III  Model variant 1b (complex correlation, purely imaginary and stationary Pomeron)

<table>
<thead>
<tr>
<th>$/t/ = .55$</th>
<th>$/t/ = .65$</th>
<th>$/t/ = .8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_\rho$</td>
<td>.001 e^{200}$</td>
<td>-</td>
</tr>
<tr>
<td>$T_c$</td>
<td>.059 e^{\delta 0.15i}</td>
<td>-</td>
</tr>
<tr>
<td>$T_{\lambda}$</td>
<td>.046 e^{\delta 0.6i}</td>
<td>-</td>
</tr>
<tr>
<td>$T_{\rho}$</td>
<td>.055 e^{2}</td>
<td>-</td>
</tr>
</tbody>
</table>

$/t/ = 1.00$ in this region: $T_c \sim T_{\lambda}$

$/t/ = 1.4$

| $T_{\lambda}$ | .0177 e^{223.26i}     |           | $T_{\lambda}$ | .0072 e^{363.5i}     |           | $T_{\lambda}$ | .0056 e^{357.07i}     |

$/t/ = 1.200$

| $T_{\lambda}$ | .0113 e^{233.53i}     |           | $T_{\lambda}$ | .0056 e^{357.07i}     |           | $T_{\lambda}$ | .0056 e^{357.07i}     |
Polarization chart for model variant Tb with complex correlation.

| \( |d| \) | \( \text{Re} T^* \times \text{Im} T' - \text{Im} T^* \times \text{Re} T' \) | \( \frac{d\delta}{dt} \) | \( \frac{d\phi}{dt} \) | Polarization | \( \frac{d\phi}{dt} \) |
|---|---|---|---|---|---|
| .00 | \( .458 \times .00 \) - \( .255 \times .00 \) | .275 | .346 | .275 | .275 |
| .05 | \( .282 \times .387 \) - \( .137 \times .373 \) | .387 | .123 | .123 | .123 |
| .10 | \( .109 \times .519 \) - \( .004 \times .306 \) | .251 | .266 | .266 | .266 |
| .15 | \( .053 \times .399 \) - \( .033 \times .163 \) | .127 | .110 | .110 | .110 |
| .20 | \( .035 \times .125 \) - \( .037 \times .059 \) | .051 | .039 | .039 | .039 |
| .25 | \( .027 \times .861 \) - \( .035 \times .004 \) | .041 | .013 | .013 | .013 |
| .30 | \( .025 \times .021 \) - \( .031 \times .012 \) | .005 | .009 | .009 | .009 |
| .35 | \( .023 \times (-.032) \) - \( .032 \times (-.023) \) | .004 | .005 | .005 | .005 |
| .40 | \( .019 \times (-.057) \) - \( .057 \times (-.019) \) | .004 | .008 | .008 | .008 |
| .45 | \( .015 \times (-.05) \) - \( .05 \times (-.015) \) | .003 | .008 | .008 | .008 |
| .50 | \( .010 \times (-.01) \) - \( .01 \times (-.010) \) | .001 | .003 | .003 | .003 |
| .55 | \( .005 \times (-.013) \) - \( .013 \times (-.005) \) | .002 | .005 | .005 | .005 |
| .60 | \( .00 \times (-.005) \) - \( .005 \times 0.00 \) | .000 | .001 | .001 | .001 |
| .65 | \( .000 \times (-.001) \) - \( .001 \times 0.000 \) | .000 | .000 | .000 | .000 |
| .70 | \( .000 \times 0.001 \) - \( .001 \times (-.000) \) | .000 | .000 | .000 | .000 |
Traditional reggeized absorption model with \( \lambda \) cut = 1.00

Differential cross-section measured in \( \text{mb}/(\text{GeV}/c)^2 \)

\[ \pi^- p \rightarrow \pi^+ \eta \]
Degree

Phase of helicity nonflip iso vector amplitude after traditional absorption

K → I, U

Phase of helicity flip isovector amplitude after traditional absorption

Phase of helicity nonflip isovector amplitude after modified absorption

Model variant Ib (complex correlation)

fig. 11
Relative phase between helicity flip vector amplitude and helicity nonflip vector amplitude as obtained from the correlation modified model variant 1b.

Degree

Fig. 13.
Traditional absorption model (purely imaginary and stationary Pomeron and $\lambda$ cut = 1.00)

See Table IIa for numerical values

---

Helicity nonflip pole
Absorbed amplitude

Numbers indicate $t$ values

Scale =

$1 \text{ cm} = 0.019 \text{ (mb)}^{1/2}/\text{GeV}/c$

Correlation modified model variant Ib

See Table III for numerical values
Argand diagram for the helicity nonflip isovector amplitude obtained with the traditional absorption model (purely imaginary and stationary Pomeron and conclusion modified model)
VI. III. 3 - PURELY REAL AND NEGATIVE CORRELATION MODIFIED MODELS AS A CROSSING SYMMETRY PRESERVING SOLUTION TO THE PHASE PROBLEM

A more convenient way to manipulate the terms of the Gribov cut in complex vector representation as written on pages 107 to 119, is to rewrite it as on pages 145 and 146 with the relevant terms explicitly expressed for fixed and variable energies as done on pages 147 to 152. For certain purposes it is specially instructive to write the energy dependence term in powers of \( \ln s \) as done for example for the non-rotating part on pages 151 and 152. The vector representations of pole and cut enable us to obtain a great deal of insight into how the phases behave, and we learn that the necessary modifications in order to restore the most profound failures of the absorption cut, as discussed on page 127, can be accomplished for either a purely imaginary and stationary pomeron together with a complex "c" as we have seen, or for a pomeron with a significant real part at all \( |t| \) values, e.g. an initial phase of 101° and a slope of \( \alpha'p = 0.6 \), together with a purely real "c". Indeed, as one can see from the explicit formulas of the Gribov cut represented in vector form, the imaginary part of the "c" can, to a certain extent, be exchanged against the slope of the Pomeron.

A concrete model (Model V introduced on page 154) with the parameters as in Table I and in addition -

\[
\begin{align*}
\text{Re } c^0 &= -1.5 \\
\text{Im } c^0 &= 0 \\
\text{Re } c^1 &= -1 \\
\text{Im } c^1 &= -1
\end{align*}
\]

has been given with numerical details. Rather than describe all the details of the combined effect due to the "interaction" of the "Gribov c" and the real part of the Pomeron, we are going to draw several possible variations II, III, IV, V, VI, on the theme: "Gribov c" and Pomeron real part, while demonstrating how these joint effects as purely real c and real part of the Pomeron accomplish the necessary phase modification. This has been shown in figs. 18 to 23.

\* e.g. the stabilisation of the energy dependence (see p. 160)
Helicity monopole

\[ \Psi_i^{\circ} = \frac{(2\pi)^{\frac{3}{2}} \Lambda_{\alpha_i} \beta_{\nu} \beta_{\nu} (S_{\alpha_i})^{\delta_{\nu, \nu} - 1}}{\sqrt{\left( \text{Re} \ k_i^{\circ} \right)^2 + \left( \text{Im} \ k_i^{\circ} \right)^2}} \]

\[ \Psi_i^{\circ} = \frac{\text{Re} k_i^{\circ} \text{Re} k_i^{\circ} + \text{Im} k_i^{\circ} \text{Im} k_i^{\circ}}{(\text{Re} k_i^{\circ})^2 + (\text{Im} k_i^{\circ})^2} \]

\[ \psi_i^{\circ} = \frac{\text{Re} k_i^{\circ}}{\text{Re} k_i^{\circ}} - \frac{\text{Im} k_i^{\circ}}{\text{Re} k_i^{\circ}} \text{sgn} \theta \]

\[ \Psi_i^{\circ} = -\theta \text{sgn} + \phi_i^{\circ} - \text{ph} \text{Im} \frac{\text{Im} k_i^{\circ}}{\text{Re} k_i^{\circ}} \]
Helicity flip

\[ \tilde{\varphi}^0_i = \frac{\left(2\gamma^0\right)^{-1} \lambda^i_\mu \tilde{\varphi}^i_\mu \tilde{B}_i \tilde{B}_i^* \left(\text{Re}^2 \tilde{B}_i^*\right)^2 + C \text{Im}^2 \tilde{B}_i^* \frac{1}{\gamma^0} \left(\frac{\gamma^0}{\gamma^+}\right) \phi^0_{\gamma^0 i} - 1}{\left(\text{Re}^2 \tilde{B}_i^*\right)^2 + C \text{Im}^2 \tilde{B}_i^*} \]

\[ \phi^0_i = \frac{\text{Re}^2 \tilde{B}_i^* \text{Re} \tilde{B}_i^* + \text{Im}^2 \tilde{B}_i^* \text{Im} \tilde{B}_i^*}{\left(\text{Re}^2 \tilde{B}_i^*\right)^2 + C \text{Im}^2 \tilde{B}_i^*} \]

