NUCLEAR COLLECTIVE STATES IN
\(^{110}\text{Cd},^{192}\text{Pt},^{192}\text{Os}\) AND \(^{166}\text{Er}\)

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TO MY PARENTS
ABSTRACT

The properties of the low-lying energy states of $^{110}$Cd, $^{192}$Pt, $^{192}$Os and $^{166}$Er following the radioactive decays of $^{110}$mAg, $^{192}$Ir and $^{166}$mHo were investigated.

Two high resolution Ge(Li) detectors, one intrinsic germanium detector and a Compton suppression system were employed for the measurements of the γ-ray energies and relative intensities. γ-γ coincidence measurements using a conventional fast-slow coincidence technique were performed by coupling the Ge(Li) detectors to a 4096X4096 Dual Parameter Data Collection System.

The level schemes of the above nuclei were authenticated on the basis of the coincidence measurements. The properties of the levels are discussed, logft values and branching ratios deduced, spins and parities assigned and the lifetime for the first excited state of $^{166}$Er was determined using the method of delayed coincidence between a plastic and a NaI(Tl) detector.

Each established level scheme is compared with the theoretical predictions of current nuclear models, notably the interacting boson model (IBM). The Program-Package PHINT was run on the University of London CDC 7600 Computer in order to calculate energies and transition rates on the basis of this model. The nuclei investigated test the SU(5) limit for $^{110}$Cd, the O(6) limit for $^{192}$Pt and the SU(3) -- O(6) transitional region for $^{192}$Os and $^{166}$Er.
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CHAPTER I

GENERAL

1.1 Introduction

The exploration of nuclear levels by radioactivity has for more than two decades entered a new dimension. The tremendous developments in solid-state detectors and computers are in a short time-span having an impact on nuclear spectroscopy where a great impetus for research has come from the shell model of Mayer and Jensen\(^1\), the Bohr-Mottelson extension\(^2\) and more recently the interacting boson model (IBM) of Arima and Iachello\(^3\).

In the present work, investigations have been made of the decay schemes of \(^{110}\text{Cd}\), \(^{192}\text{Pt}\), \(^{192}\text{Os}\) and \(^{166}\text{Er}\) following the radioactive decays of \(^{110m}\text{Ag}\), \(^{192}\text{Ir}\) and \(^{166m}\text{Ho}\). For this purpose, Ge(Li) detectors have been employed both for singles and \(\gamma-\gamma\) coincidence measurements. The latter has been achieved by coupling the conventional fast-slow coincidence circuit to a more versatile Dual Parameter Data Collection System (DPDCS). For lifetime measurement a NaI(Tl)-plastic combination has been used; it has thus enabled the measurement of the half-life of the first excited state in \(^{166}\text{Er}\).

The experimentally obtained results were compared with the current nuclear models. A particular emphasis has been placed on the role that the (IBM) now play in understanding the nuclear structure of the nuclei under considerations.

The accessibility of fourth-generation computers has increased and highly developed coding techniques have been become available. The computer code PHINT which determines the eigenenergies and eigenstates of the (IBM) presents us with the opportunity of comparing the experimental data in the hope of deepning our theoretical understanding.

In this chapter, theoretical considerations relevant to the present work are introduced. The next section considers the radioactive decay law with a few words about the (n,\(\gamma\)) reaction. A brief review of the theory of \(\beta\)-decay and \(\gamma\)-emission is given in sections 1.3 and 1.4, respectively. Internal conversion process will be discussed in the last section.

Chapter II considers the main characteristics of few, but important nuclear models mentioned above. Experimental arrangements necessary for this investigation will be dealt with in Chapter III. Chapter IV, V and VI treat the decay of \(^{110m}\text{Ag}\), \(^{192}\text{Ir}\) and \(^{166m}\text{Ho}\), respectively. A conclusion is drawn in Chapter VII.
1.2 Radioactivity

When the nucleus of an atom emits an alpha particle, a beta particle, a gamma ray or any other particle, or when it captures an electron from an extra nuclear shell, the process is called radioactive decay.

It was shown by Rutherford\(^4\) that the amounts of the primary substance and the different products in a given quantity of radioactive matter vary according to the system of differential equations:

\[
\frac{dP}{dt} = -\lambda_1 P \\
\frac{dQ}{dt} = \lambda_1 P - \lambda_2 Q \\
\frac{dR}{dt} = \lambda_2 Q - \lambda_3 R
\]  

(1.2.1)

where \(P, Q, R, \ldots\) denote the number of atoms of the primary substance and successive products which are present at a time \(t\). The decay was observed to follow an exponential law with great accuracy. Then, this exponential requires (in general case) if an individual atom-by-atom process is assumed:

\[
N = N_0 e^{-\lambda t}
\]  

(1.2.2)

where \(N\) is the activity at a time \(t\),

\(N_0\) is the activity at a time \(t=0\), and

\(\lambda\) is the radioactive decay constant and is just the probability/sec of an atom decaying.

A characteristic of a particular radionuclide is the time \(t_\frac{1}{2}\), known as the half-life, and defined as the time required for the activity to fall to half its initial value. Thus:

\[
t_\frac{1}{2} = \frac{0.693}{\lambda}
\]  

(1.2.3)

so that Eq. (1.2.2) becomes

\[
N = N_0 e^{-0.693 \frac{t}{t_\frac{1}{2}}}
\]  

(1.2.4)

or

\[
\log_e (N/N_0) = -0.693 \frac{t}{t_\frac{1}{2}}
\]  

(1.2.5)

From the knowledge of the half-life of a radioisotope, the maximum specific activity \(S_{\text{max}}\) of that isotope can be derived. The radioisotopes are obtained either by bombardment of suitable target materials with neutrons or charged particles or extraction from naturally occurring radioactive substances or from fission products. Of major importance in this work are those radioisotopes produced by neutron bombardment in a nuclear reactor which is the most useful source of neutrons. In such a situation, the reaction is denoted by \(A(n,\gamma)B\), where \(n\) represents the capture neutron and \(\gamma\) represents the photon. \(A\) and \(B\) are the initial and final nuclides, respectively.
Since the product of this type of reaction is an isotope of the target element, a chemical separation cannot generally be carried out. Thus the specific activity obtainable by the \((n,\gamma)\) reaction is limited. For short irradiations, and irradiation in low fluxes, the burn up of the target nuclei can be neglected and frequently the neutron cross-section of the product isotope is negligible. The specific activity \(S\) of the product is thus given by:

\[
S = \frac{0.6 \phi \sigma (1 - e^{-0.693 t/t_\frac{1}{2}})}{3.7 \times 10^{10} W}
\]

where \(S\) is in Ci g\(^{-1}\) of the target element, 
\(\phi\) is the effective neutron flux in sample, in n cm\(^{-2}\) sec\(^{-1}\), 
\(\sigma\) is the activation cross-section of the target material in barns, 
\(W\) is the atomic weight of target material, 
\(t\) is the radiation time, and 
\(t_\frac{1}{2}\) is the half-life of the product isotope.

It thus appears appropriate to employ Eq. (1.2.6) for the determination of the activities of \(^{110}\text{Ag}\), \(^{192}\text{Ir}\) and \(^{156}\text{Ho}\). It should be noted that the \(^{110}\text{Ag}\) and \(^{192}\text{Ir}\) sources were obtained by respective irradiations of \(^{109}\text{Ag}\) and \(^{191}\text{Ir}\) in the thermal neutron, low flux CONSORT reactor at the University of London Reactor Centre (ULRC). The \(^{156}\text{Ho}\) source is the product of the thermal neutron irradiation of \(^{155}\text{Ho}\) in the DIDO reactor at Harwell (see, respectively, Chapters IV, V and VI for the preparation of these sources).

### 1.3 Beta decay

The purpose of studying \(\beta\)-decay are twofold. The first is to understand the intrinsic nature of the \(\beta\)-interaction and its possible connections with other known weak interactions. The correlation study between \(f_t\)-values and the known changes of spin and parity of \(\beta\)-transitions give the supplementary check of the theory. The second purpose of studying \(\beta\)-decay is to use it as a tool to investigate decay schemes and to assign the pertinent properties to each nuclear state involved, such as its decay rate, spin and parity.

In this section, the tools that may be employed in the treatment of the theory of \(\beta\)-decay are discussed. The log\(f_t\) values, resulted from this treatment, have played a very crucial role in the assignment of spin and parity to the nuclear states involved in the present investigations.

The long mean life of the neutron, \(\tau \approx 16\) minutes, shows that the interaction causing \(\beta\) emission is very weak indeed. According to Pauli's neutrino hypothesis, the basic processes involved in \(\beta\)-decay are:
\begin{align*}
\text{n} & \rightarrow \text{p} + \beta^- + \bar{\nu} & \beta^- \text{ decay} \\
\text{p} & \rightarrow \text{n} + \beta^+ + \nu & \beta^+ \text{ decay} \\
\text{p} + \text{e}^- & \rightarrow \text{n} + \nu & \text{electron capture}
\end{align*}

where n, p, \nu and \bar{\nu} denote neutron, proton, neutrino and anti-neutrino, respectively.

The foundations of the theory of $\beta$-decay were laid down originally by Fermi\(^6\). The description of nuclear $\beta$-decay has not changed essentially, except for the modifications necessary since the discovery by T.Lee and C.Yang in 1957 of parity non-conservation for the weak interaction\(^7\).

Fermi's theory is based on the formal analogy which one can establish with the description of the electromagnetic interaction with the "electron-neutrino field" acting in place of the electromagnetic field.

Since the relevant interaction (1.3.1) is very weak, it is quite justified to use first order perturbation theory for the calculation of transition rates. Thus, the probability per unit time for the emission of a $\beta$ particle within the momentum range p to (p+dp) can be written as\(^8\):

\begin{equation}
N(p) \, dp = \frac{2\pi}{h} \left| \int \bar{\psi}_f H_{\text{op}} \psi_i \, dv \right|^2 \frac{dn}{dE}\beta
\end{equation}

or

\begin{equation}
N(p) \, dp = \frac{2\pi}{h} \left| H_{\text{if}} \right|^2 \frac{dn}{dE}\beta
\end{equation}

where

\begin{equation}
H_{\text{if}} = \int \bar{\psi}_f H_{\text{op}} \psi_i \, dv
\end{equation}

In these equations, $\psi_i$ and $\psi_f$ are the time independent wave functions of the initial and final states, respectively, $H_{\text{op}}$, the Hamiltonian operator, is the operator associated with the interaction energy that causes the transition, $H_{\text{if}}$ is the expectation value, $dv$ is a small volume and $\frac{dn}{dE}$ is the statistical weight-factor that indicates the number of states per unit energy and is given by\(^9\):

\begin{equation}
\frac{dn}{dE} = \frac{p^2_{\beta}}{4 \pi^2 c^3 \hbar^6} \frac{(E_{\text{max}} - E_{\beta})^2 \, dp_{\beta}}{dE_{\beta}}
\end{equation}

where $p_\beta$ and $E_{\beta}$ are the momentum and energy of the electron, respectively.

Note that, $E_{\beta} + E_{\nu} = E_{\text{max}}$, where $E_{\nu}$ is the neutrino energy.

In the derivation of Eq. (1.3.5), the interaction was assumed to take place in a unit volume, i.e. $\Delta x \Delta y \Delta z = 1$.

Because the wave functions $\psi_i$ and $\psi_f$ of the initial and final states of the nucleus are not known, the whole theory depends, therefore, on the choice of $H_{\text{op}}$.

From analogy with the electrostatic field, the expression of $H_{\text{op}}$ can be written as:

\begin{equation}
H_{\text{op}} = g \, \phi_{\beta} \, \phi_{\nu}
\end{equation}
where $g$ is a constant of the weak interaction and was found to be $g = 1.4 \times 10^{-49}$ erg cm$^3$; $\phi_\beta$ and $\phi_\nu$ are the time-dependent wave functions that characterize the electron and the neutrino fields and are given by:

$$\phi_\nu = e^{i\mathbf{q} \cdot \mathbf{r}}$$

$$\phi_\beta = e^{i\mathbf{k} \cdot \mathbf{r}}$$

where $\mathbf{q} = \mathbf{p}_\nu/\hbar$ and $\mathbf{k} = \mathbf{p}_\beta/\hbar$ are the neutrino, electron propagation constants.

Note that, the derivation of these equations resulted from the fact that the neutrino and electron are treated (for high velocity) as free charges so that they are not affected by the nuclear Coulomb-field.

Combining Eqs. (1.3.4), (1.3.6), (1.3.7) and (1.3.8), one gets:

$$H_{1f}^2 = g^2 |\int \psi_\nu^* e^{i(k+q) \cdot \mathbf{r}} \psi_\nu |^2$$

Combining Eqs. (1.3.5) and (1.3.9) with (1.3.2), one obtains:

$$N(p) \frac{dp}{dE} = \frac{g^2 |M|^2}{2\pi^3 c^3 \hbar^7} (E_{\text{max}} - E_B)^2 p_B^2 dp_B$$

where $M = \int \psi_\nu^* e^{i(k+q) \cdot \mathbf{r}} \psi_\nu |^2$

This matrix element $M$ is the only unknown factor in Eq. (1.3.10) and its discussion provides useful information on whether a $\beta$-transition is classified either allowed or forbidden.

### 1.3a Allowed transitions

In lowest order of approximation the nucleons are considered to be nonrelativistic particles and the lepton (electron and neutrino) wave functions are evaluated at the origin $r=0$. In these cases, the exponential term $e^{i(k+q) \cdot \mathbf{r}}$ is of the order unity, so that the matrix element is independent of energy and therefore the transitions are termed allowed $\beta$-transitions. In such a situation, Eq. (1.3.10) can take the form:

$$\left(\frac{N(p)}{p^2}\right)^{1/2} = C \left(E_{\text{max}} - E_B\right) = C \left(K_{\text{max}} - K_B\right)$$

where $C = g |M| / (2\pi^3 c^3 \hbar^7)^{1/2}$.

Eq. (1.3.12), provides, if the theory given above is correct, a straight line when $\left(\frac{N(p)}{p^2}\right)^{1/2}$ is plotted against $K_B$. Such a plot is referred to as Kurie or Fermi plot.

It is however necessary to take into account the effect of the nuclear electrostatic field on the motion of the electron. This effect is provided by the Coulomb correction factor $F(Z, E)$ known as the Fermi function with $Z$ denoting the atomic number of the residual nucleus. Then the shape of the allowed $\beta$-spectrum is simply given by the statistical factor $\left((E_{\text{max}} - E_B)^2 p_B^2\right)^{1/2} F(Z, E)$. 

$$F(Z, E)$$
and Eq. (1.3.10) becomes

$$N(p) \, dp = \frac{g^2 |M|^2}{2\pi^3 c^3\hbar^7} \, F(Z,E) \, (E_{\text{max}} - E_{\beta})^2 \, p_{\beta}^2 \, dp_{\beta} \quad (1.3.14)$$

In this case the Kurie plot of an allowed spectrum should yield a straight line.