\[ \Phi^0_i = \frac{\text{Im}^2 \tilde{B}_i^* \text{Re} \tilde{B}_i^* - \text{Re} \tilde{B}_i^*}{\left(\text{Re}^2 \tilde{B}_i^*\right)^2 + C \text{Im}^2 \tilde{B}_i^*} \]

\[ \phi^0_i = \delta^0_i - 2 \gamma^0 \text{Re} \tilde{B}_i^* \tilde{B}_i^* + \mu \gamma^0 \frac{\text{Im} \tilde{B}_i^*}{\text{Re}^2 \tilde{B}_i^*} \]
At a fixed energy however the relevant quantities read as follows:

\[ \Re a_i^{oo} = \Re \lambda_i \gamma_i \gamma_i + \Re c (\Re \lambda_i \gamma_i + \lambda_i \gamma_i) + \Im c \frac{\gamma}{2} \]

\[ \Im a_i^{oo} = \Im c (\lambda_i \gamma_i + \lambda_i \gamma_i) - \frac{\gamma}{2} (\lambda_i \gamma_i + \Re c) \]

\[ \Re b_i^{oo} = \Re \lambda_i \gamma_i + \lambda_i \gamma_i + 4 \Re c \]

\[ \Im b_i^{oo} = 4 \Im c - \frac{\gamma}{2} \]

\[ \delta_{i/1} = \delta_{i/1} - \frac{\gamma}{2} \]

\[ \Re a_i^{oo} = \Re a_i^{oo} + \Re c (\Im c - \frac{\gamma}{2}) \]

\[ \Im a_i^{oo} = \Im a_i^{oo} - \Im c (\Re \lambda_i \gamma_i + \Re c) \]

\[ \Re b_i^{oo} = \Re b_i^{oo} \]

\[ \Im b_i^{oo} = \Im b_i^{oo} - \Re c \]

\[ \delta_{i/1}^\prime = \delta_{i/1}^\prime - \frac{\gamma}{2} \delta_{i/1}^\prime \]
In the case of fixed energy -

Non rotating

\[ \text{Re } a^0_i - \text{Re } \lambda^0 P \lambda^0 P + \text{Re } c(\text{Re } \lambda^0 P + \lambda^0 P) + \bar{J} \text{mc } \frac{\delta \phi}{2} \]

\[ \text{Im } a^0_i - \bar{J} \text{mc}(\text{Re } \lambda^0 P + \lambda^0 P) - \frac{\delta \phi}{2} (\lambda^0 P + \text{Re } c) \]

\[ \text{Re } b^0_i = \lambda^0 P + \lambda^0 P + 4 \text{Re } c \]

\[ \text{Im } b^0_i = 4 \bar{J} \text{mc } - \frac{\delta \phi}{2} \]

\[ \delta^0 P = \delta^0 P - \frac{\delta \phi}{2} \]

\[ \text{Re } e^0_i = \text{Re } \lambda^0 P + 2 \text{Re } c \]

\[ \text{Im } e^0_i = \bar{J} \text{mc } - \frac{\delta \phi}{2} \]

Rotating

\[ \text{Re } a^0_i = \text{Re } a^0_i + \tau \phi (\bar{J} \text{mc}) \]

\[ \text{Im } a^0_i = \text{Im } a^0_i - \tau \phi (\text{Re } \lambda^0 P) \]

\[ \text{Re } b^0_i = \text{Re } b^0_i \]

\[ \text{Im } b^0_i = \text{Im } b^0_i - \tau \phi \]

\[ \delta^0 P = \delta^0 P - \tau \phi \]

\[ \text{Re } e^0_i = \text{Re } e^0_i \]

\[ \text{Im } e^0_i = \text{Im } e^0_i \]
In order to stabilize the energy dependence of the phase we provide the parameter $C$ with a slow energy dependence.

\[ \text{Re} \alpha_i^o = \left( \chi_0^+ + \chi_0^- \ln(C \alpha_0) \right) \left( \chi_0^+ + \chi_0^- \ln(C \alpha_0) \right) + \text{Re} \left( C_i^+ \chi_0^- \ln(C \alpha_0) \right) \left( \chi_0^+ + \chi_0^- \ln(C \alpha_0) \right) + \text{Im} \left( C_i^+ \right) \]

\[ \text{Im} \alpha_i^o = \text{Im} \left( C_i^+ \chi_0^- \ln(C \alpha_0) \right) \left( \chi_0^+ + \chi_0^- \ln(C \alpha_0) \right) + \text{Re} \left( C_i^+ \chi_0^- \ln(C \alpha_0) \right) - \chi_0^+ + \chi_0^- \ln(C \alpha_0) + \text{Re} \left( C_i^+ \chi_0^- \ln(C \alpha_0) \right) \]

\[ \text{Re} b_i^o = \chi_0^+ + \chi_0^- \ln(C \alpha_0) + \chi_0^+ + \chi_0^- \ln(C \alpha_0) + \text{Re} \left( C_i^+ \chi_0^- \ln(C \alpha_0) \right) \]

\[ \text{Im} b_i^o = \text{Im} \left( C_i^+ \chi_0^- \ln(C \alpha_0) \right) - \frac{\varphi}{2} \]

\[ \text{Re} a_i^o = \text{Re} a_i^o + \frac{\varphi}{2} \left( \text{Im} \left( C_i^+ \chi_0^- \ln(C \alpha_0) \right) \right) - \frac{\varphi}{2} \]

\[ \text{Im} a_i^o = \text{Im} a_i^o - \frac{\varphi}{2} \left( \chi_0^+ + \chi_0^- \ln(C \alpha_0) + \text{Re} \left( C_i^+ \chi_0^- \ln(C \alpha_0) \right) \right) \]

\[ \text{Re} b_i^2 = \text{Re} b_i^o \]

\[ \text{Im} b_i^2 = \text{Im} b_i^o - \frac{\varphi}{2} \]
With explicit energy dependence for the 'non rotating' contribution

\[
\text{Re } a_{11}^{01} = (\lambda_{\rho} + \alpha_{\rho} \ln(\theta_{11})) (\lambda_{\rho} + \alpha_{\rho} \ln(\theta_{11})) + \text{Re } C_{1} + C_{2} \ln(\theta_{11})^2
\]

\[
\text{Im } a_{11}^{01} = \text{Im } C_{1} + C_{2} \ln(\theta_{11}) (\lambda_{\rho} + \alpha_{\rho} \ln(\theta_{11})) + \lambda_{\rho} + \alpha_{\rho} \ln(\theta_{11}) \cdot \frac{\pi \alpha_{\rho}}{2} (\lambda_{\rho} + \alpha_{\rho} \ln(\theta_{11}) + \text{Re } C_{1} + C_{2} \ln(\theta_{11})^2)
\]

\[
\text{Re } b_{11}^{01} = \lambda_{\rho} + \alpha_{\rho} \ln(\theta_{11}) + \lambda_{\rho} + \alpha_{\rho} \ln(\theta_{11}) + 2 \text{Re } C_{1} + C_{2} \ln(\theta_{11})^2
\]

\[
\text{Im } b_{11}^{01} = 4 \text{Im } C_{1} + C_{2} \ln(\theta_{11}) - \frac{\pi \alpha_{\rho}}{2}
\]

\[
\text{Re } c_{11}^{01} = \lambda_{\rho} + \alpha_{\rho} \ln(\theta_{11}) + 2 \text{Re } C_{1} + C_{2} \ln(\theta_{11})^2
\]

\[
\text{Im } c_{11}^{01} = 2 \text{Im } C_{1} + C_{2} \ln(\theta_{11}) - \frac{\pi \alpha_{\rho}}{2}
\]

\[
\delta_{\rho} = \delta_{\rho} - \pi/2
\]
For the "rotating" contribution

\[
\begin{align*}
\text{Re} a_2^0 & = \text{Re} a_1^0 + \text{Re} \phi \left( \text{Im} \left( c_1 + c_2 \phi \sigma_{58} \right) \right) - \frac{\text{Re} \phi}{2} \\
\text{Im} a_2^0 & = - \text{Im} a_1^0 - \text{Re} \phi \left( \text{Re} \phi + \text{Re} \phi \sigma_{58} \sigma_{58} \right) + \text{Re} \left( c_1 + c_2 \phi \sigma_{58} \right) \\
\text{Re} b_2^0 & = \text{Re} b_1^0 \\
\text{Im} b_2^0 & = - \text{Im} b_1^0 - \text{Re} \phi \\
\text{Re} d_2^0 & = \text{Re} d_1^0 \\
\text{Im} d_2^0 & = - \text{Im} d_1^0 \\
\delta^0 & = - \frac{\text{Re} \phi}{2} + \delta^0
\end{align*}
\]
\[ \text{Re} \, q_1^\circ = \lambda_1^p \lambda_2^p + \text{Re} \, c_i \{ (\lambda_1^p + \lambda_2^p)^2 \} + \text{Im} \, c_i \{ \overline{\text{Re} \, q_2^\circ} \} \]

\[ + \text{Im} \{ (\lambda_1^p + \lambda_2^p) \} + \text{Re} \, c_i \{ (\lambda_1^p + \lambda_2^p)^2 \} + \text{Re} \, c_i \{ (\lambda_1^p + \lambda_2^p) \} \]

\[ + (\text{Im} \{ \} )^2 \{ (\lambda_1^p + \lambda_2^p) \} \]

\[ \text{Im} \, q_1^\circ = - \lambda_1^p \overline{\lambda_2^p} - \text{Re} \, c_i \{ \overline{\text{Re} \, q_2^\circ} \} + \text{Im} \, c_i \{ (\lambda_1^p + \lambda_2^p) \} \]

\[ + \text{Im} \{ - \lambda_1^p \overline{\lambda_2^p} - \text{Re} \, c_i \{ \overline{\text{Re} \, q_2^\circ} \} + \text{Im} \, c_i \{ (\lambda_1^p + \lambda_2^p) \} + \text{Re} \, c_i \{ (\lambda_1^p + \lambda_2^p) \} \}

\[ + (\text{Im} \{ \} )^2 \{ \text{Im} \, c_i \{ (\lambda_1^p + \lambda_2^p) \} \}

\text{Re} \, b_1^\circ = (\lambda_1^p + \lambda_2^p) + 4 \text{Re} \, c_i + \text{Im} \{ (\lambda_1^p + \lambda_2^p) \} + \text{Im} \, c_i \{ (\lambda_1^p + \lambda_2^p) \} + 4 \text{Re} \, c_i \]

\[ \text{Im} \, b_1^\circ - \frac{\overline{\lambda_2^p}}{2} + 4 \text{Im} \, c_i + \text{Im} \{ \text{Im} \, c_i \{ (\lambda_1^p + \lambda_2^p) \} \}

\[ \text{Re} \, b_1^\circ - (\lambda_1^p + \lambda_2^p) + 4 \text{Re} \, c_i + \text{Im} \{ (\lambda_1^p + \lambda_2^p) \} + 4 \text{Re} \, c_i \]

\[ \text{Im} \, b_1^\circ - \frac{\overline{\lambda_2^p}}{2} + 4 \text{Im} \, c_i + \text{Im} \{ 4 \text{Im} \, c_i \} \]
\[ \text{Re } a_0 = X_P \lambda_P^\prime + \text{Re } c_1 \left\{ \left( X_P + \lambda_P^\prime \right) J \right\} + \text{Im } c_1 \left\{ -i \lambda_P^\prime \right\} + \text{Im } \left[ \left( X_P^2 + \lambda_P^\prime \lambda_P^\prime \right) J \right] \]

\[ + \text{Im } \left[ \left( X_P^2 + \lambda_P^\prime \lambda_P^\prime \right) J \right] \]

\[ \text{Im } a_0 = -X_P \lambda_P^\prime \lambda_P^\prime \lambda_P \lambda_P^\prime + \text{Re } c_1 \left\{ \left( \lambda_P^\prime \lambda_P^\prime \right) J \right\} + \text{Im } c_1 \left\{ \left( \lambda_P^\prime \lambda_P^\prime \right) J \right\} + \text{Im } \left[ \left( \lambda_P^\prime \lambda_P^\prime \right) J \right] \]

\[ + \text{Im } \left[ \left( \lambda_P^\prime \lambda_P^\prime \right) J \right] \]

\[ \text{Re } b_0 = \left( \lambda_P^\prime + X_P \right) J + 4 \text{Re } c_1 + \text{Im } J c_1 \left\{ \left( \lambda_P^\prime + X_P \right) J \right\} + \text{Im } \left[ \left( \lambda_P^\prime + X_P \right) J \right] \]

\[ \text{Im } b_0 = \left( \lambda_P^\prime + X_P \right) J + 4 \text{Im } c_1 + \text{Im } J c_1 \left\{ \left( \lambda_P^\prime + X_P \right) J \right\} + \text{Im } \left[ \left( \lambda_P^\prime + X_P \right) J \right] \]

\[ \text{Re } c_0 = \lambda_P^\prime + 2 \text{Re } c_1 + \left[ \left( X_P + \lambda_P \lambda_P^\prime \right) J \right] \]

\[ \text{Im } c_0 = -X_P \lambda_P^\prime \lambda_P^\prime + 2 \text{Im } c_1 + \text{Im } J c_1 \left\{ \left( X_P + \lambda_P \lambda_P^\prime \right) J \right\} + \text{Im } \left[ \left( X_P + \lambda_P \lambda_P^\prime \right) J \right] \]
\[ T_p^0 = 1.042 \times 10^{-3} \left( \frac{1}{2} \times (0.52 - 0.80) \right) \times (-5.00) \times \left( 33.20 + 72.36 \right) \times [k] \]

\[ T_c^0 = -1.582 \times 10^{-3} \times \left( 20.39 ^ 2 + 60.44 \right) + 127 \times 2 \times \left( -39.82 + 62.2 \right) \times [k] \]

\[ T_p^1 = 4.656 \times (1k)^{0.5} \times \sin \frac{\pi}{\lambda} \times \left( 0.52 - 0.80 \right) \times [-6.00] \times \left( 33.20 + 72.36 \right) \times [k] \]

\[ T_c^1 = -1.512 \times (1k)^{0.5} \times \left( 0.98 \right) \times [-69.67 + 13 \times [k]] \]

\[ + 1.917 \times (1k)^{0.5} \times [k] \times [-17.56 + 36 \times [k]] \]
### MODEL V  Helicity nonflip

<table>
<thead>
<tr>
<th>$t/L = .00$</th>
<th>$t/L = .05$</th>
<th>$t/L = .15$</th>
<th>$t/L = .25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_p$</td>
<td>.76° $43.20^\circ$</td>
<td>.97° $44.39^\circ$</td>
<td>.18° $51.03^\circ$</td>
</tr>
<tr>
<td>$T_c$</td>
<td>.13° $216.75^\circ$</td>
<td>.15° $257.00^\circ$</td>
<td>.15° $257.00^\circ$</td>
</tr>
<tr>
<td>$T_k$</td>
<td>.69° $36.10^\circ$</td>
<td>.34° $36.15^\circ$</td>
<td>.34° $36.15^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t/L = .35$</th>
<th>$t/L = .45$</th>
<th>$t/L = .55$</th>
<th>$t/L = .65$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_p$</td>
<td>.03° $65.34^\circ$</td>
<td>.00° $75.76^\circ$</td>
<td>.00° $65.34^\circ$</td>
</tr>
<tr>
<td>$T_c$</td>
<td>.05° $264.12^\circ$</td>
<td>.06° $264.30^\circ$</td>
<td>.05° $264.30^\circ$</td>
</tr>
<tr>
<td>$T_k$</td>
<td>.05° $83.75^\circ$</td>
<td>.05° $83.30^\circ$</td>
<td>.05° $83.30^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t/L = .8$</th>
<th>$t/L = 1.00$</th>
<th>$t/L = 1.2$</th>
<th>$t/L = 1.5$</th>
<th>$t/L = 2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_p$</td>
<td>.0003° $101.05^\circ$</td>
<td>.00° $100.00^\circ$</td>
<td>.00° $100.00^\circ$</td>
<td>.00° $100.00^\circ$</td>
</tr>
<tr>
<td>$T_c$</td>
<td>.02° $64.64^\circ$</td>
<td>.01° $64.15^\circ$</td>
<td>.01° $64.31^\circ$</td>
<td>.00° $64.31^\circ$</td>
</tr>
<tr>
<td>$T_k$</td>
<td>.02° $75.01^\circ$</td>
<td>.02° $75.46^\circ$</td>
<td>.02° $75.46^\circ$</td>
<td>.02° $75.46^\circ$</td>
</tr>
</tbody>
</table>
\[
\begin{array}{c|c|c|c|c|c}
\hline
\frac{1}{t_l} & .05 & .15 & .25 & .35 \\
\hline
T_p & .626 & .623 & .69 & .767 \\
\hline
T_c & .066 & .106 & .15 & .25 \\
\hline
T_h & .578 & .533 & .43 & .29 \\
\hline
\frac{1}{t_l} & .45 & .55 & .65 & .8 \\
\hline
T_p & .136 & .05 & .03 & .012 \\
\hline
T_c & .062 & .052 & .05 & .04 \\
\hline
T_h & .049 & .082 & .12 & .15 \\
\hline
\frac{1}{t_l} & 1.00 & 1.2 & 1.5 & 2.0 \\
\hline
T_p & .037 & .025 & .012 & .012 \\
\hline
T_c & .392 & .272 & .15 & .085 \\
\hline
T_h & .074 & .065 & .05 & .045 \\
\hline
\end{array}
\]
### Table 1: Model V Parameters