1.3b Forbidden transitions, logft values and selection rules

If the assumptions of nonrelativistic nucleons is dropped (i.e. also the small components of the wave functions are considered) and the variation of the lepton wave function over the nuclear volume is taken into account, one obtains the so-called forbidden $\beta$-transitions. These transitions are actually the ones that provide the test of the Fermi theory because, unlike allowed transitions, their shape does depend on the matrix element $M$.

Beta transition probabilities are calculated, just like electromagnetic transition probabilities, in terms of perturbation theory. The total $\beta$-decay rate of one nuclear state into another is obtained after integration over the spectrum of the electron energy. This total rate is nothing but the disintegration constant, $\lambda$. Thus, as a result one obtains:

$$\lambda = \frac{\ln 2}{t_0} = \frac{g^2 |M|^2 m^5 c^4}{2\pi^3 \hbar^7} \, f(Z, K_{\text{max}}) \quad (1.3.15)$$

where

$$f(Z, K_{\text{max}}) = \int_0^{p_{\text{max}}/mc} F(Z, E) \left(\frac{K_{\text{max}} - K}{mc^2}\right)^2 \frac{p^2}{mc} \, dp \quad (1.3.16)$$

In Eq. (1.3.16), the momentum and energy of the emitted electron are expressed in units of $mc$ and $mc^2$, respectively. The dimensionless function $f(Z, K_{\text{max}})$ describes the dependence of the nuclear charge $Ze$ and the available energy $K_{\text{max}}$.

From Eq. (1.3.15), one finds for the comparative half-life, i.e. the product of the statistical function $f(Z, K_{\text{max}})$ and the actual half-life, denoted by $t$ instead $t_0$, that

$$f(Z, K_{\text{max}}) \, t = f(Z, K_{\text{max}}) \frac{\ln 2}{\lambda} = \frac{2\pi^3 \hbar^7 \ln 2}{g^2 m^5 c^4 |M|^2} = \frac{\text{cte}}{|M|^2}$$

or

$$ft = \text{cte} \frac{1}{|M|^2} \quad (1.3.17)$$

As the comparative half-lives $ft$ vary over several order of magnitudes, it is customary to quote the logft (expressed in seconds). The evaluation of logft involves the calculation of the function $f$ from Eq. (1.3.16) which is not so simple. Alternatively, once the $E_{\text{max}}$ and $t$ are known, the numerical values of the logft can be read from a set of graphs prepared by S. Moszkowski\(^{10}\) (see also ref. 11).

For the allowed transitions, the logft values are mostly in the
range 3 to 6, while higher values correspond to the forbidden transitions. For example, those transitions which lie between 6 and 9 are classified as first forbidden. For second forbidden transitions log_{10t} is greater than 9, and so forth.

Finally, a brief discussion is given to the selection rules that govern $\beta$-decay. The angular momentum selection rules can be obtained when one thinks of the $\beta$-decay process as a reaction in which one electron and a neutrino are emitted. The conservation of angular momentum is then given by:

$$J_i = J_f + L_{ev} + S_{ev}$$  \hspace{1cm} (1.3.18)

where $J_i$ and $J_f$ are the nuclear initial and final spins, respectively. The symbols $L_{ev}$ and $S_{ev}$ denote the orbital angular momentum and spin of the emitted electron-neutrino pair. If the electron and neutrino, each with a spin 1/2, are emitted antiparallel, the total spin is zero and they are called Fermi-transitions. Similarly, if they emitted with spins parallel resulting in a total spin of 1, they are called Gamow-Teller transitions, after the physicists who first suggested them. In allowed $\beta$-decay, only $L_{ev} = 0$ appears with even-parity, and for Fermi transitions $S_{ev} = 0$. This leads to the selection rule $\Delta J = 0$. For a Gamow-Teller transition, $S_{ev} = 1$ from which $J_i = J_f + 1$ follows. This leads to $\Delta J = 0$ or $\pm 1$ (except $0 \rightarrow 0$).

The selection rules for first forbidden transitions may be arrived at by considering the expansion of the lepton wave functions \( e^{i(k + q) \cdot r} = 1 + i(k + q) \cdot r + \ldots \). This brings in a factor of $r$ and the particle cannot be emitted as an S-wave. This in turn, leads to the P-wave emission of the particle thereby resulting in $L_{ev} = 1$ and a change in parity. Tables giving the selection rules for $\beta$-decay can be found in many references, see for example, ref. 9.

It is interesting to note that there is an additional useful source of information which is not directly derived from the observation of $\beta$-decay, that is the knowledge of the multipole order of the $\gamma$-radiations in the daughter nucleus. Since, as a result of the $\beta$-decay, the excited states in the daughter nucleus always de-excite to lower or ground states by gamma-emissions. The beta and gamma investigations always complement each other in studies of nuclear levels. Therefore, the next section is devoted to the study of gamma emission.

1.4 Electromagnetic transitions

A large part of the knowledge of nuclei is obtained from the study of electromagnetic transitions, since the electromagnetic interaction is well understood, in contrast with nuclear force. It is, for example, the
main source of information about the spin assignments of nuclear states. The nuclear multipole moments and the transition rates for the various multipole radiations can be calculated theoretically, once the nuclear wave functions are known.

In this section, a detailed description of the theory of gamma emission will not be given. Instead only some of the most important steps that lead to explicit expressions of some measurable quantities such as reduced transition rates, lifetimes, branching and mixing ratios will be summarized. The much used Weisskopf single-particle estimates of transitions strengths will be briefly dealt with. Angular momentum and parity selection rules will be discussed. For more details, one is referred to, for example, Blatt and Weisskopf (1952)\textsuperscript{12}, Jackson (1962)\textsuperscript{13}, Morse and Feshbach (1953)\textsuperscript{14} and Roy and Nigam (1967)\textsuperscript{15}.

The description of the process of emission or absorption of gamma radiation by nuclei requires the use of quantum mechanics, since unlike classical situations, the structure of the nuclei and their wave functions are not known. But by assuming a certain model of the nucleus, it is possible to estimate the transition probabilities.

The interaction between the nuclear currents and charges on the one hand and the radiation field on the other is considered as a perturbation causing transitions between the stationary states of the nuclear Hamiltonian. The emitted gamma radiation can be characterized by its energy, angular momentum and parity. The angular momentum carried away by the gamma quantum determines the multipolarity of the radiation. Angular momentum $L$ corresponds to $2L$ pole radiation with its characteristic radiation pattern, i.e. the angular distribution of the intensity of the emitted gamma rays. Two identical intensity distributions for one particular value of $L$ still may correspond to different parities of the fields. The two different possibilities are distinguished as electric $2L$ pole radiation and magnetic $2L$ pole radiation.

In the long-wavelength approximation, the wavelength $\lambda$ of the emitted radiation should be much larger than the nuclear radius, $R_0 = 1.2A^{1/3}$ fm. Thus, one finds\textsuperscript{16}:

$$qR_0 = \frac{E_Y}{197 \text{ MeV fm}} \times 1.2A^{1/3} \text{ fm} = 6 \times 10^{-3} \frac{E_Y}{\text{MeV}} A^{1/3}$$

This leads to the condition:

$$\frac{E_Y}{\text{MeV}} \ll 160 A^{-1/3}$$

where $E_Y = h\omega$ is the gamma radiation energy and $q = \frac{\omega}{c} = \frac{E_Y}{197 \text{ MeV fm}}$ is the momentum transferred by the gamma radiation. Thus, in this long-wavelength
approximation, the magnetic multipole transition and the electric multipole
transition operators are given by a sum over the single-particle operators
for the \( k \) nucleons (de-Shalit and Talmi (1963)\(^{17}\); Bohr and Mottelson (1969)\(^{18}\))

\[
\mathcal{M}(EL,\mu) = \sum_k e(k) r_k^L Y_{LM}(\Theta_k, \Phi_k) \tag{1.4.1}
\]

\[
\mathcal{M}(ML,\mu) = \sum_k (g_s(k)\Sigma_k + (2g_o(k)/(L+1))\Sigma_k), \Sigma_k (r_k^L Y_{LM}(\Theta_k, \Phi_k)) \Omega, \tag{1.4.2}
\]

The \( e(k) \) is an effective charge for the \( k \)th nucleon, \( \mu_0 \) is the nuclear magneto and the \( g_s(k) \) and \( g_o(k) \) are the spin and orbital \( g \) factors for
the \( k \)th nucleon, respectively. In Eq. (1.4.2), \( \Sigma(p) = r(p)X(p)\Omega, \) is the
operator which is supposed not to act on \( Y_\ell(k)Y_{LM}(\Theta_k, \Phi_k) \) but only on
the nuclear wave functions when the matrix element is taken. Moreover, the
gradient operator \( \Sigma(k) \) acts on \( r_\ell(k)Y_{LM}(\Theta_k, \Phi_k) \) only.

The transition probability for a gamma transition of multipolarity \( L \)
from an excited state \( \psi_a \) to a final state \( \psi_b \) is given by\(^{12,19}\):

\[
\lambda(L) = \frac{8\pi(L+1)}{L(2L+1)!} \frac{(1/\hbar)}{(E_\gamma/hc)^{2L+1}} B(L) \tag{1.4.3}
\]

where the reduced transition probability for initial stat \( J_a \) to final state
\( J_b \) is:

\[
B(L) = \frac{1}{(2J_a+1)} |<\psi_b|\mathcal{M}(L,\mu)|\psi_a>|^2 \tag{1.4.4}
\]

This \( B(L) \) represents the sum of squared \( \mathcal{M}(L,\mu) \) matrix elements over the
\( m \) substates of the photon and the final state and an average over the initial
substates. Electric transitions are usually expressed in units of \( e^2 \text{ fm}^{2L} \)
or \( e^2 \text{ b}^{L-1} \). For magnetic transitions one commonly employs units of \( \mu_0^2 \text{ fm}^{2L-2} \)
or \( \mu_o^2 \text{ b}^{L-1} \). The \( B(L) \) for the first few electric and magnetic multipoles
as calculated from Eq. (1.4.3) are:

\[
\begin{align*}
B(E1) &= 6.29 \times 10^{-16} (E_\gamma)^{-3} \lambda(E1) \quad (e^2 \text{ fm}^{2}) \\
B(E2) &= 8.20 \times 10^{-10} (E_\gamma)^{-5} \lambda(E2) \quad (e^2 \text{ fm}^{4}) \\
B(E3) &= 1.76 \times 10^{-3} (E_\gamma)^{-7} \lambda(E3) \quad (e^2 \text{ fm}^{6}) \\
&\vdots \\
B(M1) &= 5.68 \times 10^{-14} (E_\gamma)^{-3} \lambda(M1) \quad (\mu_0^2) \\
B(M2) &= 7.41 \times 10^{-8} (E_\gamma)^{-5} \lambda(M2) \quad (\mu_0^2 \text{ fm}^{2}) \\
\end{align*}
\tag{1.4.5}
\]

where \( E_\gamma \) is in MeV and the \( \lambda(L) \) in sec\(^{-1}\).

A mean lifetime \( \tau \) corresponds to the transition probability \( \lambda(L) \) of the
decaying state with \( \tau=1/\lambda(L) \). When \( \gamma \)-decay of multipolarity \( L \) to state \( b \)
is the only decay mode, then the mean lifetime of the initial state \( a \) is
directly related to \( \lambda(L) \). However, if state \( a \) decays to other final states
or more than one multipole is involved, then information in addition to the
lifetime is needed to extract the \( B(L) \). In the first case, where state \( a \)
decays to other final states, the partial $\gamma$-ray transition probability $\lambda_\gamma(L)$ is obtained from the total transition probability $\lambda(L)$ by:

$$\lambda_\gamma(L) = \lambda(L) \frac{N_\gamma(L)}{\sum_i N_i} \quad (1.4.6)$$

where $\sum_i N_i$ is the sum of the intensities of all transitions depopulating the level of interest in the same relative units as the intensity $N_\gamma(L)$ of the $\gamma$-ray transition with multipolarity $L$ for which $\lambda_\gamma(L)$ is to be calculated.

In the second case, where more than one multipole radiation from initial state to one final state is involved, one introduces the mixing ratio $\delta$, the magnitude of which is defined by the relation:

$$\delta^2 = \frac{\lambda(L+1)}{\lambda(L)} \quad (1.4.7)$$

This definition of $\delta^2$ stems from the fact that one usually encounters the mixing of no more than two multipole radiations that differ one unit in angular momentum, say $L$ and $L+1$.

In order to obey parity conservation, one finds from Eq. (1.4.7) that the two competing radiative transitions must be $EL+1$ and $ML$ or $ML+1$ and $EL$.

From experimental data on angular distribution of mixed gamma radiation, one can extract the mixing ratio.

In what follows, a brief treatment of the selection rules governing the emission of gamma radiations is given. These selection rules are the results of conservation laws. The two most important conservation laws applicable in the present situation are those of angular momentum and parity. Thus, when a nucleus emits a photon, the initial total nuclear momentum should be equal to the sum of the final total nuclear momentum and the angular momentum carried by the radiation. Thus:

$$J_i = J_f + L \quad (1.4.8)$$

This implies that the multipole matrix elements $\langle \psi_f | M(X) | \psi_i \rangle$ with $X=EL$ or $ML$ denoting electric or magnetic multipole radiation, can differ from zero if the selection rule

$$|J_i - J_f| < L < J_i + J_f \quad (sometimes \ referred \ as \ \Delta(J_i,J_f,L))$$

is satisfied. The relation (1.4.9) is known as the triangle condition, because the three angular momentum vectors must be such that they can form a triangle. One of the consequences of this rule is that the gamma transitions $J_i + J_f = 0$ do not occur since monopole radiation ($L=0$) does not exist.

The other important rule results from the conservation of parity. In gamma emission the system involves the parity of the initial state wave function, $\psi_i$, the parity of the final state wave function, $\psi_f$ and the parity
of the multipole radiation field. It can be shown\textsuperscript{16} that the parity selection rule for electromagnetic transitions can be formulated as:

\[ \pi_f \pi_\gamma \pi_i = 1 \quad (1.4.10) \]

where \( \pi_f \), \( \pi_\gamma \) and \( \pi_i \) represent the parities of the final state, the emitted radiation and the initial state, respectively. Thus, one sees that:

\[ \Delta \pi = (-1)^L \text{ for (EL)} \]
\[ \Delta \pi = (-1)^{L+1} \text{ for (ML)} \]

where \( \Delta \pi \) is the change in parity in going from state \( i \) to state \( j \). The electromagnetic interaction decreases rapidly with \( L \) with the results that only the lowest allowed \( L \) for the electric and for the magnetic multipoles are important.