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\Re T^0 \times \Im T^1$ - $\Im T^0 \times \Re T^1$</th>
<th>$k_{eff}$</th>
<th>$k_{pt}(\text{in mb/mb})$</th>
<th>$\lambda_{cut}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>0.191 - 0.00 - 0.367 + 0.00</td>
<td>0.369</td>
<td>0.362 + 0.02</td>
<td>-0.00</td>
</tr>
<tr>
<td>0.65</td>
<td>0.276 - 0.132 + 0.519 + 0.019</td>
<td>0.416</td>
<td>0.423 + 0.03</td>
<td>0.108 + 0.006</td>
</tr>
<tr>
<td>0.55</td>
<td>0.399 - 0.029 - 0.353 - 0.03</td>
<td>0.491</td>
<td>0.266 + 0.02</td>
<td>-0.14</td>
</tr>
<tr>
<td>0.45</td>
<td>0.405 - 0.278 + 0.114 + 0.039</td>
<td>0.519</td>
<td>0.328 + 0.01</td>
<td>0.14 + 0.06</td>
</tr>
<tr>
<td>0.35</td>
<td>0.459 - 0.136 - 0.051 - 0.009</td>
<td>0.557</td>
<td>0.173 + 0.002</td>
<td>0.54 + 0.06</td>
</tr>
<tr>
<td>0.25</td>
<td>0.505 - 0.038 - 0.009 + 0.036</td>
<td>0.535</td>
<td>0.009 + 0.001</td>
<td>-0.53</td>
</tr>
<tr>
<td>0.15</td>
<td>0.556 - 0.137 - 0.006 + 0.003</td>
<td>0.519</td>
<td>0.009 + 0.002</td>
<td>0.55 + 0.05</td>
</tr>
<tr>
<td>0.05</td>
<td>0.575 - 0.017 - 0.005 + 0.001</td>
<td>0.519</td>
<td>0.009 + 0.002</td>
<td>0.60 + 0.05</td>
</tr>
<tr>
<td>0.01</td>
<td>0.585 - 0.017 - 0.005 + 0.001</td>
<td>0.519</td>
<td>0.009 + 0.002</td>
<td>0.60 + 0.05</td>
</tr>
</tbody>
</table>

**Notes:**
- $\Re C^0 = -1.5$, $\Re C' = -1$, $\Im C' = -1$.
<table>
<thead>
<tr>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>6.07</td>
<td>6.07</td>
<td>6.07</td>
<td>6.07</td>
<td>6.07</td>
<td>6.07</td>
</tr>
</tbody>
</table>

MODEL V - Elastic amplitude differential cross section - Polarization chart
### Model V: Elastic Amplitude Differential Cross Section Chart

| $| T|_0$ | $| T|_t$ | $(| T|_0^2 - | T|_t^2) + (| T|_0 | T|_t) + (| T|_t | T|_0)$ | $| T|_{07}$ | $| T|_{07} | T|_0$ | $| T|_{07} | T|_t$ | $| T|_{07} | T|_t^2$ |
|------|------|---------------------------------|------|------|------|------|------|
| 2   | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 3   | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 4   | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 5   | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 6   | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 7   | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 8   | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 9   | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 10  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 11  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 12  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 13  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 14  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 15  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 16  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 17  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 18  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 19  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 20  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 21  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 22  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 23  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 24  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 25  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 26  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 27  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 28  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 29  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
| 30  | 1.25 | 1.25                              | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |
VI. III. 4 A PHASE ENERGY DESCRIPTION OF $\gamma^- p \rightarrow \pi^- n$ AT 6 GEV/C AND FNAL ENERGIES WITH A GOOD PREDICTION OF BOTH INELASTIC AND ELASTIC POLARIZATION OF $^{12}$N SCATTERING (MODEL V AND MODEL VI).

In Model V we have constructed (see Ardill et al. (45)) the isovector helicity amplitude by means of the corrected Gribov absorption model. We absorb with a helicity conserving pomeron pole which possesses a real part $A_0 = 10^{10}$ and a slope of $\alpha'_p = 0.6$ - see fig. 32 page 179. The same pomeron is used as isoscalar helicity amplitude for the elastic polarization see fig. 33 page 179. By describing the elastic polarization we encounter the same problem in (22, 23). We were obliged to give the helicity flip shower factor a strong $t/t^*$ dependence, such that the deuteron turns for $t/t^* < 0.6$ into a constructive one.

The imaginary part of the helicity flip amplitude is in that way not only prevented from becoming negative as in "untreated" Model variant V, fig. 27 and 28, but grows and falls rapidly in magnitude - fig. 29 - so as to account for the rise and fall of the elastic polarization beyond the double zero - fig. 33 and 34. The parameters have been taken from Table I, but with the addition of $\rho_0 = 0.6$ for $c_0 = |c_0| = 0.6$, $\lambda_1 = 1$, $\lambda_2 = 2.5$, $\rho_1 = 2.5$, $\rho_2 = 1$, $\lambda_1 = 1.5$, $\lambda_2 = 1$. When extrapolating our amplitudes to FNAL energies we stabilize the energy dependence of the helicity nonflip amplitude such that the initial cut phase has minimal energy dependence* and the rotational phase has only a small shrinkage. We accomplish this by providing the Gribov 'c' with an energy dependence such that

$$c = c_1 + c_2 \ln t$$

which results in a considerably stable polarization over a wide range in energy (fig. 36, 37). The smaller $c_1$ traditional shrinking problem further out in $t$ causing a slower fall of $\sigma_{\gamma p}^{\pi^- n}$ ($\pi^- p \rightarrow \pi^- n$) and a spread trajectories at 6 and 200 GeV/c (fig. 39, 40). The parameter values for the energy dependence of $c$ are

$$\begin{align*}
\text{Re } c_1^0 &= -1.225 \\
\text{Re } c_1^1 &= -0.5125 \\
\text{Im } c_1^1 &= -1
\end{align*}$$

$$\begin{align*}
\text{Re } c_2^0 &= -0.5125 \\
\text{Re } c_2^1 &= -0.5125 \\
\text{Im } c_2^1 &= 0
\end{align*}$$

which coincide with the parameter values found at 6 GeV/c.

* Since our absorptive factor is the full elastic amplitude the initial phase of the Pomeron possesses the experimentally observed This rotates the initial cut phase by a few degrees in clockwise direction and is the only energy dependence of the initial cut phase.
$14 = 35$ scaled by $\frac{1}{9}$
### MODEL VI  Helicity flip

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>$T_p$</th>
<th>$T_c$</th>
<th>$T_h$</th>
<th>$\lambda_1$</th>
<th>$T_p$</th>
<th>$T_c$</th>
<th>$T_h$</th>
<th>$\lambda_1$</th>
<th>$T_p$</th>
<th>$T_c$</th>
<th>$T_h$</th>
<th>$\lambda_1$</th>
<th>$T_p$</th>
<th>$T_c$</th>
<th>$T_h$</th>
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<td>0.67</td>
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MODEL VI Elastic amplitude differential cross section - Polarization chart
MODEL VI: as MODEL V but $\lambda'_{\text{cut}} = \lambda'_{c}(1/t) = (1-1.8/\epsilon) \epsilon^2 (2.5-\epsilon/\epsilon) \epsilon^2$.

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<th>$P_t$</th>
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Modulus

\[ \frac{\sqrt{mb}}{(\text{GeV}/c)} \]
Argand diagram for the helicity nonflip isovector amplitude

\[ \sqrt{mb/\text{(GeV/c)}} \]

- Traditional absorption
- Correlation modified absorption
- Amplitude analysis
- Pole
- Traditional absorption
Argand diagram - Helicity nonflip isovector amplitude

- for large $|t|$ region
Argand diagram for the helicity flip isovector amplitude for small $|s|$

The numbers on the curve indicate values in $|s|$. 
Previous diagram for helicity flip isovector amplitude enlarged for large s/t region.
Model VI

Correlation modified together with /H/ dep shower factor to see sharp rise and fall in asymmetrical part of elastic polarization but the double zero.

Argand diagram of helicity flip isovector amplitude for large /l/ region.