The evaluation of the electromagnetic matrix element with various nuclear wave functions for a comparison with experimental results is simplified by the fact that the electromagnetic operators are sums of the single-particle operators. A simple approximation of the single-particle matrix elements is often used to calculate from Eq. (1.4.4) an approximate unit of strength, the so-called Weisskopf unit:\textsuperscript{12}

\[ B(EL)_W = \left( \frac{1}{4\pi} \right) \left\{ \frac{3}{(3+1)} \right\}^2 (1.2 A^{1/3})^{2L} \left\{ e^2 \text{ fm}^2 L \right\} \]
\[ B(ML)_W = \left( \frac{10}{\pi} \right) \left\{ \frac{3}{(3+1)} \right\}^2 (1.2 A^{1/3})^{2L-2} \left\{ \mu_0^2 \text{ fm}^2 L-2 \right\} \]

These estimates for radiation of multipolarity \( 2^L \) are based on a very simple model:

(i) The nucleus consists of an inert core plus one active particle.
(ii) The transition takes place between states \( J_i = L+1/2 \) and \( J_f = 1/2 \).
(iii) The radial parts of the initial and final state wave functions are both given by \( u(r) = \text{constant for } r < R \) and \( u(r) = 0 \) for \( r > R \), where \( R \) denotes the nuclear radius.

It is common practice to compare an experimentally determined transition strength with the corresponding Weisskopf estimate, the Weisskopf unit. The main reason for expressing transition rates in these units is the removal of the strong dependence on the transition energy.

In making such a general review of the theory of gamma emission, a more sensitive check on the nuclear structure of a given state can be achieved by comparing values obtained from the above equations with the corresponding ones deduced from theoretical nuclear models (see Chapters IV, V and VI).
1.5 Internal conversion

An electromagnetic decay of the atomic nucleus can proceed by competing processes: emission of gamma radiations, production of electron-positron pairs \((e^-e^+)\) or emission of orbital electron \((e^-)\), that is internal conversion. If a given radioactive sample emits \(I_\gamma\) gamma rays, in a given time and \(I_e\) conversion electrons in the same time, the ratio \(I_e/I_\gamma\) is called the conversion coefficient, \(\alpha\), that is:

\[
\alpha = \frac{I_e}{I_\gamma} = \frac{\lambda_e}{\lambda_\gamma} \tag{1.5.1}
\]

where \(\lambda_e\) and \(\lambda_\gamma\) are the transition probabilities for the conversion electron-emission and gamma-emission, respectively.

Because of the compeitity of the processes of gamma-emission and orbital electron-emission, the total transition probability \(\lambda\) for a given state can be written as:

\[
\lambda = \lambda_\gamma + \lambda_e \tag{1.5.2}
\]

and

\[
\lambda_e = \lambda_K + \lambda_L + \lambda_M + \ldots \tag{1.5.3}
\]

where \(\lambda_K\), \(\lambda_L\) and \(\lambda_M\) are the transition probabilities for K, L and M conversion electron-emission. Thus, Eq. (1.5.1) can be written as:

\[
\alpha = \frac{\lambda_K + \lambda_L + \lambda_M}{I_\gamma} = \frac{\lambda_K}{I_\gamma} + \frac{\lambda_L}{I_\gamma} + \frac{\lambda_M}{I_\gamma} = \alpha_K + \alpha_L + \alpha_M \tag{1.5.4}
\]

where \(\alpha_K\), \(\alpha_L\) and \(\alpha_M\) are called the K, L and M conversion coefficients, respectively. From the above equations, one can relate the mean lifetime \(\tau\) of the total transition and \(\tau_\gamma\) for the gamma transition in the following way:

\[
\tau = \frac{1}{\lambda} = \frac{1}{(\lambda_\gamma + \lambda_e)} = \frac{1/\lambda_\gamma}{1 + \lambda_e/\lambda_\gamma}
\]

or

\[
\tau = \frac{\tau_\gamma}{1 + \alpha}
\]

i.e.

\[
\tau_\gamma = \tau (1 + \alpha) \tag{1.5.5}
\]

In the analysis of Eqs. (1.4.5), one should correct for a possible internal conversion by making use of Eq. (1.5.4). It should be noted that the knowledge of the coefficients is one of the most important tools for the determination of parity and multipolarity of electromagnetic nuclear transitions and the construction of nuclear decay schemes. Theoretical evaluation of the K, L, M, \ldots shell conversion-coefficients, basically depend on four factors: (1) the energy of the nuclear transition, (2) the atomic subshell out of which the orbital electron is ejected, (3) the charge of the decaying nucleus and (4) the multipolarity and parity of nuclear transition.
With the help of high-speed computers, tables of internal conversion coefficients have been made available and are thus of great importance. Such tables were produced by many workers\textsuperscript{21–23}. They cover different atomic subshells, nuclear charges and transition energies. The present theoretical $\alpha(K)$, which are needed in Chapters IV and V, are obtained from ref. 23.
CHAPTER II
NUCLEAR MODELS

2.1 Introduction

It would be an ideal situation if one could find a single model of the nucleus that would explain all the nuclear properties. Unlike atomic models, where the law of force is known and the models are on firm footing, no such definiteness exists in the nuclear models because of a lack of knowledge of nuclear forces.

The development of nuclear models has taken place along two lines. There are those in which the constituents of the nucleus are treated on a statistical basis, as in the case of a liquid drop or a volume gas. The second type of models are constructed in analogy with the shell model of the atom. The nucleons are treated as individual particles in the system.

Much effort has been made in the previous thirty years or more to understand the nature of the collective properties in nuclei in terms of geometrical models. The problem with these models is that there is no well-defined procedure for making transition between the different models, e.g., the transition from a spherical vibrator to a deformed rotor. It has thus been necessary to develop more models in order to explain the properties of the excited states in different energy ranges and different transitional cases. The interacting boson model (IBM), recently developed by Arima and Iachello, is destined to make such a description of the transition between different models (see section 2.4). At present, however, these models are helpful in explaining certain properties of the nuclei. Some of these models are: (1) the shell model, (2) the collective model, (3) the liquid drop model, (4) the pairing plus quadrupole model, (5) the moment of inertia model and the (IBM).

In this Chapter, the nuclear shell model is discussed first. Sections 2.3 and 2.4 describe the collective model and the (IBM), respectively.

2.2 The nuclear shell model

The basic assumption of the nuclear shell model is that to a first approximation each nucleon (proton or neutron) moves independently in a potential that represents the average interaction with the other nucleons in the nucleus.
Before 1945 progress in the development of the nuclear shell model was rather slow. This was mainly due to the failure of the model to reproduce the binding energy, which were the most extensive data of that time. After the introduction of a strong spin-orbit term to the single particle potential (Mayer (1949)^9; Haxel, Jensen and Suess (1949)^10), the usefulness of the shell model in correlating many experimental data began to be widely accepted. Moreover, it is only after the inclusion of this term that the so-called magic numbers as they observed in nature can be reproduced. These numbers at 2, 8, 20, 28, 50, 82 and 126 that correspond to the number of protons or neutrons in the nucleus are the counterpart in nuclear structure to the atomic numbers (Z = 2, 10, 18, 36, 54 and 86) characterizing the noble gases. As in the atomic case, these numbers correspond to the closing of shells that have an especially large energy separation from the next higher orbits (major shell closings). Due to the Pauli principle each state or orbit of given total angular momentum \( j \) can be occupied by a restricted number of identical fermions, thus forming a subshell. A group of orbits lying close in energy is referred to as a shell, or rather a major shell. Each orbit is characterized by a particular value of the radial quantum number, the orbital angular momentum and the total angular momentum, denoted by \( n \), \( \ell \) and \( j \), respectively.

The existence of the shell model predicts the spins and parities of the nuclei in the ground states if one considers the following:

1. In a completely filled level (subshell or shell), the orbital angular momenta and spins of the nucleons add in such a way as to give a zero-resultant total angular momentum.
2. In the levels that are not completely filled, the nucleons form pairs (proton pairs and neutron pairs, but no proton-neutron pairs: "pairing effects"). These two assumptions lead to the following coupling rules:

   **Rule 1** The ground states of all nuclei with an even number of protons and of neutrons have zero angular momentum and even parity. This is an outstanding empirical fact to which no exception is known.

   **Rule 2** In a nucleus with an even number of neutrons and odd number of protons, the ground-state properties are determined by the protons only, i.e., because the total angular momentum of the neutrons is zero. The spin of the nucleus is determined by the last odd proton. Similarly, in nuclei with an even number of protons and odd number of neutrons only the neutrons need be considered.

It is interesting to note that, in the usual formalism of the shell model the potential is assumed to be isotropic, but it was found that nuclei with proton and neutron numbers very different from those corresponding to
closed shells have large deformations, as evidenced, for example, by large quadrupole moments. It may be possible to explain these moments by assuming that they result from a mixture of many states, but such calculations will be difficult and even impossible. A simpler explanation was given on the basis of another model, the collective model, which forms the subject of the next section.

2.3 The collective model

According to the collective model\(^2,24,31\), as in the shell model, the nucleons in a nucleus move independently in a real potential. But, unlike the shell model, the spherically symmetric potential is capable of undergoing a deformation in its shape as a result of the motion of the nucleon or nucleons around the core. The collective motion of the nucleons may be described as a vibrational motion about the equilibrium and a rotational motion that maintains the deformation shape of the nucleus.

The behaviour of the quadrupole moments finds a simple explanation\(^24\) if one considers the motion of the individual particles in a deformable nucleus. Due to the centrifugal pressure exerted by the particles on the nuclear walls, the nucleus may acquire a considerable deformation. The quadrupole moments thus induced have the same sign as those observed and appear also to have the right order of magnitude.

Apart from the behaviour of the quadrupole moments, the collective model may find application in the analysis of the energy spectrum of the nucleus and of transitions between nuclear states. Such transitions not only involve a change of state of the individual nucleus, but must be expected to be accompanied in general, by changes in vibrational and rotational state of the nucleus.

2.3a The rotational model

For nuclei whose equilibrium shape deviates strongly from spherical symmetry, one can distinguish approximately between two essentially different modes of excitations, rotational and intrinsic. The former is associated with a collective motion which affects only the orientation in space while preserving the internal structure of the nucleus, the latter may be associated with the excitation of the individual particles or with collective vibrations of the nuclear shape (\(8\)- and \(\gamma\)-vibrations).

The rotational spectrum depends essentially on the nuclear equilibrium shape, and is especially simple for axially symmetric nuclei. The rotational motion can then be characterized by the quantum numbers \(I\), \(K\) and \(M\) representing the total angular momentum, its projection on the nuclear symmetry
axis, and its projection on the space fixed axis, respectively.

The separation of the nuclear motion into rotational and intrinsic modes corresponds to the existence of approximate solutions of the nuclear wave equation of the simple product type:

$$\mathbf{\psi} = \sqrt{\frac{2I + 1}{8\pi^2}} \psi_{tK} \mathcal{D}^I_{MK} (\theta)$$

(2.3.1)

Here $\psi_{tK}$ represents the intrinsic motion of the nucleons characterized by $K$, and the additional set of quantum numbers, $\tau$. $\mathcal{D}^I_{MK} (\theta)$ represents the collective rotational motion of the system as a whole which depends on the Eulerian angle $\theta$ of the nuclear coordinate system.

Beside the rotational system around the nuclear axis, one also assumes that the nucleus has reflection symmetry through a plane perpendicular to this axis. The states in a rotational band are characterized by the same intrinsic wave function $\psi_{tK}$ and are labelled by different values of $I$. In odd-A nucleus, where $K$ is a positive half integer number, $I$ may take on the values:

$$I = K, K+1, K+2, \ldots$$

(2.3.2)

all of the same parity as the odd particle configuration. In an even-even nucleus, the ground state has $K=0$ and the symmetrization of the wave function limits the rotational band to $I=0, 2, 4, 6 \ldots$ even parity. In an odd-odd nucleus or in excited intrinsic states of even-even nuclei with $K=0$, the rotational spectra is again given by (2.3.2).

The energies of the states in a rotational band are given by the expression:

$$E_{\text{rot}} = \frac{\hbar^2}{2I} \left\{ I(I+1) - K^2 + a (-1)^{I+\frac{1}{2}} \right\}$$

(2.3.3)

where $\hbar$ is the moment of inertia and $\hbar$ is Plank's constant divided by $2\pi$. The two first terms of Eq. (2.3.3) just represent the energy levels of a symmetric top (the constant term $\hbar K^2/2I$ does not affect the level spacings). The last term which occurs only for odd-A nuclei with $K=\frac{1}{2}$ is associated with the symmetrization of Eq. (2.3.1). The parameter $a$ can be expressed in terms of the properties of the intrinsic wave function and is called the decoupling parameter. Its value is given by:

$$a = \sum_{j} (-1)^{j+\frac{1}{2}} (j+\frac{1}{2}) |c_j|^2$$

(2.3.4)

where $|c_j|^2$ is the probability that the last odd particle has angular momentum $j$.

Rotational spectra of the type (2.3.3), characteristic of axially symmetric nuclei, are found to occur systematically in certain regions of
elements, \( A = 24 \) and especially for \( 155 < A < 185 \) and \( A > 225 \). It should be noted that, although intrinsic particle excitations and rotational excitations may have comparable energies, the two types of excitations can be rather easily distinguished, partly on the basis of their essentially different transition probabilities. However with decreasing deformation and increasing rotational frequency, the intrinsic nuclear structure is excited by the rotational motion, and the quantum numbers \( K, \tau \) are no longer exact constants of motion. This implies a modification in the rotational spectrum of Eq. (2.3.3), which may often be described by a term proportional to \( I^2(I+1)^2 \) as is characteristic of the rotation-vibration interaction in molecules. Hence, one can expand the rotational energy in powers of the angular momentum. This yields:

\[
E(I(I+1)) = E_K + A I (I + 1) + B I^2 (I + 1)^2 + \ldots \quad (2.3.5)
\]

where \( E_K \) is the intrinsic energy and is the same for all members of the band. \( A \) and \( B \) are two normalizing parameters which can be determined from the experimental values of the energy levels. For the ground-state, their values can be deduced from the two lowest members of the band. It should be mentioned that Eq. (2.3.5) has played a very sensitive role in the interpretation of the collective states of the \(^{166}\text{Er} \) nucleus (see Chapter VI).

The gamma transitions within a rotational family are of \( E2 \) and \( M1 \) type. The \( E2 \) transition probabilities are strongly enhanced as compared to those corresponding to the transitions of a single proton. This enhancement is observed in some cases to exceed a factor of one hundred. The \( M1 \) rotational probability is related to the static magnetic moment of the nucleus and is of the order of magnitudes of estimates for single-particle transitions.

The strength of the gamma-transition of multipole order, \( L \), between an initial state \( i \) and a final state \( f \) may be characterized by the reduced transition probability:

\[
B(L, I_i \rightarrow I_f) = \sum_{\mu, M_f} |\langle I_i, M_i | \mathcal{M}(L, \mu) | I_f, M_f \rangle|^2
\]

where \( \mathcal{M}(L, \mu) \) is the \( \mu \)-component of the transition operator of multipole order \( L \). The collective \( E2 \) moment connects states (with \( \Delta I \leq 2 \)) belonging to the same rotational band. Thus, one obtains the \( E2 \) reduced transition probabilities:

\[
B(E2, K I_i \rightarrow I_f) = \frac{5}{16\pi} e^2 Q_i^2 <I_i K20|I_f K>^2 \quad (2.3.7)
\]

The vector addition coefficient \( <I_i K20|I_f K> \) represents the coupling of the angular momenta in the intrinsic frame.
The diagonal matrix element \( (I^=I^=) \) gives the static moment \( Q \), with

\[
Q = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} \quad Q_0
\]

(2.3.8)

where \( Q_0 \) is the intrinsic electric quadrupole moment. For \( E2 \) transitions between two different bands, say the gamma-band with \( K=2 \) and the ground state band with \( K=0 \), one gets\(^3^3^3\):

\[
B(E2, K=2, I^=I^=K=0, I^=) = 2 \quad <I^2 I^2-2 \quad I^0>^2 \quad (M_1 + M_2 \quad (I^+1) - I^0 \quad (I^+1)) + \ldots)^2
\]

(2.3.9)

On this basis, a plot of the square root of the \( B(E2) \) values divided by their respective Clebsch-Gordan coefficients should exhibit a linear dependence on \( I^2 I^2-2 \quad I^0 \quad (I^+1) - I^0 \quad (I^+1) \) (Mikhailov plot\(^3^4^4\)). The quantities \( M_1 \) and \( M_2 \) are directly related to the unmixed \( \Delta K=2 \) transition matrix element and the gamma-ground mixing amplitude, respectively. They can be determined from a least-square fitting procedure to the experimental data (see Chapter VI).

It is interesting to note that the quadrupole deformation may be characterized by the parameter \( \beta_2 \), called the deformation parameter and is associated with the expansion of the density distribution in spherical harmonics. The advantage of \( \beta_2 \) is its rather direct relationship to the experimental quadrupole moment.

2.3b The vibrational model

For many-body (many particles, many holes), it is possible to describe the excitations spectra in terms of elementary modes of excitations representing the different, approximately independent fluctuations about equilibrium\(^3^3\).

In order to classify the stationary states of an oscillating nucleus, the Hamiltonian of the system (nuclear surface) must be known. This surface shape can be described by five parameters \( a_{\lambda,\mu} (-2 \leq \mu \leq 2) \) and can be given in polar coordinates, by \( R(\theta,\phi) \). It is convenient to use the multipole expansion:

\[
R(\theta,\phi) = R_o (1 + \sum_{\lambda,\mu} a_{\lambda,\mu} Y_{\lambda,\mu}(\theta,\phi))
\]

(2.3.10)

where \( R_o \) is the radius of the nucleus in its spherical equilibrium shape. The function \( Y_{\lambda,\mu} \) is the normalized harmonic of order \( \lambda,\mu \).

Vibrations of the surface imply that the deformations \( a_{\lambda,\mu} \) are time dependent, which together with their time derivative \( \delta_{\lambda,\mu} \) are considered as dynamical variables. The former play the role of position coordinates and the latter the role of velocities. Thus for small values of \( a \), the
collective Hamiltonian of the nuclear surface is given by:

$$H = T + V = 1/2 \sum_{\lambda,\mu} B_{\lambda} |d_{\lambda,\mu}|^2 + 1/2 \sum_{\lambda,\mu} C_{\lambda} |a_{\lambda,\mu}|^2$$

where $T$ and $V$ are, respectively, the kinetic and potential energies of the deformations. $B_{\lambda}$ represents the inertial parameter and $C_{\lambda}$ is the stiffness parameter or restoring potential for the nuclear surface, both depend on more detailed assumptions regarding the properties of nuclear matter.

Eq. (2.3.11) represents a set of harmonic oscillators with frequencies $\omega = \sqrt{C_{\lambda}/B_{\lambda}}$ and energy values, $E(n) = nE_1$, where $n$ is the degree of the phonon excitation and $E_1$ is the energy of the first excited state. As the phonons are bosons, the states must be completely symmetric. Each quadrupole phonon carries angular momentum $2^+$. Then, the ground state represents no phonons, spherical shape, the first excited state one phonon $2^+$, the second excited state two phonons each of $2^+$, giving a degenerate triplet of states $0^+, 2^+, 4^+$ and so on.

Anharmonic effects in the vibrational motion can be treated in terms of interaction between the phonons and one expects the above crude model to be obeyed only roughly. This anharmonicity will, for example, remove the degeneracies in the 2-phonon states as it is the case in $^{110}\text{Cd}$ nucleus (see Chapter IV). Moreover, one can expect contribution from the vibrating core to the electric multipole moments because it is seen that pure $n$ phonons vibrational states do not possess electric moments. This is not unexpected since the nuclei have been taken to oscillate about a spherical equilibrium shape.

2.4 The interacting boson model (IBM)

The low-lying energy spectra of many medium-to-heavy-mass nuclei show regular features which can be explained remarkably well by such collective models of the nucleus as the geometrical model of Bohr and Mottelson and recently the interacting boson model of Arima and Iachello. The latter model, whose characteristics are discussed in this section, was introduced as an alternative algebraic description of the former.

In the (IBM), the spectroscopies of the low-lying collective properties of even-even nuclei are described in terms of a system of interacting $L=0$ and $L=2$ bosons (s and d bosons). Furthermore, the model assumes that the structure of the low-lying levels is dominated by excitations among the valence particles outside closed major shells. In the particle space, the number of proton $N_p$ and neutron $N_n$ bosons is counted from the nearest closed shells and the resulting number of bosons is a strictly conserved
quantity. The underlying structure of the six-dimensional unitary group SU(6) of the model basis leads to a simple Hamiltonian which is capable of describing both the three specific types of collective structure with classical geometrical analogs, vibrational\(^{35}\), rotational\(^{36}\) and gamma-unstable\(^{37}\) and the transitional nuclei whose structures are intermediate (see Chapters V and VI).

In its simplest form, the so called (IBM-1) which makes no distinction between neutron and proton bosons, the most general two-body Hamiltonian for a system of s and d bosons can be written in terms of bosons creation and annihilation operators. Thus\(^{35}\):

\[
H = \varepsilon_s (s^+.s) + \varepsilon_d (d^+.d) + \sum_{L=0,2,4} \frac{1}{2(2L+1)^{1/2}} C_L [(d^+ X d^+)^L X (d X d)^L]^{(0)} + \frac{1}{2} \tilde{V}_2 [(d^+ X d^+)^{(2)} X (s X s)^{(2)} + (d^+ X s^+)^{(2)} X (d X d)^{(2)}]^{(0)} + \frac{1}{2} \tilde{V}_0 [(d^+ X s^+)^{(0)} X (s X s)^{(0)} + (s^+ X s^+)^{(0)} X (d X d)^{(0)}]^{(0)} + U_2 [(d^+ X s^+)^{(2)} X (d X d)^{(2)}]^{(0)} + \frac{1}{2} U_0 [(s^+ X s^+)^{(0)} X (s X s)^{(0)}]^{(0)},
\]

(2.4.1)

where \((s^+,d^+)\) and \(s,d\) are the creation and annihilation operators for s and d bosons. The \((\ ), [ ]\), respectively denote the scalar, tensor product of two tensor operators.

The above Hamiltonian contains nine parameters: the two single bosons energies \(\varepsilon_s\) and \(\varepsilon_d\) and the seven two-body terms \(C_L (L=0,2,4), \tilde{V}_L (L=0,2)\) and \(U_L (L=0,2)\). Since the total number of bosons is conserved, \(N=n_s+n_d\) (\(n_s\) and \(n_d\) are the number operators for s and d bosons, respectively), the Hamiltonian can be written so that the excitation energies are independent of \(n_s\) and depend upon only six parameters\(^{38-39}\).

Another convenient form often used in phenomenological analyses is obtained by writing the Hamiltonian \(H\) as\(^{38}\):

\[
H = \varepsilon \sum_m d^+_m d_m + k'' P P - k' L L - k Q Q + k_3 T_3 T_3 + k_4 T_4 T_4
\]

(2.4.2)

where \(\varepsilon = \varepsilon_d - \varepsilon_s\) is the boson energy, \(k'', k', k, k_3\) and \(k_4\) designate the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecapole interaction between bosons, respectively. The operators of Eq. (2.4.2) are defined by:

\[
P = \frac{1}{2} (d.d) - \frac{1}{2} (s.s),
\]

\[
L = \sqrt{10} [d^+ X d]^{(1)},
\]

\[
Q = [d^+ X s + s^+ X d]^{(2)} - \frac{1}{2} \sqrt{10} [d^+ X d]^{(2)},
\]

(2.4.2a)

\[
T_3 = [d^+ X d]^{(3)},
\]

\[
T_5 = [d^+ X d]^{(4)}.
\]
In the three limiting cases\(^{35-37}\)(see later), different terms of the Hamiltonian (2.4.2) are used. Before going into the discussion of those limits, it is worthwhile to consider the electromagnetic transition rates in the (IBM-1). In order, however, to calculate these transition rates, one must specify the transition operators. In the simplest form of the (IBM-1), the one body transition operator which has the second quantized form is\(^{38}\):

\[
T^{(E2)}_m = \alpha_2 [d^+ X s + s^+ X d]^{(2)} + \beta_2 [d^+ X d]^{(2)} + \gamma_0 \delta_{20} \delta_{m0} [s^+ X s]^{(0)}
\]

(2.4.3)

This equation yields transition operators for EO, M1, E2, M3 and M4 transitions with appropriate values of the corresponding parameters. The \(T^{(E2)}\) operator, which has enjoyed a widespread application in the analysis of \(\gamma\)-ray transitions, can thus take the form:

\[
T^{(E2)}_m = \alpha_2 [d^+ X s + s^+ X d]^{(2)} + \beta_2 [d^+ X d]^{(2)}
\]

(2.4.4)

It is clear that, for the E2 multipolarity, two parameters \(\alpha_2\) and \(\beta_2\) are needed in addition to the wave functions of the initial and final states. The \(B(E2)\) values are obtained in the usual way as:

\[
B(E2, J_i \rightarrow J_f) = \frac{1}{2J_i + 1} |<J_f| T^{(E2)}_m |J_i>|^2
\]

(2.4.5)

where \(J_i\) and \(J_f\) are the initial and final spins of the states, respectively.

2.4a The SU(5) or vibrational limit\(^{35}\)

The Hamiltonian of Eq. (2.4.1) can take the form:

\[
H = \sum_m d^+ d_m + \sum_{L=0,2,4} \frac{1}{2(2L+1)} C_L [d^+ X d]^{(L)} X [d X d]^{(L)}
\]

(0)


\[+ n_d \text{ changing terms} \]

(2.4.6)

where the "\(n_d\) changing terms" contain factors such as \([d^+ X d] X [d X d]\).

For various choices of the parameters \(\epsilon, C_L, \ldots\) the SU(6) model spans the entire variety of the observed spectra. The situation in which the spectrum is dominated by values of \(\epsilon\) larger in comparison with the other parameters and in which the \(\Delta n_d \neq 0\) terms are neglected is referred to as the SU(5) limit of the (IBM-1) (SU(5) is the group of unitary transformation in five dimensions). The energy levels in this limit are given by the formula\(^{35}\):

\[
E(N,n_d,n\alpha,L,M) = \epsilon n_d + \frac{1}{2} \alpha n_d (n_d - 1) + \beta (n_d - \nu) (n_d + \nu + 3)
\]

\[+ \gamma [L(L+1) - 6n_d]\]

(2.4.7)
where the parameters $\alpha, \beta$ and $\gamma$ are related to the $C_L (L=0,2,4)$ by considering states with $n_d=2$. Thus:

\[
\begin{align*}
C_0 &= \alpha + 10\beta - 12\gamma \\
C_2 &= \alpha - 6\gamma \\
C_4 &= \alpha + 8\gamma
\end{align*}
\]

(2.4.8)

In Eq. (2.4.7), the symbols in parentheses on the left-hand side denote the quantum numbers which are needed to specify uniquely the states of each subgroup chain\(^\text{38}\) (a subgroup is defined by a subset of generators of the larger group which close under commutation). In the SU(5) limit, the chain involved is

\[
SU(6) \supset SU(5) \supset O(5) \supset O(3) \supset O(2)
\]

(2.4.9)

Thus, the quantum numbers of Eq. (2.4.7) are given the following meanings:

- $N$ is the total number of bosons, which is the SU(6) quantum number;
- $n_d$ is the number of d-bosons, which is the SU(5) quantum number;
- $v$ is the d-boson seniority, which is the $O(5)$ quantum number ($O(5)$ is the orthogonal group in five dimensions);
- $L$ is the angular momentum, which is the $O(3)$ quantum number ($O(3)$ is the ordinary rotation group);
- $M$ is the $Z$-projection of $L$, which is the $O(2)$ quantum number ($O(2)$ is the group of rotations around the $Z$-axis).