Fig. 24
Figure 50
fig. 31

Polarization $\pi^+\pi^-$, $\pi^-\pi^+$

6 GeV/c

- --- pure Regge pole
- --- traditional
- weak absorption

fig. 31
P_{\text{Sym}} \text{ polarization } \pi^\pm \rightarrow \pi^\pm

\text{Model VI}

P_{\text{lab}} = 6 \text{ GeV}

fig. 34
fig. 35

Data at 6 GeV/c
\[ \text{Polarization: } \pi^+ p \rightarrow \pi^0 n \]

Model:

- \( \lambda_{\text{cav}} = 1, \lambda_{\text{sym}} = \lambda_{\text{asy}}(1) \)
fig. 39
VII - SOME PURELY REAL AND POSITIVE CORRELATION MODIFIED MODELS

VII.1. - THE LARGE POMERON PHASE MODEL

VII.1.1. - The Zero Residue Version (Model VII)

An excellent \( \chi^2 \) fit can be achieved if we provide the Pomeron with a large real part in forward direction, and also a large Regge pole like slope. If, in addition, we introduce a Regge pole-residue parameterized exponentials, the minimisation programme then chooses amplitudes and observables as shown in figs. 41, 42, and happens is that in this case the programme can arrange for an 'ad hoc' zero in the imaginary part of the amplitude to the crossover position without dragging the zero of the real part too closely behind.

We show, in fig. 41, the parallel and perpendicular part (41a and 41b helicity nonflip, and 41c and 41d helicity flip) amplitude and our theoretical curve obtained by means of correlation modified absorption. The transformation complex plane in which we find our theoretical curve and the plane in which amplitude analysis works has been considered the circumstances that, first Ambats reference depends on a model assumption for the helicity flip isovector amplitude becomes less valid with increasing \( /t/ \) from \( /t/ = .35 \) onwards, and, second, our theory assumes a different phase for the transformation one has to respect these complications. Fig. 41b and 41d show the orthogonal component of the components near the minimum of the modulus decides unambiguously the sense of rotation of the amplitude origin of the Argand diagram with increasing \( /t/ \). The failure of the absorption model to obtain the correct sign and hence obtain the correct sense of rotation is displayed in fig. 8 and 6, pages 103 and 101. The incorrect relation the two helicity amplitudes as seen in fig. 5 and 4, page 100 and 99, causes the polarization to become so disastro the proportionality between fig. 2 and fig. 5.

VII.1.2. - The Non-Zero Residue Version (Model VIII)

Case 1
fig. 46
p. 193
Gribov \( c = 0 \)
In this case we do not force a "crossover zero" but still the large phase of the pomeron moves the zero out in \( /t/ \) with respect to the traditional absorption model.

Case 2
fig. 47
p. 194
Gribov \( c = 1.15 \)
The Gribov \( c \) switched on. Had not to arrange the basic modification although it still impre
\pi^- + p \rightarrow \pi^0 + n \text{ at } P_{\text{lab}} = 6.0 (\text{GeV/c})

\chi^2 = 93.676

\text{Fig. 42}
$\pi^{-} + p \rightarrow \pi^{0} + n$ at $P_{lab} = 6.0$ (GeV/c)

$\chi^{2} = 39.947$
$\pi^- + p \rightarrow \pi^0 + n$ at $P_{lab} = 6.0 \text{(GeV/c)}$

$\chi^2 = 127.058$

Fig. 4.5
$\pi^- p \rightarrow \pi^0 n$ at $P_{lab} = 6.0$ (GeV)

$\chi^2 = 54$

Fig. 14.1

Hill
Drobnis
Bonamy (5.9)
Bonamy (4.9)
VIII - THE CORRELATION MODIFIED "DERIVATIVE RULE" MODEL (MODEL IX) (46)

We have given, on page 74 the result of the derivative rule as applied to the helicity isovector nonflip amplitude of our correlation modified absorption model.

The parameters, c, A and B as given on page 73 for the helicity nonflip amplitude, are determined by the amplitude analysis data. We find $Re\, c = -1.067\, (GeV/c)^2$, $Im\, c = -0.629\, (GeV/c)^2$, $A = 0.36467\, (mb/GeV/c)$ $B = 6.58\, (GeV/c)^2$ with a $x^2 = 2.832$ for 14 data points. A comparison of our result with the amplitude analysis data is shown in fig. 48. Our choice of parameters gives a helicity nonflip amplitude which goes through the origin in a clockwise direction as shown in fig. 49. From these figures we note the imaginary zero is at $t = -0.228\, (GeV/c)$ while the real zero is at $t = -0.228\, (GeV/c)^2$.

Schrempp and Schrempp (47) have shown, in a model-independent way, for the same reaction and energy that one can obtain the s-channel helicity flip amplitude by means of the derivative of the helicity nonflip amplitude. This is a consequence of the peripheral nature of the process. The proper use of this rule, however, demands an exact relation for the helicity nonflip amplitude to begin with since the derivative rule tends to exaggerate any deviation from the curve which goes through the centre points of the experimental data. How these deviations are amplified can be seen in fig. 50 on page 198 with fig. 48 on page 199. We illustrate in fig. 50 again the parallel and the perpendicular helicity flip amplitude which one obtains by rotating the amplitude in the complex plane relative to the isoscalar amplitude.

As we have already noted, the helicity flip amplitude is extremely sensitive to any deviation in curvature of the isoscalar amplitude. Thus, the property of the helicity flip amplitude that it should have a constant phase $-t\pi/2\, 0.4\, (GeV/c)^2$ is only maintained out to $-t^2 = -0.175\, (GeV/c)^2$ and is rapidly lost beyond this value. This can be seen in fig. 51 the behaviour of the isoscalar helicity nonflip amplitude having the same $t^2\pi/2\, 0.4\, (GeV/c)^2$ property as the isovector helicity flip amplitude of the isovector exchange as a function of the trajectory. In principle this feature is indicated in fig. 51. However, the larger $t$ value loop should never reach into the first quadrant. Of course, fitting data that the Pomeron has a slope as large as the $s$ pole could make our effort with a purely imaginary Pomeron only tolerable. We have already seen that once we have included a nonflat Pomeron the fit will be considerably improved.
The fit to the observables as shown in fig. 32 suffers naturally from the inaccuracy in the nonflip amplitude. In a derivative rule imposes its own nature on to the polarization as can be seen by a comparison between our theoretical polarization curve and the one obtained by Barger and Phillips (48), which are strikingly similar in structure. From can see that the zeros in our polarization occur exactly where the phase of the nonflip helicity amplitude has its points, namely between \(-t = 0.025\) (GeV/c)\(^2\) and \(-t = 0.05\) (GeV/c)\(^2\) and shortly after \(-t = 0.35\) (GeV/c)\(^2\). The of the polarization results from the effects of competition between a shifted peak and a stationary point of the phase apparent peak is caused by the turning point of the phase but is prevented from developing into an actual polarization because of the differential cross section which is, at this point, still large but is falling off very fast such that the minimum is shifted further out in \(/t/\). This does not have time to develop a broad shape as we would like to see because stationary point is forcing the polarization to change sign. Finally, we observe a phenomenon which, although not consequence of the derivative rule, is also connected with peripherality (49). This is that the rate of change of cross section with respect to scattering angle is equal to the polarization up to a factor which is approximately of the range \(0 \xi \leq /t/ < 1.0\) (GeV/c)\(^2\).

We have seen that by employing a Pomeron which is more rich in structure we could not only improve our fit to also exchange the actual unwanted imaginary part of the parameter in the correlation kernel for a real part of the correlation kernel could be retained as purely real in character and could, therefore, if it were positive a form factor describing the extended structure of the hadrons.

The persistently negative nature of \(c\) (apart from the cosmetic \(c\) models) leads us to an interpretation within the quark parton framework, where care is taken of the particular sub-hadronic nature of the interaction (see. IX).

* Model VII and VIII
\( (F^1_{\perp})_{ll} \) 
\( (F^1_{\perp})_l \)

Total amplitude
Pole amplitude
Cut amplitude \((-1)\)

Fig. 48

\( |t| (\text{GeV/c})^2 \)
\[(F_{1-})_{ll} \over \sqrt{mb/(GeV/c)}\]

- Total amplitude
- Pole amplitude
- Cut amplitude

Fig. 15c
Fig. 51.

Each unlabelled division corresponds to an integer multiple of 0.1 for t.

- Total amplitude
- Pole amplitude
- Cut amplitude x 3
Phase of $\phi_{\pi^+}(\text{degrees})$

Polarization

Differential cross section $\frac{d\alpha}{d\Omega} (\text{mb}/(\text{GeV}/c)^2)$

Fig. 52
IX - AN INTERPRETATION OF THE NEGATIVE SIGN OF THE CORRELATION PARAMETER

We have, in the introduction, remarked upon the need to do the Reggeon-Pomeron convolution with both Reggeon and Pomeron renormalized. The effective interaction range needed to be considerably shorter than is the case for the single exchange. This has been borne out by the necessity to include into the Regge-particle coupling function the mutual orientation of the transverse components of the Reggeons (Pomeron). This in turn provided a way to account for the effective contribution of inelastic intermediate states besides the elastic pole state in the Gribov-Migdal R unitarity condition. Although the formal analogy to nuclear physics suggested parameterization of the modified absorption model by including a parameter which seemed to simulate the oscillator length of a quark-antiquark bound state, the actual persistently negative value of this parameter implying the shortening of the interaction range defied such a simple picture.