The symbol $n_\Delta$ is an extra quantum number required in order to fully decompose $O(5)$ in going to $O(3)$. This number is chosen to be the number of boson triplet coupled to zero angular momentum and is related to $n_d$ by\(^\text{35}\):

\[
n_d = 2n_\beta + 3n_\Delta + \lambda
\]

(2.4.10)

where $n_\beta = (n_d - v)/2$

(2.4.11)

with $n_d = 0, 1, \ldots, N$

(2.4.12)

and $v = n_d, n_d - 2, \ldots, 1 \text{ or } 0$; $n_d$ odd or even

(2.4.13)

Then, the values of $L$ contained in each representation $n_d$ of SU(5) are given by:

\[
L = \lambda, \lambda + 1, \lambda + 2, \ldots, 2\lambda - 2, 2\lambda
\]

(2.4.14)

It is interesting to arrange the different states in this limit into various bands. According to Arima and Iachello\(^\text{35}\), the important bands are given by the names $Y, X, Z, X', \beta, \Delta$ and are defined by the following representations:

\[
Y\text{-band } |n_d, v, n_\Delta, L, M\rangle \equiv |n_d, n_d, 0, L=2n_d, N\rangle
\]
X-band \equiv |n_d^+, n_d^+, 0, L=2n_d^-2, M> \\
Z-band \equiv |n_d^+, n_d^+, 0, L=2n_d^-3, M> \\
X'-band \equiv |n_d^+, n_d^+, 0, L=2n_d^-4, M> \quad (2.4.15) \\
\beta\text{-band} \equiv |n_d^+, n_d^-2, 0, L=2n_d^-4, M> \\
\Delta\text{-band} \equiv |n_d^+, n_d^+, 1, L=2n_d^-6, M>

The level scheme alone is not sufficient to determine whether or not a state belongs to the quadrupole mode. Detailed knowledge of their electromagnetic decays is also necessary. In the SU(5) limit the $T^{(E2)}_m$ operator of Eq. (2.4.4), when taken between states $|N, n_d, \nu, n_\Delta, L, M>$ has the selection rules: $\Delta n_d=0, \pm 1$.

Since the states of the group chain of Eq. (2.4.9) are characterized by a fixed number of $d$-bosons, $n_d$, the $B(E2)$ values along the ground state band with the selection rules $\Delta n_d=\pm 1$, are given by the first term of Eq. (2.4.4). Their explicit expression is\textsuperscript{36}:

$$B(E2, n_d+1, \nu=n_d+1, n_\Delta=0, L'=2n_d+2 \rightarrow n_d, \nu=n_d, n_\Delta=0, L=2n_d) \equiv a_2 \frac{L+2}{2} \frac{2N-L}{2}$$ \quad (2.4.16)

Thus $B(E2, 2^+_1 \rightarrow 0^+_1) = a_2^2 N$, in SU(5) \quad (2.4.17)

It should be noted that the above discussion has been considered on the basis that there is no breaking of the SU(5) symmetry. However, a small breaking will be always present, even in the best of the cases, and Arima and Iachello deduced analytical expressions for energies and electromagnetic transitions by performing first-order perturbation calculations based on the vibrational limit. In this case, the full Hamiltonian of Eq. (2.4.6) was taken with the "$n_d$ changing terms" treated as the perturbations and the first two terms diagonalized by the interacting $d$-boson model.

A test of the $d$-boson model with perturbation is meaningful only when many of the matrix elements of the perturbed states are known experimentally. In the case of $^{110}$Cd, this information are available and a proper treatment of the decay scheme of this nucleus in the frame work of the SU(5) limit will be given in Chapter IV.

It was noticed that nuclei with $N$ (or $Z$) only 4-6 particles away from a closed-shell, and $Z$ (or $N$) 8-10 particles away, show a vibrational-like structure. The $0^+, 2^+, 4^+$ states of the two-phonon triplet are present in the neighborhood of twice the energy of the first excited $2^+$ state. They gradually tend through the transition region to the $4^+$ state of the ground band ($\gamma$-band), to the $2^+$ head of the $\gamma$-vibrational band ($X$-band).
and to the $0^+$ state of the $\beta$-vibrational band ($\beta$-band), respectively.

2.4b The SU(3) or rotational limit

The group chain here is:

$$\text{SU}(6) \supset \text{SU}(3) \supset \text{O}(3) \supset \text{O}(2)$$

(2.4.18)

and the (IBM-1) Hamiltonian has the simple form (see Eq. (2.4.2)):

$$H = -kQQ - k'L'L$$

(2.4.19)

In such a case, the energy levels can be described by the expression

$$E(N, (\lambda, \mu), \kappa, L, M) = (0.75k - k')L(L+1) - k C(\lambda, \mu)$$

(2.4.20)

where $\kappa$ is the quantum number which is closely related to the projection of $L$ along the body-fixed axis as in the geometrical description. $C(\lambda, \mu)$ is the quadratic Casimir operator with $(\lambda, \mu)$ labels the representation of SU(3). The eigenvalue of this operator is given by

$$C(\lambda, \mu) = \lambda^2 + \mu^2 + \lambda \mu + 3(\lambda, \mu)$$

(2.4.21)

where $\lambda$ is the number of valence particles, that is $\lambda=2N$.

The first term of Eq. (2.4.20) describes the spacing within rotational bands, while the second, dependent only on the $QQ$ interaction determines the intrinsic energies of the collective bands. The ground band representation is denoted by $(\lambda, \mu)=(2N, 0)$, while the next representation appears as $(\lambda-4,2)=(2N-4,2)$ and includes the $\beta$ and $\gamma$ bands of the geometrical description with $\kappa=0$ and $\kappa=2$, respectively. The parameters $k$ and $k'$ of Eq. (2.4.20) can be deduced, by combining Eqs. (2.4.20) and (2.4.21), in the following way:

$$k = \frac{(E_{22}^+ - E_{21}^+)}{6(\lambda-1)}$$

(2.4.22)

$$k' = 0.75k - E_{21}^+/6$$

where $E_{22}^+$ and $E_{21}^+$ are the energy values of the second and first $2^+$ states, respectively.

It is important to note that, because the SU(3) eigenvalues do not depend on $\kappa$, states of the $\beta$ and $\gamma$ bands of identical angular momentum ($L=0,2,4,\ldots$) are degenerate in SU(3). Therefore, the Hamiltonian of Eq. (2.4.19) is not sufficient to describe each deformed nucleus in detail. Certainly, additional terms have to be introduced to remove the degeneracy (see Chapter VI).

For the calculations of the E2 transitions in the SU(3) limit, it is convenient to rewrite the $T_{m}^{(E2)}$ operator of Eq. (2.4.4) as:

$$T_{m}^{(E2)} = a_{2} Q_{m}^{(2)} + a_{2}^{'} Q_{m}^{'}(2)$$

(2.4.23)
where:
\[ \alpha_2 = \beta_2 + \frac{\sqrt{7}}{2} \alpha_2 \]  
(2.4.24)

\[ Q_m^{(2)} = [d^+x^s + s^+x^d]^{(2)} - \frac{1}{2} \sqrt{7} [d^+x^d]^{(2)} \]  
(2.4.25)

\[ Q_m^{(2)} = [d^+x^d]^{(2)} \]  
(2.4.26)

It turns out that the first term of Eq. (2.4.24) is much larger than the second one in regions where the SU(3) limit applies. The selection rules are thus given by:

\[ \Delta \lambda = 0, \Delta \mu = 0 \]  
(2.4.27)

and, therefore, the operator of Eq. (2.4.23) cannot connect states belonging to different representations \((\lambda, \mu)\) of SU(3). The two parameters \(\alpha_2\) and \(\beta_2\) can be determined from the measured \(B(E2, 2^+ \rightarrow 0^+)\) values for excitations of \(2^+\) members of the ground band and the \(\gamma\)-vibrational band, respectively. The \(B(E2)\) values for the ground-state band are given by:

\[ B(E2, L+2 \rightarrow L) = \alpha_2 \frac{3}{4} \frac{(L+2)(L+1)}{(2L+3)(2L+5)} (2N-L)(2N+L+3) \]  
(2.4.28)

Thus, \(B(E2, 2^+_1 \rightarrow 0^+_1) = \alpha_2 \frac{1}{5} N(2N+3), \) in SU(3)  
(2.4.29)

It should be noted, however, that the limiting situation of the \(T_m^{(E2)}\) operator described by the selection rules of Eq. (2.4.27) is never reached in practice and perturbation must be added to it. Because of the particular structure of the SU(3) limit, the perturbed situation always appears to have the following properties: transitions between the excited \(\kappa=2\) and the ground state acquire a sizeable value \(\approx 5B_{sp}\), where \(B_{sp} = 0.3A^{4/3} e^2fm^4\), while transitions between the excited \(\kappa=0\) band and the ground state band remain small \(\approx 1B_{sp}\). Finally, transitions between the excited bands decrease but still remain sizeable \(\approx 5B_{sp}\). This situation is schematically summarized in Fig. (2.1).

A necessary condition for describing nuclei in terms of the SU(3) symmetry is that the energies of the ground-state band behave like the \(L(L+1)\) rule (referred as \(l(l+1)\) in the rotational limit of section 2.3a). This behaviour implies a ratio \(E_{4^+_1}/E_{2^+_1} = 10/3\). It may be noted that a more careful measurement of \(B(E2)\) values for large \(L\) would indicate whether or not the SU(3) symmetry is an appropriate description for these nuclei.

2.4c The \(O(6)\) or \(\gamma\) unstable limit

The group chain is:

\[ SU(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2) \]  
(2.4.30)
Fig. (2.1). Schematic illustration of the electromagnetic properties of excited collective bands in different models: (a) the empirical rotational model; (b) the SU(3) limit of the interacting boson model-1; (c) the perturbed SU(3) limit of the interacting boson model-1.
This symmetry corresponds to the vanishing of the coefficients \( k \) and \( k \) in Eq. (2.4.2), and therefore is very useful in describing nuclei at the end of major shells. Again, in this case, the levels divide naturally into families characterized by the quantum numbers \( \sigma \) of \( O(6) \), \( \tau \) of \( O(5) \) in addition to \( N \), \( L \) and \( M \) defined in section 2.4a. Thus, one gets:

\[
\sigma = N, N-2, \ldots, 0 \text{ or } 1 \quad \text{for } N=\text{even or } N=\text{odd} \quad (2.4.31)
\]

\[
\tau = \sigma, \sigma-1, \ldots, 0 \quad (2.4.32)
\]

The values of \( L \) contained in each representation \( \tau \) of \( O(5) \) are obtained by partitioning \( \tau \) as

\[
\tau = 3\nu + \lambda, \quad \nu = 0, 1, \ldots \quad (2.4.33)
\]

and taking:

\[
L = 2\lambda, 2\lambda-2, \ldots, \lambda+1, \lambda \quad (2.4.34)
\]

The quantum number \( \nu \) is related to the number of zero-coupled triplets of bosons, and further subdivides the levels.

The eigenvalues of this limit is given by:

\[
E(N, \sigma, \tau, \nu, L, M) = \frac{A}{4} (N-\sigma)(N+\sigma+4) + \frac{B}{6} \tau(\tau+1) + C L(L+1) \quad (2.4.35)
\]

where the quantities \( A \), \( B \) and \( C \) are constants. The effect of a positive \( A \) is that of placing the representation \( \sigma=\sigma_{\text{max}}=N \) lowest in energy, while a positive \( B \) gives the ordering \( O^+, 2^+, 4^+, \ldots \). Finally, the effect of the \( C \)-term is monotonic in breaking the degeneracy of each \( \tau \)-multiplet. One notes, however, that for the lowest lying levels with \( \sigma=N \), \( A \) is zero.

A very characteristic signature of the \( O(6) \) limit is a repeating \( O^+-2^+-2^+ \) sequence with strong cascading \( E2 \) transitions within the group. In this limit, the \( E2 \) operator has the form (see Eq. (2.4.4)):

\[
\frac{\hbar}{2m} E2 = a_2 \left[ d^+_m s + s^+_m d \right]^{(2)} m, \quad \beta_2 = 0 \quad (2.4.36)
\]

This operator satisfies the selection rules \( \Delta \sigma = 0 \) and \( \Delta \tau = \pm 1 \). The \( B(E2) \) values along the ground state band are given by:

\[
B(E2, \sigma=N, \tau+1, \nu=0, L'=2\tau+2 \rightarrow \sigma=N, \tau, \nu=0, L=2\tau) = a_2^2 \frac{L+2}{2(L+1)} \frac{1}{4} (2N-L)(2N+L+8) \quad (2.4.37)
\]

Thus \( B(E2, 2^+_1 \rightarrow 0^+_1) = a_2^2 \frac{1}{5} N(N+4), \text{ in } O(6) \) \quad (2.4.38)

It is interesting to note that, because of the selection rule \( \Delta \tau=\pm 1 \), all quadrupole moments are zero in \( O(6) \) if the \( E2 \) operator is
strictly given by Eq. (2.4.36). Furthermore, the $O(6)$ limit provides an explanation for the systematic decays of the $0^+$ levels. There are two types of $0^+$ states: those with high $\tau$ and $\sigma=\sigma_{\text{max}}=N$ and those bandhead $0^+$ states with $\tau=0$ and $\sigma=N$. The latter cannot decay in the pure $O(6)$ limit because of the $\Delta\sigma=0$ selection rule. For higher lying $\sigma<N$, $\tau=0$, $0^+$ states will decay preferentially to the $\tau=1$ $2^+_1$ state rather than to the $2^+_2$ state with $\tau=2$ (see ref. 40).

2.4d Transitional classes

The three limiting situations described above have never been reached in practice. Most nuclei do not belong to any of these limiting but are somewhere in between two of them. In order to describe the transitional nuclei, one must refer to the full Hamiltonian of Eq. (2.4.1) and diagonalize it numerically. For the purpose of classification, these transitional nuclei are conveniently divided into four classes:

A) nuclei with spectra intermediate between $SU(5)$ and $SU(3)$,
B) nuclei with spectra intermediate between $SU(3)$ and $O(6)$,
C) nuclei with spectra intermediate between $O(6)$ and $SU(5)$, and
D) nuclei with spectra intermediate among all three limiting cases.

Obviously, class D seems to be the most difficult to treat phenomenologically as in this case all the operators of Eq. (2.4.2) must be taken into account. Much simpler phenomenological studies can be done for nuclei belonging to the transitional classes A)\textsuperscript{41}, B)\textsuperscript{40} and C)\textsuperscript{42} (see also Chapters V and VI).

2.4e Program codes PHINT and FBEM

The (IBM-1) calculations for the present even-even nuclei were carried out by the program codes PHINT and FBEM which are incorporated in the Program-Package PHINT\textsuperscript{43}. The former calculates the energies and eigenvectors for positive and negative parity states by exactly diagonalizing the full Hamiltonian of Eq. (2.4.1) and the latter calculates the electromagnetic transitions. Both programs are coded in Fortran IV and were run on the University of London CDC 7600 computer.