There has meanwhile emerged an intuitive interpretation (15, 50, 51) of the Gribov graphs based on the simultaneity of multiperipheral and diffractive discontinuities in the two body amplitude. Seen in the light of several converging views, such as the multiperipheral model, the parton picture and the diffusion analogy via the Green's function for it might be possible to understand the meaning of the negative sign of \( c \). In the intuitive picture of hadronic interactions it is understood that fast hadrons at distance \( b \) in impact parameter and at a certain "time" \( t \) cannot interact and reduce their energy by emitting a shower of virtual particles which populate the Impact parameter plane and are separated distances of the order of a Compton wave length due to Gribov's finite mass hypothesis.

Every produced particle is a step in the random walk across the impact parameter plane performed by the Regge reducing the energy of the colliding hadrons so as to bring them closer to each other. The higher the initial energy steps have to be done, i.e. the more virtual particles are produced in a multiperipheral chain. The initial position of the colliding hadrons is the vector sum of the distances the produced particles are apart from each other. They define the interaction range which has a specific Regge component and boundaries due to the colliding hadrons. Due to the approximative absence of long range correlations it happens that the Regge interaction region is growing linearly with initial energy: the more steps to go, i.e. more particles produced. The average squared value of the vector by the impact parameter values of the produced particles is then the defined effective interaction region between
colliding hadrons. The steepness of the exponentially parameterized Regge residues is then the non-shrink of the interaction region. The introduction of the Gribov "c" seems to reduce the size of this region in the. It reduces the size by comparison to the single graph and also by comparison to the traditional absorption n is preserved.

Why is this? In general the colliding hadrons produce several showers from which slower partons can e of hadrons then takes place via the simultaneous interaction of the partons. The parton is reduced in ene; Each has $\sqrt{s}$ in the c.m. system.

![Fig. 53](image)

They are also closer in impact parameter. They enter the diffusion slowing down process from their "s; by contrast to the "space, time" components of the initial single Regge exchange, where the hadron interact It seems to be plausible that the effective interaction range of each individual parton interaction is now bei with the first order interactions. That is to say it is the "size" of the partons which in higher orders is m size of the hadron.
X - AN OUTLOOK TOWARDS A UNITED DESCRIPTION OF HADRONIC TWO BODY INTERACTIONS

We do not wish to end on the speculative note of the last paragraph without emphasizing that the good results on the phase modifying nature of the negative Gribov "c" encouraged us to pursue the subject further, in particular to construct the isoscalar amplitude which could shed new light on the phase of the pomeron as probed by the elastic polarization scattering around and beyond the residue zero. That is to say a correlation modified moving Pomeron might be the elastic polarization beyond the residue zeros and a helicity amplitude which is regge behaved even for |t| ≈ 0 be more satisfying than the adhoc-introduced showerfactor which converts the destructive cut into a constructive one.

Thus, in conclusion we can say we have found a natural solution for the shortcomings of the optical model listed in the introduction together with a rather adhoc approach to point (2) in order to tide us over until we have constructed an amplitude by means of the Gribov "c". The inelastic differential-cross section is still unsatisfying in the region and we are still left with the shrinkage problem, see figs. 39 and 40, p. 185 and 186, but see the pole dominated model.

Work is now in progress to investigate the introduction of Gribov "c"-like couplings into diagrams of the enhanced type, (see fig. 54)
so as to correct the deviations of

\[
\frac{\partial \sigma}{\partial \Omega |_{\Omega \rightarrow 1}} (\pi^- p \rightarrow \pi^n n)
\]

and \(\alpha_{\text{eff}}\) from the FNAL data, while preserving the maximal energy stability of the phase as obtained in our Gribov "c" tre of the unenhanced diagrams.

With the isoscalar amplitude constructed, we could genuinely describe elastic scattering, and, in particular, the cross section and the dip problems in elastic scattering, but see also (37, 53) and for a \(\mathcal{L} \rightarrow \gamma\) model see (54). Another feature of our correlation modified model is that it does not have to treat the real and imaginary part of \(\Omega\) differently in order to modify the real and imaginary part of the amplitude differently. Thus the consequences of modification due to correlation for HCEX will be of interest. See for those reactions in phase modified model as (55) and Egli (5).

The description of the Pion-Nucleon system over a wide range in energy i.e. \(0 \lesssim Q \lesssim 2000 \) GeV

and a relatively large range in momentum transfer i.e. \(0.0 \lesssim |t| \lesssim (2 GeV/c)^2\)

should provide the basis for a unified description of all two body reactions connected via various symmetry schemes. For example, see (53), once we have obtained the residues for \(P\) and \(P'\) from \(\pi^- \pi^+ \rightarrow \pi^+ \pi^-\), the \(\sigma\) from the \(O\) from \(K^- p \rightarrow K^- p\)

one can link, via \(\psi\) factorization, quark model and the experimental necessity to account for the fact that Pomeran is not a singlet (use \(\mathcal{L} \rightarrow \gamma\) dominance model), the \(\pi N\) system with the \(\pi N\) system elastic NN scattering i.e. all reactions dominated by the five leading poles.

There lies a vast and fascinating field ahead and models developed within the frame of the Reggeon diagram techni are better suited to explore the systematics of hadronic interactions than traditional absorption.
XI  APPENDIX
NORMALIZATION AND UNITS. THE POMERON AND OPACITY

We express, in order to compare with the amplitude analysis by Ambelas et al., the relativistic invariant scattering amplitude in the centre of mass-system such that:

\[ T (E^*, \Theta^*) = \frac{q^*}{a^*} \frac{1}{1 - \frac{E^*}{E_0}} \left| \frac{p^*}{a^*} \left( C E^*, \Theta^* \right) \right|^2 \]

With \( q^*, p^* \) the three dimensional initial and final momentum, \( E^* \) the total energy and \( \Theta^* \) the scattering angle, centre of mass-system. \( S \) and \( t \) are the invariant total energy and transferred momentum respectively.

The differential cross-section in the centre of mass-system reads:

\[ \frac{d^2 \sigma}{dE \ d\Omega^*} = \frac{1}{C \cdot S_{1} \cdot S_{2} \cdot \frac{q^*}{a^*} \left( C E^*, \Theta^* \right) \left| \frac{p^*}{a^*} \right|^2} \]

with \( \Omega^* \) the solid angle in the c.m. system, \( s_1 \) and \( s_2 \) are the spins of the colliding particles, replacing the invariant amplitude we obtain the invariant differential cross-section:

\[ \frac{d^2 \sigma}{dE \ dt} = \frac{1}{C \cdot S_{1} \cdot S_{2} \cdot \frac{q^*}{a^*} \left( C E^*, \Theta^* \right) \left| \frac{p^*}{a^*} \right|^2} \left| T (E, t) \right|^2 \]
Our normalization is therefore \( N = 1 \) and in consequence the optical theorem reads:

\[
\sigma_{\text{tot}} = \frac{1}{4} \sqrt{\pi} \int_{\text{final}} T^0(\epsilon, t=0)
\]

where the upper right hand o indicates helicity nonflip. We decompose now the elastic helicity nonflip amplitude by the stars. We denote the elastic amplitude by \( P \) (Pomeron). Thus we find:

\[
\frac{q}{T^0} \frac{T^0(c, t)}{P} = \frac{i}{2 \xi} \sum_{j=0}^{+\infty} (2j+1) T^0(c, t) \frac{T^0(c, t)}{P} \cos \theta
\]

projecting out the partial waves and putting \( \cos \theta = z \) we write:

\[
\int_{-1}^{+1} \frac{q}{T^0} \frac{T^0(c, t)}{P} \frac{T^0(c, t)}{P} \cos \theta \, dz = \frac{i}{2 \xi} \int_{-1}^{+1} (2j+1) T^0(c, t) \frac{T^0(c, t)}{P} \frac{T^0(c, t)}{P} \cos \theta \, dz
\]

and obtain with the help of the orthogonal relation:

\[
\delta_{ij} \frac{2 \delta_{ij}}{2j+1} \frac{\frac{q}{T^0}}{\frac{T^0(c, t)}{P}} \frac{T^0(c, t)}{P} \cos \theta \, dz
\]
the partial wave of the elastic amplitude 

\[ \mathcal{T}_{\varphi}^{(s)}(s) = -i q \int_{-1}^{1} \frac{dt}{t} \mathcal{T}_{\varphi}^{(s, t)} \mathcal{T}_{\varphi}^{(t)} dt \]