In order to specify the parameters for the (IBM-1) Hamiltonian, the completely equivalent form "multipole expansion" to Eq. (2.4.2):

$$H = \text{EPS} \ n_d + \text{PAIR} \ (P \cdot P) + \frac{\text{ELL}}{2} \ (L \cdot L) + \frac{\text{QQ}}{4} \ (Q \cdot Q)$$

$$- 10 \sqrt{7} \ \frac{\text{OCT}}{2} \ [[d^+Xd]^3 (d^+Xd)^3]_0 ^{(3)} + 30 \ \frac{\text{HEX}}{2} \ [[d^+Xd]^4 (d^+Xd)]_0 ^{(4)}$$

(2.4.39)
was used. In this equation, "EPS, PAIR, ELL, QQ, OCT, HEX" refer to the
Variable Names as employed in the PHINT program, and are related to the
parameters of Eq. (2.4.2) by:

\[
\begin{align*}
EPS &= \epsilon, \\
PAIR &= k'', \\
\frac{ELL}{2} &= -k', \\
\frac{QQ}{4} &= -k, \\
OCT &= \sqrt{\frac{7}{5}} k_3, \\
HEX &= \frac{k_4}{15}
\end{align*}
\]

(2.4.40)

With the program FBEM, it is possible to calculate electromagnetic
transition rates. The possible E2 operator is given by the equivalent
form to Eq. (2.4.4). Thus:

\[
T_m^{(E2)} = E2SD \left( s^+ Xd + d^+ Xs \right)^{(2)} + \frac{1}{\sqrt{5}} E2DD \left( d^+ Xd \right)^{(2)}
\]

(2.4.41)

where the parameters E2SD and E2DD are given by:

\[
E2SD = \alpha_2, \quad E2DD = \sqrt{5} \beta_2
\]

(2.4.42)

It is important to note that, in its standard version, the program
PHINT can handle up to 16 bosons. Furthermore, FBEM cannot function
independently without PHINT but not vice versa.
CHAPTER III

EXPERIMENTAL ARRANGEMENTS

So far, analytical expressions related to the nuclear properties of the states, have been mainly discussed. In fact, reasonable understandings of these expressions cannot be arrived at without a proper experimental treatment of the decay schemes of the nuclei under considerations. The aim of this Chapter is to discuss the experimental techniques that could lead to the construction of comprehensive decay schemes.

The experimental arrangements are outlined below. The first section deals with the single spectra measurements, while the discussion in section 3.2 refers to the energy and efficiency calibrations of the detectors employed in this study. A brief description of the Compton suppression system is considered in section 3.3. The more detailed treatment of the time spectroscopy is taken up in section 3.4, as a basis for interpreting a wide variety of effects associated with the electronics used. This section is divided into six parts: scintillators-photomultiplier detectors, solid-state detectors, walk and jitter, the constant fraction timing technique, timing spectrometer, and resolving time. The experimental arrangement of the Dual Parameter Data Collection System (DPDCS) in section 3.5 makes it possible to bring together the discussion of the previous section as well as to measure a two dimensional time-energy spectra and to write the unstored data on a large storage medium.

3.1 Single spectra measurements

During the course of this work, three true-coaxial Ge(Li) detectors and one intrinsic germanium detector were employed for the gamma ray singles measurements. The Ge detector was used for the detection of gamma rays in the region 50–350 keV. Two other detectors, a plastic and a NaI(Tl) were employed in a coincidence configuration for lifetime measurements. The specifications of the above-mentioned detectors are given in Table (3.1). The standard method for obtaining the relative efficiency of each of the Ge(Li) detectors was to compare its counting rate for the 1.33 MeV line of $^{60}\text{Co}$ with that of a 3''x3'' NaI(Tl) detector using a standard distance of 25cm from the source-to-detector (the efficiency of the above size NaI(Tl) detector for the 25cm distance is given as $1.2 \times 10^{-3}$ (see ref. 44)).
Table (3.1). Specifications of the detectors used in this work.

<table>
<thead>
<tr>
<th>No.</th>
<th>Detector</th>
<th>Approximate volume</th>
<th>Relative efficiency</th>
<th>Energy resolution*</th>
<th>Peak/Compton ratio*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ORTEC Ge(Li)</td>
<td>70cc</td>
<td>12%</td>
<td>2.14</td>
<td>37.5</td>
</tr>
<tr>
<td>2</td>
<td>ORTEC Ge(Li)</td>
<td>60cc</td>
<td>10%</td>
<td>1.91</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>HARSHAW Ge(Li)</td>
<td>70cc</td>
<td>11%</td>
<td>2.4</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>ORTEC Ge</td>
<td>2cm² x 2.5cm</td>
<td></td>
<td></td>
<td>0.5 ** ,1.62</td>
</tr>
<tr>
<td>5</td>
<td>NE 102A (Plastic)</td>
<td>&quot;xl&quot;</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>NE 5288R (NaI(Tl))</td>
<td>2.2 &quot; x 1.56&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* For the 1.33 MeV peak of ³³Co.
** The FWHM of the 122 KeV γ-ray line.

Each of the first four detectors of Table (3.1) is equipped with a cooled Field Effect Transistor FET preamplifier which has brought out the amazing resolving power of the detector in the analysis of multicomponent photon spectra. The simple pulse counting system of this work is illustrated in the block diagram of Fig. (3.1). The linear output pulses from the preamplifier were fed into a Spectroscopy Amplifier (ORTEC 572) whose shape time constant of 2 µsec indicated a good signal-to-noise ratio. The signals were then fed through an Analog To Digital Converter ADC (NS 628) into a Memory Unit (NS 630) whose conversion gain was set at 4096 channels.

For each of the single spectra measurements, the radioactive source of about 10 µCi of activity was placed at 25 cm from the face of the detector. In such an arrangement, the counting rates were kept below 2000 counts/sec and the dead time of the ADC was always less than 10%. As a result, the pile-up effect were avoided and the coincidence summing corrections were minimized.

3.2 Energy and efficiency calibrations

Calibration of germanium detectors can be performed using a number of absolutely standardized gamma emitting sources. Before quantitative data on the relative intensities of gamma rays can be obtained from a γ-ray spectrum, the efficiency of the detector must be known.

In the present work, the energy and efficiency calibrations of detectors Nos. 1-4 in Table (3.1) were performed by making use of the sources:
Fig. (3.1). Block diagram of the single measurements arrangement.
$^{241}\text{Am}$, $^{57}\text{Co}$, $^{133}\text{Ba}$, $^{137}\text{Cs}$, $^{22}\text{Na}$ and $^{60}\text{Co}$ obtained from the Radiochemical Centre, Amersham$^{45}$, in a form of standard sources suitable for immediate use. They cover the range from 60 to 1332 keV and are calibrated to better than 1-2%. High energy calibration lines, whose specifications are given in ref. 46, were obtained from a 9.9 µCi of $^{226}\text{Ra}$ prepared at the (ULRC).

For energy calibrations, the least squares fit to an nth degree polynomial was carried out using the code program SAMPO$^{47-48}$. The errors associated with the energy determinations are estimated by combining in the root mean square sense the error in the peak centroid and the errors in the calibration points.

For efficiency calibrations, two methods are incorporated into SAMPO. The first scheme uses a number of calibration points and interpolates logarithmically between these points. The second method employs a functional representation of the efficiency curve:

$$
\epsilon = P_1 \left( E^{P_2} + P_3 \exp(P_4 E) \right)
$$

(3.1)

where the four parameters $P_1$, $P_2$, $P_3$ and $P_4$ are determined from the fit of the efficiency $\epsilon$ versus the energy $E$ expressed in keV.

Although Eq. (3.1) was shown to be inferior to other forms$^{49}$, the general applicability of the program SAMPO seems to outweigh this limitation. Recently, however, Ahmad et al.$^{50-51}$ introduced a 6-parameter function:

$$
\epsilon = \left\{ P_1 + P_2 \ln E + P_3 (\ln E)^2 + P_4 (\ln E)^3 + P_5 (\ln E)^5 + P_6 (\ln E)^7 \right\}/E
$$

(3.2)

which is linearly dependent on the parameters $P_i$ (i=1-6). The least squares fitting procedure applied to Eq. (3.2) appears to fit experimental data with deviations of no more than 1-2%, resulting in an uncertainty band that can only be attributed to the errors in the calibration points. As a consequence, Eq. (3.2) has been employed in this work. Figs. (3.2) and (3.3) illustrate the graphs of $\epsilon$ versus $E$ for the 12% Ge(Li) and the Ge detectors, respectively. The 11% efficient Harshaw detector shows an identical curve to Fig. (3.2).

It should be noted that, in the calculations of the relative intensity of the transition involved, the efficiency was deduced from Eq. (3.2) and the net peak area from SAMPO modified to run on the CDC 6600 computer at the University of London Computer Centre. In this context, it is worthwhile to consider the main features of this program. The search of the photopeaks is based on a modification of the smoothed second differences method of Mariscotti$^{52}$. The peak areas are calculated by fitting them with functions which are integrated analytically. The functional form used for peak fitting
Fig. (3.2). Absolute efficiency of the 12% Ge(Li) detector as a function of γ-ray energy.
analysis of a Gaussian function with exponentially decreasing tails and
the background has a linear or quadratic slope. Three parameters, the
width of the Gaussian function and the distance from the peak center to
the starting point of the exponential tails, are calculated as a function
of the pulseheights in the individual scintillator channels above
and a nonlinear least-squares. The results also include routine
and efficiency calibrations and for nuclear identification in the
and calculation.

3.3. Quantum absorption system

In the analysis of complex spectra in particle spectroscopy, the
is known because of its high resolution which is a result of
the small energy required to produce an electron-positron pair (Eγ ≈
300 keV) and the resulting Ge(II) spectra are characterized by the prominent peaks which
obscure the low intensity peaks from other γ-ray energies. In order
to detect a weak radiation in the presence of strong interfering γ-rays of the
an absorption spectroscopy spectrometer. (Figure 3.4.42), was used in
this work (see Fig. 3.4.45).

The Ge(II) detector (Table 3.3.1) location is at very
the detector, which is distant from the scattering point and is
viewed by four photomultiplier tubes and comes out as a signal on one of the
detector channels. The signals from the scintillation
of the Ge(II) detectors are routed to a multichannel analyzer (MCA)
and a TDC Analyzer (MCA's TDC). Meanwhile, the pulses
from the NaI(Tl) detectors are fed through a fast word & detector
channel into the TDC Analyzer those main functions are measuring the time
between the two detectors and providing a logic output that
indicates a coincidence event. It is, however, possible to
be treated. The coincidence requirements between all Ge(II) units
are about 50 ns and the NaI(Tl) units, that is no valid Ge(II) events from the output of the
Gamma Ray Spectrometer (LANSCE).

3.4. Time Spectroscopy

Time spectra involve the measurements of the time relative
between the occurrence of two events. The results of events by
a pair of γ-rays or a combination of γ-rays and / or charged particles
consists of a Gaussian function with exponentially decreasing tails and the background has a linear or quadratic shape. Three parameters, the width of the Gaussian function and the distances from the peak centroid to the starting points of the exponential tails, are calculated as a function of channel using a few statistically good singlet peaks. The fitting of the full spectrum is carried out using these precalibrated shape parameters and a nonlinear least-squares. The program also includes routines for energy and efficiency calibrations and for nuclides identification and activity calculation.

3.3 Compton suppression system

In the analysis of complex spectra in γ-ray spectroscopy, the Ge(Li) detector is used because of its high resolution which is a result of the small energy required to produce an electron-hole pair. Unfortunately, Ge(Li) spectra are characterized by very prominent continua, which can obscure low-intensity peaks from other γ-ray energies. In order to detect weak radiation in the presence of strong interfering γ-rays of high energy, a Compton suppression spectrometer, installed at the (ULRC), was used in this work (see Fig. (3.4)).

The Ge(Li) detector (No. 3 in Table (3.1)) housing is enclosed by a very large 200 mm long × 200 mm diameter NaI(Tl) scintillator which is viewed by four photomultipliers and machined in a manner as to allow maximum closure of the Ge(Li) detector. The pulses from the preamplifier output of the Ge(Li) detector are routed to both a Spectroscopy Amplifier (ORTEC 472) and a Time Analyser (HARSHAW NC 26). Meanwhile, the pulses from the NaI(Tl) detector are fed through a Dual Sum & Inverter Unit (ORTEC 433A) into the Time Analyser whose main functions are measuring the time relationships between the two detectors and providing a logic output that represents a Ge(Li)-NaI(Tl) coincidence event. It is, however, this event which is to be rejected. The coincidence requirement between all Ge(Li) events that are fed through the Spectroscopy Amplifier into the Linear Gate (CANBERRA 1451) and just the valid Ge(Li) events from the output of the Time Analyser selects the latter. Finally, the anticoincidence requirement of the Linear Gate allows full energy events of the selected ones to be displayed on the Multichannel Analyser (LABEN 8000).

3.4 Time spectroscopy

Time spectroscopy involves the measurements of the time relation between the occurrence of two events. The source of events is frequently a pair of γ-rays or a combination of γ-rays and / or charged particles in
Fig. (3.4). Block diagram of the Compton Suppression Spectrometer.
cascade, which de-excite some level in the nucleus.

The most important consideration about timing measurements is the specifications of the detectors involved, such as electric field strength in the detector, efficiency, peak-to-Compton ratio and resolution. Thus, in the following two sub-sections, considerations relevant to the detectors associated with the present timing measurements are briefly discussed.

3.4a Scintillator-photomultiplier detectors

These detectors have been widely used in the detection of nuclear radiations for timing purposes. There are two types of scintillators that are commonly employed. The first type which includes the plastic and organic (often preferred for $\beta$ spectroscopy and fast neutron detection) scintillators is characterized by a fast decay or fast light output, which implies good timing capabilities. For $\gamma$-rays, these scintillators give limited information due to the fact that they have essentially no photopeak cross-section unless doped with high Z material. The second type of scintillators is characterized by a slower light output or good energy information. The NaI(Tl) scintillator which has a decay constant of about 250 nsec is an example of this type. However, in spite of this long decay time, NaI(Tl) detectors are still very useful for fast timing because of a fast rise time and good energy information in the photopeak because of the high Z material. The photomultiplier (PM) window to which the scintillator is attached should transmit a large fraction of the scintillator light. The gain of a (PM) tube normally needs no further amplification and the preamplifier employed in conjunction with it shapes the pulses for the circuit involved.

3.4b Solid state detectors

These detectors collect the charge from ionizing events. $\gamma$-rays do not directly cause ionization in the crystal but can interact with material producing electrons by photoelectron absorption, Compton scattering and pair production for $E_\gamma > 1.022$ MeV. The secondary electrons produced by these processes create electron-hole pairs which are collected by the electric field across the detector. The efficiency of the detector will depend on the volume and on the Z of the material used. Photoelectric absorption is roughly proportional to $Z^5$ and thus a high Z is preferable. Germanium with $Z=32$, used in ion drifted detector for $\gamma$-ray measurements, is obviously preferable to silicon ($Z=14$) which is very useful for the detection of low energy $\gamma$-rays and X-rays.