We insert the Pomeron as a function of \( t \) / 

\[ \mathcal{T}_{\varphi}^{(s)}(s) = -i q \int_{-1}^{1} \left( i P_0 e^{i \delta_0} - 2 \frac{\alpha}{1 - \alpha} \right) dt \]

\[ \delta_0 \] is the Pomeron phase additional to the traditional value of \( 90^\circ \) and,

we make use of the relation

\[ \int_{-1}^{1} \mathcal{T}_{\varphi}^{(s, t)} dt \rightarrow \frac{\alpha}{q} \int_{-1}^{1} \frac{dt}{t} e^{i \delta_0} \alpha(\mathcal{T}_{\varphi}^{(t)}) dt \]
and transform into impact parameter space \( b \)

\[
T_0^0(q, b) = \frac{P_{\perp}}{L} \int d^2 p \frac{e^{i \theta_0}}{iH_0} b \frac{\chi e^{-\lambda}}{\chi e^{-\lambda}}
\]

and since

\[
\int_0^\infty x^2 e^{-\alpha x} J_0(\alpha x) dx = \frac{1}{(\alpha^2 + 1)^2} \phi - \frac{R^2}{4 \lambda}
\]

for \( Re \alpha > 0 \)

We obtain the Pomeron
in impact parameter space

\[
T_0^0(q, b) = \frac{P_{\perp}}{L^2} \frac{e^{i \theta_0}}{iH_0} b \frac{\chi e^{-\lambda}}{\chi e^{-\lambda}}
\]

This gives \( \alpha = -1.35 \) which compares with Höhler, Strauss value \( \alpha = -1.35 \pm 0.5 \)

With such a small real part its presence is negligible where the differential cross section in forward direction.

the optical point, namely

\[
\frac{\alpha^2}{L^2} (\alpha^2 + 1) = \frac{1}{6} \frac{\alpha^0}{GV} \frac{\alpha^0}{GV}
\]

here we have used the
In order to return the transformation 1.e. from b-space into s/t space we write

\[ \begin{align*}
\frac{q}{r} &\ T^0\ (s,t) = \sum_{l=0}^{\infty} T^0 (s, l) \ \mathcal{P} (l) \\
\end{align*} \]

and use the small angle impact parameter dictionary (for elastic scattering \( q = p \))

\[ \begin{align*}
&\mathcal{P}_{l+1} \rightarrow 2^n \mathcal{P} \\
&S_{l+1} \rightarrow J_0 (k \tau) \\
&\sum_{l} \rightarrow \int_{0}^{\infty} \rho \ dl \\
&T^0_p (s) \rightarrow T^0_q (s, k) \]

thus we convert the partial wave sum into an impact parameter integral

\[ \begin{align*}
T^0_q (s, t) = \int_{0}^{\infty} \rho \ dl \ \ T^0_q (s, k) \ J_0 (k \tau) \\
\end{align*} \]
We insert the Pomeron dependence on b, which we have found above such that

\[ T^0_Q(s,t) = \lambda^2 \int_0^\infty \frac{d\lambda}{\lambda^2} \left( \frac{b^2}{4\lambda^2} \right) \]

making use of the Fourier-Bessel integral as before but now with \( \lambda = \sqrt{s} \).

This results in the

\[ T^0_Q(s,t) = \lambda^2 \int_0^\infty \frac{d\lambda}{\lambda^2} \left( \frac{b^2}{4\lambda^2} \right) \]

Pomeron in \( t/t' \) space

The elastic differential cross section has been fitted by Ambats et al. Ref ( ).

\[ \frac{d\sigma}{d^2p} = A \cdot B \frac{m_t^2}{G_F V^2} \]

\[ A = 40.2 \pm 1.6 \frac{m_t^2}{G_F V^2} \]

\[ B = 7.7 \pm 0.8 \ (6GeV)^{-2} \]
The isospin decomposition for $\pi^+ \rightarrow \pi^0$ reads $T^0 = T^0_\sigma + T^0_\pi$.

and the differential cross section at forward direction -

$$\frac{\text{d}^2\sigma}{\text{d}^2\Omega} = \left|\Re e \left( T^0 e^{i\theta} \right) + i \Im m \left( T^0 e^{i\theta} \right)\right|^2 \frac{m^2 B}{G_F V^2} = \frac{G^2 f^2}{16\pi} (\alpha^2 + 1)$$

The last line follows from the optical theorem which gives in our normalization $N = 1$ the imaginary part amplitude in forward direction such that -

$$\Im m \left( T^0 e^{i\theta} \right) = \frac{\sqrt{\pi}}{2} \alpha_{tot}$$

Furthermore

$$\alpha = \frac{\Re e \left( T^0 e^{i\theta} \right)}{\Im m \left( T^0 e^{i\theta} \right)}$$

Ambats et al's value for all amplitudes are represented with respect to $T^0$ which possesses at $/t/ = 0$ a peak the amplitude from Table XX at page 1206 into the complex plane one obtains at $/t/ = 0$

$$R^e T^0 = -0.055, \quad \Im m T^0 = 0.319 \quad \text{measured in}$$
for $\Gamma_{\text{tot}}(\pi^p,6.6\text{eV}) = 0.2\,\text{mb}$. Since, however $d\Gamma/dt$, is measured by Ambats in

\[d\Gamma = 2.78/\text{GeV}\]
\[d\Gamma/\text{mb} = 6.81\,\text{mb} \times 5.37\,\text{mb}\]
\[1.00\,\text{mb} \rightarrow ?/\text{GeV}\]
\[1.00\,\text{mb} = 2.67 \,(\text{GeV})^{-2}\]
\[1.00\,\text{mb} = 1.60 \,(\text{GeV})^{-1}\]
\[\varepsilon = 5.37\,\text{mb} = 5.53 \,(\text{GeV})^{-1}\]

\[\varepsilon = 0.2\,\text{mb} \rightarrow 45.608 \,\text{mb/GeV}\]

\[\alpha = \left(\frac{d\Gamma}{dt}\right)_{1.0} = 41.873 \,\text{mb/GeV}^2\]
If we neglect \( Q > 1 \) we obtain \( \gamma \). See also Ambars (5) 10 page 1187.

And finally, Ambits amplitude gives:

\[
\frac{\gamma}{k} = \frac{2}{k} + \text{critical}
\]

with \( \gamma \) a poly. So: 615 m/s.

\[
\frac{\gamma}{k} = \frac{16}{k} + \text{critical}
\]

whereas the actual measurement lies at \( \frac{\gamma}{k} = 3 \).
We found the Pomeron (isoscalar amplitude) in b and ln/t-space:

\[
T^0_\rho(s, b) = \frac{\lambda^0_\rho}{2\sqrt{s} \, \lambda^0_\rho} \rho - \frac{\lambda^0_\rho}{4 \lambda^0_\rho} \exp(-\lambda^0_\rho b^2) \quad T^0_\rho(s, t_1) = \rho - \frac{\lambda^0_\rho}{4 \lambda^0_\rho} \exp(-\lambda^0_\rho t_1^2)
\]

On the other hand

\[
T^0_\rho(s, b) = 1 - S(b), \quad S(b) = 1 - C^0_\rho \, \rho - \frac{\lambda^0_\rho}{4 \lambda^0_\rho} \exp(-\lambda^0_\rho b^2) \quad T^0_\rho(s, b) = C^0_\rho
\]

With \( C^0_\rho \) the opacity coefficient and \( R^2 \) the radius of the absorbing region.

Thus

\[
\tilde{S}^0_\rho = \frac{\rho}{2 \sqrt{s} \, \lambda^0_\rho} \quad \text{and} \quad \tilde{Q}^2 = 4 \frac{\lambda^0_\rho}{\rho}
\]

from the parameterization of the elastic differential cross section by a forward diffraction peak which is to a very good approximation exponential over a small range

\[
\tilde{Q}^2 \rho = (GeV)^2
\]
such that \( \frac{d\sigma}{d\Omega} = A e^{-\frac{3}{2} \Omega} \), which reads as well as \( \frac{6^2}{16\pi} \left( \alpha^4 + 1 \right) \) thus \( a_n = \frac{6^{\frac{1}{3}}}{8\pi^\frac{1}{3}} \).

because of the optical theorem we can determine \( C_{o.p.} \), roughly by the exp. parameterization \( A C_{o.p.} \sim 77 \) for \( \frac{\hbar}{\sqrt{\alpha}} \sim 0.1 \) \( \alpha \sim 1 \), \( M \sim 0.1 \).

\[ (\text{GeV})^{-2} \rightarrow 3 \text{ m}^2 \]

\[ A = \frac{\kappa (\lambda_p C_{o.p.})^2 m^3}{\text{GeV}^2} \]

\[ \sim 2 \mu \rightarrow 2.17 (\text{GeV})^{-1} \]

If \( C_{o.p.} \sim 79, \lambda_p = 6 \text{GeV}^2 \), \( A = 15.33 \frac{m^3}{\text{GeV}^2} \).
We now consider the range \( |t| = 0 \) for the differential cross section. Although the isovector amplitude contributes we assume complete isoscalar dominance in this case we write:

\[
\frac{d^{2} \sigma}{d\Omega} = \left| T_{\varphi}(|t|) \right|^{2} = (b_{\varphi}^{0})^{2} \rho - 2 \lambda_{\varphi} |t|^{2}\]

and it follows \( B^{} = \lambda_{\varphi} \) and \( A^{} = \).