The application of lithium drifted technique to germanium and the development of large volume detectors made possible the use of lithium-drifted germanium detectors or Ge(Li) as high resolution $\gamma$-ray spectrometers.
There are two types of Ge(Li) detectors: the planar and the coaxial. Because of the limited volume and large exposed intrinsic region of a planar detector, coaxial detector has the advantage of detecting high energy photons and can have higher efficiency by increasing its volume. Among the three major designs of coaxial Ge(Li) detectors, the true right circular cylinder is the most desirable configuration, especially for fast timing because of its shortest rise times with the least variations (the other two designs are the trapezoidal coaxial cylinder and the wraparound coaxial cylinder). These detectors must always be cooled to 77° K, the boiling point of liquid nitrogen, in order to have acceptably low noise. Small volume pure germanium detectors have low efficiency for high energy γ-rays which means that the background due to Compton scattering is low.

To process the pulses provided by the detector, a system composed of two separate channels: the fast channel and the slow channel must be used. The purpose of the fast channel is to derive the best possible signal from each detector when a set of two detectors is involved, and to change the time information into amplitude information in a Time To Pulse Height Converter TPHC. The time spectrum from the output of the TPHC is essentially of Gaussian shape. For measuring time differences, the timing peak must be narrow, that is the timing resolution or FWHM(full width at half maximum) must be high. It is important that the narrow peak be maintained down to a small fraction of the total peak height to make certain that all true coincident events are recorded (the figure of merit is the FW(1/10)M).

It is interesting to note that, for the scintillation detectors where the rise times of the signals are approximately the same, the time distribution is governed by the decay times of the scintillators. On the other hand, the rise times of semiconductor detector signals are not only a function of the geometry of the detector but also dependent on the location of the point of interaction.

In order to understand the techniques used to derive good timing signals, it is important to understand some of the characteristics that affect the time resolution of the system.

3.4c Walk and jitter

Characteristics that strongly affect timing in germanium are the walk due to amplitude variations or rise time changes and the intrinsic time jitter of the detector signal. These effects are shown by an ideal discriminator with threshold $V_T$ operating on detector signals (see Figs. (3.5), (3.6) and (3.7)). The discriminators used in properly designed electronics have minimized the charge necessary to trigger an output, and
Fig. (3.5). Time walk caused by the amplitude variation and the charge sensitivity of the discriminator.

Fig. (3.6). Time walk caused by the rise time effect and the charge sensitivity of the discriminator.

Fig. (3.7). Time jitter due to noise.
most of the apparent walk is contributed by variations in pulse amplitudes and rise times. In Fig. (3.6), the width of the timing uncertainty which is statistical in its nature is given by the triangle rule:

\[ \sigma_T = \frac{\sigma_V}{dV(t)|_{t=T}} \]

where \( \sigma_V \) is the amplitude of the steady-state noise and \( \sigma_T \) is the timing uncertainty in crossing the threshold \( V_T \).

3.4d The constant fraction timing technique

Due to the varying rise times of germanium detector signals, a modified constant fraction of pulse height timing is usually used with this detector. It is necessary to understand the constant fraction technique in order to fully appreciate the benefits that can be gained by compensating for both the varying pulse heights and the varying rise times found in germanium detectors. Moreover, many fast-slow coincidence systems utilize fast plastic scintillators and constant fraction timing in one branch of the system.

Experimental works\(^{55-56} \) had shown that for a signal of given rise time and height there is a value at which the discriminator can be set to give the optimum timing results, especially with scintillators/photomultiplier systems. The constant fraction method is then a technique for greatly improving time walk and resolution characteristics over a wide range of input signal amplitudes, when compared with other types of discriminator. The basic technique is to convert the negative input mode signal into a zero-crossing signal by summing a fraction \( f_V \) of the input with a delayed and inverted portion of the input itself as simply shown in Fig. (3.8). All signals will cross zero at the same time, independent of rise times and amplitudes. With fast plastic scintillators, this technique can reduce the walk to less than 120 psec for 100:1 dynamic range of pulse heights (a dynamic range is defined as the ratio of the largest pulse to the smallest pulse that can be accepted by the timing system). For applications in which better energy characteristics than those of fast plastic scintillators are required, NaI(Tl) often used in the time reference branch.

3.4e Timing spectrometer

Fig. (3.9) shows the block diagram of the timing spectrometer employed in this work. The source was sandwiched between two fast NaI(Tl) and plastic scintillators. The start branch of the fast channel is composed of a Constant Fraction Discriminator CFD (ORTEC 473A) integrally mounted in
Fig. (3.8). Pulse shape and time considerations for the constant fraction timing technique.
Fig. (3.9). Block diagram of the timing Spectrometer.
a photomultiplier tube base and looking at the output of the fast plastic scintillator. The actual timing signal output is obtained from a modular control unit that supplies power and bias to the (PM) base. This signal is used as the start signal for the TPHC (ORTEC 467). The NaI(Tl) side of the fast channel is used as the stop signal for the TPHC; for optimum timing results the time signal is fed into a CFD (ORTEC 463) and a Nanosecond Delay (ORTEC 425A) used to compensate between the difference in rise times of the plastic (~5 nsec) and the NaI(Tl) (~25 nsec). This Delay was also employed to perform time calibration of the Multichannel Analyser MCA. Fig. (3.10) shows the block diagram of the time calibration, while Figs. (3.11) and (3.12) illustrate the results.

The Gaussian shape of the time spectrum from the output of the TPHC is very important for simple coincidence determination. It shows a very high coincidence counting efficiency (about 75% when the time window of the Gaussian shape spans the FWHM and about 95% when it spans the FW(1/10)M). This accuracy can be affected by the contribution from undesired events. These arise from accidental (chance) coincidences between two pulses since detector events occur at random time, producing a background in the coincidence counting (see section 3.4f).

It is the purpose of the slow channel of the coincidence system to set energy restrictions on each branch of the fast channel and to gate the signal from the TPHC so that only those events that fall within the desired energy ranges are recorded in the MCA (NS 710). More specifically, a Timing Single Channel Analyser TSCA (ORTEC 551) was used to look at the output of a Spectroscopy Amplifier SA (ORTEC 451) connected to the linear signal of the NaI(Tl) detector. If this signal falls within the energy range, determined by the upper and lower level discriminators of the TSCA, then a time-related output pulse is generated and is used to gate the input of the MCA. The Delay Amplifier (ORTEC 427A) at the output of the TPHC delays the presentation of the linear signal to the MCA until a crossover coincidence has been achieved.

The source most frequently used for determining time resolution is $^{60}$Co. Fig. (3.13) illustrates the prompt timing curve measured on the 1.33 MeV $\gamma$-ray from a 10 $\mu$Ci $^{60}$Co source using the system in Fig. (3.9). This curve contains data of the resolving time $2\tau_0$=FWHM=3.31 nsec, and the FW(1/10)M=6.64 nsec, of the two scintillators.

For lifetime measurement, a 10 $\mu$Ci $^{22}$Na source was employed. The $^{22}$Na has two coincident positron annihilation gamma rays, each with an energy of 0.511 MeV. Thus, by measuring the time difference between the emission of the prompt 1.275 MeV $\gamma$-ray from this source and one of the 0.511 MeV gamma
Fig. (3.10). Block diagram of the time calibration of the MCA.
Fig. (3.11). Time calibration of the Northern Scientific MCA (Model 710) for different ranges of the TPHC. Conversion Gain of the MCA was set at 1024.
rays, the lifetime of the positron can be obtained. In the present work, the window of the TSCA of Fig. (3.9) was set on the 0.511 MeV γ-ray and the time spectrum accumulated in the MCA for over a month period is shown in Fig. (3.14). From the steepness of the tail of this time spectrum plotted in a semi-log scale, the half life $t_\frac{1}{2}$ of the positron, where the coincidence rate drops by 50%, was deduced (slope method$^5$) and is given by $t_\frac{1}{2}=1.23\pm0.05$ nsec. This value is in rather good agreement with the values $1.18\pm0.14$ and $1.32\pm0.03$ nsec reported by refs. 58 and 59, respectively.

3.4f Resolving time

Coincidence equipments may be classified according to their resolving time. If a two-detector system is used, each detector applies a gate pulse of width $\tau_0$ to the coincidence circuit; therefore to be in coincidence, the two gate pulses must fall within the time interval $2\tau_0$ which is the resolving time of the coincidence system. A resolving time of less than a few tenths of a nsec is termed "fast", and larger resolving time are called "slow".

Short resolving times are required wherever high counting rates are involved because the random nature of radioactive decay leads to a chance that two uncorrelated pulses will happen to occur within the coincidence resolving time. The rate of accidental or random coincidence is given by:

$$N_{acc} = 2\tau_0 N_1 N_2$$

where $N_1$ and $N_2$ are the counting rates in the two detectors. It thus appears that the best way to reduce accidental coincidences is to make $2\tau_0$ as small as possible. However, $2\tau_0$ cannot be reduced below the amount of the time jitter in the detector pulses without losing true coincidences. Because $N_1$ and $N_2$ are related to the disintegration rate $N_D$ by counting efficiencies $\varepsilon_1$ and $\varepsilon_2$ of the two detectors, the accidental coincidence rate is proportional to the square of $N_D$ so that:

$$N_{acc} = 2\tau_0 N_D^2 \varepsilon_1 \varepsilon_2$$

Since the real coincidence rate is given by:

$$N_{real} = N_D \varepsilon_1 \varepsilon_2$$

Then, the ratio of real coincidence to accidental coincidence is:

$$\frac{N_{real}}{N_{acc}} = \frac{1}{2\tau_0 N_D}$$

Thus, this ratio increases as the reciprocal of the first power of resolving time and disintegration rate.
Fig. (3.13). Prompt time distribution curve for $^{60}\text{Co}$
Fig. (3.14). Lifetime spectrum of Positron
3.5 The Dual Parameter Data Collection System (DPDCS)

In many nuclear counting problems, it is necessary to decide whether two events are time-correlated. Such information is required for investigations of nuclear decay schemes for which it may be necessary to know whether two radiations are emitted at the same time. Electronic circuits which make such decisions are called coincidence circuits and produce an output pulse only if all inputs to the device receive a pulse simultaneously.

To obtain good time and energy resolutions, a fast-slow coincidence circuit can be used by inserting a time analyser, for example, a TPHC in the fast part and by selecting with two TSCAs certain pulse height regions. Fig. (3.15) shows the block diagram of the fast-slow coincidence system for which the working principle is well established.

A more advanced technique is to measure a two dimensional time-energy spectra and to write the unstored data on a magnetic tape and sorting them for interesting regions after the measurements. The (DPDCS) employed in the present work fulfils these requirements since it provides a large amount of coincidence data in a considerably short time; thus, it overcomes the limitation caused by the fast-slow system. Fig. (3.16) gives the block diagram of the coincidence arrangement in conjunction with the (DPDCS).

In this arrangement, an analysis of stored data was made of these events in one detector which are in time coincidence with events in the other. Two fast channels of detectors, preamplifiers were each followed by two Timing Filter Amplifiers TFAs (ORTEC 474) and thence by two CFDs (ORTEC 463 and 583) which offer a timing output signal when a constant ratio of the pulse height is reached. The derived signals were then fed into the start and stop inputs of the TPHC which measures the difference in the time of occurrence of the events in the two detectors. The delay in time between the two inputs was adjusted by inserting across one channel a Nanosecond Delay Unit (ORTEC 425A) so that all genuinely coincident pulses generate an output from the TPHC. This output provides the resulting time spectrum information whose shape is essentially Gaussian. The subsequent TSCAs (ORTEC 551) offer the ultimate in time resolution, stability and linearity. One was set on the prompt time spectrum which represents the so-called total timing coincidence (true + chance) and the other on the flat chance coincidence section to the right of the time spectrum. Either of these coincidences can generate a pulse in the Gate Pulse Generator Unit, the output of which was used to gate the two ADCs (NS 628) and at the same time was coupled to the Write Interface of the (DPDCS).

The two slow channels or energy derived from the Ge(Li) detectors (the 12% efficient Ge(Li) was used as the spectrum detector and the 10% as
Fig. (3.15). Block diagram of the fast-slow coincidence system.
Fig. (3.16). Coincidence arrangement in conjunction with DPDCS.
the gating one, see Table (3.1) for the specifications of these detectors) were fed into the high level of the respective spectrum and gating ADCs through two ORTEC units, an Amplifier and a Delay Amplifier. Events can thus be analysed once the gates of the ADCs are opened by the fast timing part. The addresses from these ADCs were then coded by the Dual Parameter Interface and recorded in a 7 tracks, large storage capacity, Magnetic Tape (Write Interface). The outline of the whole arrangement is illustrated schematically in Fig. (3.17). The information on the Magnetic Tape can be read back via another unit into the MCA (NS 630) (Read Interface).

In what follows, a complete description of the (DPDCS) will not be taken up. Instead, a summary of the most important technical steps will be given. For the original work and full description, see refs. 60, 63 and 64 (see also refs. 61-62 for further applications).

In the Write Interface, the total or chance coincidence signal of the timing spectrum is furnished to the front panel of the Store Cycle Unit where it is suitably delayed to take into account of the time required for the conversion process of linear pulses by the ADCs. When the conversion is completed, the recording process is then initiated and the binary addresses from the ADCs are passed to the Multiplexer Unit through a binary-to-BCD (binary coded decimal) Converter Unit. At the Multiplexer Unit, the counter outputs are used to convert the parallel inputs to serial forms. The outputs from the Multiplexer appear as track 1, 2, 4 and 8 on the Magnetic Tape. The timing pulse is also used to identify the words on the tape according to whether a total or chance coincidence is being recorded.

The Tape Transport is an SE 8000 series synchronous type with read-after-write configuration and recording speed of 45 ips. It has a variable recording capacity and was chosen to correspond to 7-track non-return-to zero recording at density of 556 bytes per inch. The tracks are conveniently assigned as track 1, 2, 4, 8, A, B, P with track 1 adjacent to the reference edge. The Tape is designed to provide digital magnetic recording system that is compatible with the IBM recording formats for 1/2 inch computer-grade tape.

For every coincidence events, there are altogether 9 words written along the Tape with each 4-bit word recorded on the appropriate track across the width of the Tape. The first is the indicator or tag word which is used to identify whether the event following it arises from total or chance coincidence. It has all "zeroes" on tracks 1, 2, 4 and 8 and a "one" on track B. Track A is written a "one" if the words following it correspond to total coincidence event and a "zero" otherwise. Track P is
Fig. (3.17). Block diagram of the DPDCS arrangement.
the parity track and is not relevant to the present system. The remaining 8 words that follow the indicator word are addresses from the two ADCs. These addresses are first stored on the Tape mounted buffer store since coincidence events occur at random in time while efficient writing on Tape requires a regular representation of data. Then when the buffer is about full, it dumps its contents synchronously on the Magnetic Tape forming a "block" or "record" of data. During the transfer operation the system is inhibited by the dead time signal from the Tape Transport. At the end of this process, the system is live again.