\( \lambda \) (Ambats exp. value) = 3.85 (GeV\(^{-2} \)) \( \lambda_{\varphi}^{0} = 6.3 \times \frac{m_{\varphi}}{\text{GeV}^{2}} \) (exp)

We have

\[
\lambda_{\varphi}^{0} = \lambda_{\varphi}^{0} + \alpha_{\varphi}^{0} \lambda_{\varphi}^{0}(\theta_{\varphi}) - i \frac{\alpha_{\varphi}^{0}}{2} \quad \text{our value for} \quad \alpha_{\varphi}^{0} = 0.6
\]

We have fitted Ambats isoscalar amplitude by \( 6=16 \) at \( 6 \) GeV/\( c \)

\( \lambda_{\varphi}^{0} \), is the modulus of a complex radius of interaction and implies

\( \lambda_{\varphi}^{0} = 2.08 \) at

\( R_{\varphi} \lambda_{\varphi}^{0} = 3.58 \quad \text{Jev} \lambda_{\varphi}^{0} = 0.93\)
The scattering amplitude for the transition \( i \rightarrow f \) for the spinless case in terms of angular momentum is expressed via the "cikonal matrix" \( \chi^j \) such that it reads in our normalization:

\[
\frac{q}{\sqrt{s}} \ T^o_\theta (s,t) \chi^j f = \frac{i}{2q^2} \sum_{j=0}^{\infty} (2j+1) (1 - e^{i\chi^j}) P_j^\theta (\frac{q}{s})
\]

As we did in the case of the elastic amplitude, we project out the partial waves and make use of the orthogonality properties of the Legendre polynomials:

\[
\int_{-1}^{1} \frac{q}{\sqrt{s}} \ T^o_\theta (s,t) \chi^j P_{2j+1} (\cos \theta) d\cos \theta = \frac{i}{2q^2} \sum_{j=0}^{\infty} (2j+1) (1 - e^{i\chi^j}) P_j^\theta (\frac{q}{s}) P_{2j+1} (\frac{q}{s}) d\theta
\]

\[
(1 - e^{i\chi^j}) = -i q \int_{-1}^{1} \frac{q}{\sqrt{s}} \ T^o_\theta (s,t) \chi^j P_{2j+1} (\cos \theta) d\cos \theta
\]

As our notation already suggests, we identify the Born amplitude \( T^o_\theta (s,t) \chi^j \) with an amplitude for the transition parameterized in a simple Regge pole exchange model i.e. in the case of \( \pi^- p \rightarrow \pi^+ \rho^- \) a single \( J^- \)-Regge pol
We take 
\[(1 - e^{i \lambda^\nu}) \approx -i \lambda^\nu\]
and insert the Regge pole in the form

\[T^\circ_g(s, t) = (s_i^\nu) \cdot o^{1n-1} \sum_{\lambda = 1, 2} p^\circ_g \cdot e^{-\lambda^\nu \cdot l/1}\]

where we have summed over non rotating and rotating part in which the Pole has been split due to its signature such that

\[p^\circ_g \cdot e^{-\lambda^\nu \cdot l/1} \quad \text{and} \quad \lambda^\nu \cdot l/1\]

and \(\lambda^\nu \cdot l/1\) contains the energy dependence such that

\[\lambda^\nu \cdot l/1 = \lambda^\nu \cdot l/1 + \alpha^\nu \cdot l/1\]

thus we obtain by employing the relation

\[-A \chi(s, l) \rightarrow T^\circ_g(s, l) = \frac{-A}{\pi} \left(\xi^\nu \right) \cdot o^{1n-1} \sum_{\lambda = 1, 2} p^\circ_g \cdot \int_{l/1} \gamma_{1l} \cdot e^{-\lambda^\nu \cdot l/1} \cdot \int_{l/1} \gamma_{l} \cdot l/1 \cdot \int_{l/1} \gamma_{l} \cdot l/1 \cdot l/1\]
The Fourier Bessel integral gives then with

\[ \int x^{-2n} e^{-\alpha x} J_\nu(x) dx = \frac{\pi}{(2\alpha)^n} e^{-\frac{\alpha^2}{4}} \quad \alpha = \frac{2}{n} \]

\[ T^0_\nu(s, b) = -i \chi \chi(s, b) - i \sum_{l=-\infty}^{\infty} \frac{C_\nu^{(\nu)}(l)}{2 \pi} \frac{1}{\chi_{l}^{\nu}} \rho \]

Thus, for the exchange of one Rho-Reggepole and one Pomeron, we obtain

\[ \frac{\rho}{\gamma_0} \int T^0_\nu(s, t) \otimes T^0_\nu(s, t) = -i \frac{\rho}{\gamma_0} \sum_{l=0}^{\infty} (2l+1) \int T^0_\nu(s, t) \int T^0_\nu(s, t) \phi_\nu^{(l)}(t) \phi_\nu^{(l)}(t) \]

\[ = -i \int \int \int \chi \chi(s, t) \chi \chi(s, t) \phi_\nu^{(l)}(t) \phi_\nu^{(l)}(t) \]

\[ = \int_0^{\gamma_0} \int \int \chi \chi(s, t) \chi \chi(s, t) \phi_\nu^{(l)}(t) \phi_\nu^{(l)}(t) \]

\[ = \int_0^{\gamma_0} \int \int \chi \chi(s, t) \chi \chi(s, t) \phi_\nu^{(l)}(t) \phi_\nu^{(l)}(t) \]

\[ = \sum_{l=-\infty}^{\infty} \frac{1}{2 \pi} \frac{\chi_{l}^{\nu} \chi_{l}^{\nu}}{(\frac{2}{n}) \chi_{l}^{\nu} \chi_{l}^{\nu}} \]

\[ \left( \frac{\gamma_0}{\gamma_0} \right)^{\alpha(l)} - 1 \rho \quad \chi \chi(s, t) \chi \chi(s, t) \]

Absorption cut
Pomeron in momentum transfer space

\[ T_{\varrho}^0 (s, t) = \int \frac{d^3 p}{4\pi^2} \, q^2 \, \delta^2 (\rho - \lambda_{\varrho} (s)) \, |\bar{h}_{\varrho} (t)|^4 \]

\[ = \int \frac{d^3 p}{4\pi^2} \, \delta^2 (\rho - \lambda_{\varrho} (s)) \, \overline{C_{\varrho}} (\rho) \, e^{\rho} \, e^{-\lambda_{\varrho} (s)} \]

\[ = \int \frac{d^3 p}{4\pi^2} \, \overline{C_{\varrho}} (\rho) \, e^{\rho} \, e^{-\lambda_{\varrho} (s)} \]

Pomeron in impact parameter space

\[ T_{\varrho}^0 (s, b) = \frac{\beta_{\varrho} (s) \, e^{\rho} \, e^{-\frac{b^2}{4x_{\varrho} (s)}}}{2\pi \, x_{\varrho} (s)} \]

\[ = \frac{\beta_{\varrho} (s) \, e^{\rho} \, e^{-\frac{b^2}{4x_{\varrho} (s)}}}{2\pi \, x_{\varrho} (s)} \]

\[ = \frac{\beta_{\varrho} (s) \, e^{\rho} \, e^{-\frac{b^2}{4x_{\varrho} (s)}}}{2\pi \, x_{\varrho} (s)} \]

with \( \lambda_{\varrho} (s) = \lambda_{\varrho} + A_{\varrho} \, \Lambda (3/4) - \frac{\pi \, m_{\varrho} \, \rho}{2} \)

Absorption cut

\[ \sum_{j \neq i} \frac{\delta_{\varrho} \, e^{-\lambda_{\varrho} (s)}}{8\pi \, (\lambda_{\varrho} \, \lambda_{\varrho})} \]

\[ \sum_{j \neq i} \frac{\delta_{\varrho} \, e^{-\lambda_{\varrho} (s)}}{8\pi \, (\lambda_{\varrho} \, \lambda_{\varrho})} \]
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