In the Read Interface, selected coincidence data on the Tape can be directly read into the MCA, thus enabling a visual display of the coincidence spectra.

During the read sequence, a part of the serial data line (track 1, 2, 4, 8) is converted to parallel form by the Shift Register Unit whose outputs that correspond to addresses from the gating ADC are connected to the comparator on the Select Gate Unit. This Unit compares the incoming data (on the tape) with window boundaries that correspond to the lower and upper channel limits of the gated peak. The remaining outputs (addresses from the spectrum ADC) of the Shift Register are presented to the BCD-to-binary Converter Unit which converts them back to binary addresses and are then made available to the MCA. When the store process is completed, a clear signal from the MCA is generated which resets the Read Interface and permits further read data to be compared. The uncorrected coincidence spectrum thus displayed on the MCA can be corrected for chance and background coincidences. This correction is made possible by, respectively, changing the position of a switch from total to chance on the Read Total-Chance Unit, and by resetting the two thumbwell switches of the Select Gate Unit to a slightly different width (but of the same width as the energy gate) and using the subtraction mode of the MCA. Thus, the resulting spectrum represents the uncorrected one for both chance and background events.

A general view of the experimental arrangement of the (DPDCS) is illustrated in Fig. (3.18). The performance of the system was checked by measuring the coincidence spectra of the decay of $^{110m}$Ag which forms the subject of the next Chapter.
Fig. (3.18) A view of the experimental arrangement of the dual-parameter energy-time spectrometer.
CHAPTER IV
NUCLEAR COLLECTIVE STATES IN $^{110}\text{Cd}$

4.1 Introduction

The level structure of $^{110}\text{Cd}$ has been extensively investigated in recent years by many research workers$^{65-73}$. In particular the studies include the measurements of energies and intensities of gamma rays$^{66-72}$ and multipole mixing ratios$^{65}$ of the various transitions involved in the $\beta^-$-decay of $^{110}\text{Ag}$. Some conclusions about the excited states of $^{110}\text{Cd}$ were also drawn from model calculations$^{35,74}$.

The latest of the $^{110}\text{Ag}$ decay measurements done by refs. 72 and 73 indicated new gamma transitions at 648.2, 666.1, 714.9, 845.8, 927.6, 1050.1, 1465.6 and 1698.5 keV using Ge(Li) and Ge detectors. Their $\gamma-\gamma$ coincidence and sum-coincidence measurements confirmed the existence of the levels at 2078.8, 2250.7, 2287.7, 2433.1, 2539.6, 2659.9, 2706, 2793.4, 2842.5 and 2876.8 keV previously proposed on energy sum considerations.

In spite of these investigations, the spins and parities assigned to the levels at 2123.3, 2287.7, 2662.3 and 2793.4 keV are not consistent with the $\beta^-$-feeding to these levels deduced on the basis of the log ft values, and thus need further reinvestigations. Also, the study by Verma et al.$^{71}$ using a 64.1 cm$^3$ Ge(Li) detector in the singles mode presents some variances with recent compilations of Bertrand$^{69}$. Furthermore, the relative intensities of weak gamma rays need more confirmation.

On the other hand, nuclei in Cd region are commonly referred to as vibrational nuclei, mainly because of the existence of a closely spaced $0^+, 2^+, 4^+$ triplet at about twice the energy of the first $2^+$ state and because of the $\gamma$-ray decay pattern from this triplet. Thus, in the level scheme of $^{110}\text{Cd}$, the ratio $E(2_2^+)/E(2_1^+)=2.25$ is close to that of the vibrational limit of 2. The experimental ratio $R=B(E2, 2_2^+\rightarrow 2_1^+)/B(E2, 2_1^+\rightarrow 0_1^+)=1.01\pm0.12$ (see section 4.4) is considerably smaller than that of the predicted value of 2, but it is in rather good agreement with the calculations of Milner et al.$^{75}$ $R=1.08\pm0.29$. The ratio $B(E2, 2_2^+\rightarrow 0_1^+)/B(E2, 2_1^+\rightarrow 2_1^+)=0.07\pm0.01$ (see Table (4.6)) is in disagreement with the vibrational model which forbids a crossover/cascade transition. Thus, one can conclude that the $^{110}\text{Cd}$ nucleus exhibits some deviations from harmonic behaviour.

Recently, however, the (IBM), emphasizing boson degrees of freedom
has received a large amount of attention. When the SU(5) limit of this model was applied to $^{110}$Cd nucleus\textsuperscript{35}, it has extensive success in describing a number of low-spin features. In particular, this nucleus was found to be a good example of the perturbed SU(5) limit.

Keeping in view the earlier mentioned inconsistencies, it was thought worthwhile to reinvestigate the decay scheme of $^{110}$Cd using Ge(Li) and Ge detectors for the energy and intensity measurements of various gamma transitions and a Ge(Li)-Ge(Li) combination in Dual Parameter Data Collection arrangement for the coincidence measurements. Also the radioactive decay of $^{110m}$Ag provides a good check on the performance of the (DPDCS).

4.2 Experimental procedure and results

4.2a Source preparation

A 53 mg metal target of 99.998% enriched $^{109}$Ag was irradiated for 6 hours 28 minutes by thermal neutron capture (n,$\gamma$) reaction in the 100 kW CONSORT reactor at the (ULRC). The resulting radioisotope of $^{110m}$Ag ($t_1/2$=250 d.)\textsuperscript{73} of roughly 10 $\mu$Ci of activity was placed in a standard polythene capsule and was kept for two weeks before taking any measurements. This allowed any short life activity to be eliminated.

4.2b Singles spectra

A number of singles spectra were recorded using the 12% efficient Ge(Li) detector, the Compton suppression system and the intrinsic Ge detector (see Table (3.1)) at regular intervals of time following the half-life of $^{110m}$Ag. This enabled us to have a check on any impurity peak, if present, in the source. A sample pulse height spectrum taken by the 12% efficient Ge(Li) detector is shown in Fig. (4.1). The energies and relative intensities of the $\gamma$-rays were calculated employing the method described in section (3.2). An average of the values so obtained is summarized in Table (4.1) where the relative intensity results are compared with those of G.Mallet\textsuperscript{72}, Verma et al.\textsuperscript{71} and F.E.Bertrand\textsuperscript{69}. Among the sixty-eight transitions, three at 116.15, 241.84 and 598.44 are newly seen.

It should be noted that the measurements taken by the 12% Ge(Li) detector and the Compton suppression system, on the one hand, and those by the Ge detector, on the other hand, provide an overlap for energy region between 80 to 350 keV. This would give a good cross-check to be made of the measured gamma ray energies and relative intensities.
Fig. 1. 4.1. Single spectrum of Ag-110m
Table (4.1). Relative intensities of $\gamma$-rays emitted from the decay of $^{110m}$Ag

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<th>Energy E(keV)</th>
<th>Intensity related to $I(658)=100$</th>
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<td>Present work</td>
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<td>0.005(1)</td>
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Table (4.1). Relative intensities of γ-rays emitted from the decay of $^{110m}\text{Ag}$ (continued)

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* Energies not observed in this work
4.2c **Coincidence spectra and performance of the (DPDCS)**

γ-γ coincidence measurements were done using the 12% and 10% efficient Ge(Li) detectors (detectors nos. 1 and 2 in Table (3.1)) placed at 90° relative to each other. The experimental set-up is shown in Fig. (3.16) where it is described and discussed. Singles counting rates in the detectors were maintained below 2000 counts/sec by adjusting the source-to-detector distances which were kept at 10 cm and 15 cm from the 12% and 10% detectors, respectively. The effective resolution time or FWHM of the coincidence circuit was 45 nsec. The γ-γ coincidence spectra were written on three magnetic tapes. The total running time was approximately 10 days.

Corrected spectra were taken in coincidence with each of the five most intense and well separated transitions observed in the $^{110}\text{mAg}$ decay. These are the 620, 658, 764, 818 and 885 keV. The resulting coincidence spectra are shown in Figs. (4.2 to 4.13). The corrections for coincidence counts due to chance and background events accepted in the gating energy were applied by, respectively, switching on the chance toggle of the Total-Chance Unit and by moving the gated energy window on the Select Gate Unit of the Read Interface to the background region besides the peak of interest.

An illustration of the effects of background and chance corrections on coincidence spectra is provided by the spectra shown in Figs. (4.4 to 4.7). Fig. (4.4) shows the uncorrected spectrum taken in coincidence with the 658 keV transition. One notes the presence in the gating energy region of background and chance events of transitions which are in coincidence with the 658 keV line. Fig. (4.5) shows the effects of the background events only while Fig. (4.6) shows the effects of the chance events. The background spectrum was obtained by gating on the energy region slightly lower than the 658 keV line. Fig. (4.7) provides the coincidence spectrum with the 658 keV corrected for the background and the chance events. That the 658 keV line has reduced demonstrating the care that should be taken in analysing γ-γ coincidence results. The non disappearance of this line can be due, partly to the fact that this line has the strongest intensity in the gamma ray spectrum, and partly to the interfering effects of the Compton background obtained from Ge(Li) detector measurements.

A summary of the results of γ-γ coincidence experiments is given in Table (4.2). The entries VS, S, W and VW label very strong, strong, weak and very weak give the strength of an observed gamma ray relative to the other gamma rays in the coincidence spectrum. The coincidence intensities of the peaks that are in VS or S coincidence with the gating peaks exceed their relative single intensities which explain very well the feature of
Fig. (4.5). Background spectrum of Ag-110m in coincidence with 658 keV.
Fig. (4.7). Corrected spectrum of Ag-110m in coincidence with 658 keV.
Fig. (4.8). Corrected spectrum of Ag-110m in coincidence with 764 keV.
Table (4.2). Summary of γ-γ coincidence results from the decay of $^{110}$mAg

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<th>Gate (keV)</th>
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Summary of \(\gamma-\gamma\) coincidence results from the decay of \(^{110m}\text{Ag}\)

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<thead>
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<th>Transition (keV)</th>
<th>Gate (keV)</th>
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</table>

A study of the experimental \(\beta\) of \(E2\) transitions in \(^{110m}\text{Ag}\) together with the theoretical \(\beta\) values corresponding to \(E2\) and \(M1\) multipolarities is shown in Table (4.4). The first and last columns of this table give the transitions for which \(\beta\) values calculated and the adopted multipolarities, respectively.

The decay scheme, the energies of the gamma rays are presented in Table (4.3) and are consistent with the transitions shown in Table (4.4). The scheme also includes the \(\gamma-\gamma\) transitions and transition rates, which are calculated using the experimental lifetimes of the states involved. The energy levels are deduced from the 

The 1475.71 and 1301.003 keV levels are two final state levels that can be reached by the decay of the \(^{110m}\text{Ag}\) nucleus. A number of transitions to final states at 620.43, 658.37, 764.97, 725.75, 749.86, 777.4, 818.35, and 885.18 keV are also in coincidence with the 1475.71 keV transition, which allows the establishment of the level at 1475.71 keV. This level is observed to be populated through the transition of 347.72 keV and further transitions to lower energy levels. The first transition has a mixed
4.3 Decay scheme

Based on the γ-γ coincidence results (Table (4.2)) and the γ-ray energy sum relations (Table (4.3)), the decay scheme of $^{110}$Cd resulting from the 250 d decay of $^{110}$Ag was obtained and is shown in Fig. (4.14).

In this scheme, the logft values were computed from the branching ratios obtained from a detailed intensity balance for each level and by taking for $Q_\beta^-$ the value 3010.66 keV$^5$. The expression employed in the determination of the logft values is:

$$logft = logf_{0t} + logC + Alogft$$

where $logf_{0t}$ was obtained using the nomogram in appendix 19 of ref. 11, logC was read off from the appropriate curve for $\beta^-$ emission in appendix 20 and $Alogft = -log(p/100\%)$ where p is the percentage of the branching ratio. These logft values together with the experimentally available K internal conversion coefficients $\alpha(K)$ were considered for the assignments of the spins and parities to the levels. In the $\alpha(K)$ calculations, the electron conversion intensities $I_{\beta^-}$ were quoted from ref. 76 except for the gamma ray transition at 447.11 keV whose $I_{\beta^-}$ was obtained from ref. 77. Normalization was made to the theoretical $\alpha(K)$ for the E2 multipolarity of the 658 keV transition. A summary of the experimental $\alpha(K)$ of some transitions in $^{110}$Cd together with the theoretical $\alpha(K)$ values$^{23}$ corresponding to E1, E2 and M1 multipolarities is shown in Table (4.4). The first and last columns of this Table give the transitions for which $\alpha(K)$ were calculated, and the adopted multipolarities, respectively.

In the decay scheme, the energies of the gamma rays are presented vertically above the gamma ray transition lines; branching ratios, logft values, spins, parities and level energy assignments are given on the left.

An inspection of this decay scheme reveals that most of the gamma ray transitions are in coincidence with the 658 keV gate which is a pure E2 transition so the spin/parity assignment of the first excited state is $2^+$ as the ground state has $I^\pi=0^+$. The $1475.73$ and $1542.40$ keV levels

A number of gamma ray transitions at 603.42, 648.57, 686.97, 744.27, 774.83, 957.4, 1085.39 and 1186.33 keV are seen in coincidence with the gate at 818 keV; thus allow the establishment of the level at $1475.73$ keV. This level is observed to de-excite through two transitions at 818.02 and $1475.73$ keV to the $2^+_1$ and $0^+_1$ levels. The first transition has a mixed
Table (4.3). Energy relations for $^{110}$Cd

<table>
<thead>
<tr>
<th>Energy sum (keV)</th>
<th>Mean (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>657.741</td>
<td>657.741</td>
</tr>
<tr>
<td>$657.741 + 818.015 = 1475.756$</td>
<td>1475.733</td>
</tr>
<tr>
<td>1475.71</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 884.664 = 1542.405$</td>
<td>1542.405</td>
</tr>
<tr>
<td>$657.741 + 1125.726 = 1783.467$</td>
<td>1783.548</td>
</tr>
<tr>
<td>$657.741 + 884.664 + 241.84 = 1784.245$</td>
<td></td>
</tr>
<tr>
<td>1782.931</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 818.015 + 603.417 = 2079.173$</td>
<td>2078.984</td>
</tr>
<tr>
<td>$657.741 + 1421.054 = 2078.795$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 818.015 + 648.57 = 2124.326$</td>
<td>2124.533</td>
</tr>
<tr>
<td>$657.741 + 1467.0 = 2124.741$</td>
<td></td>
</tr>
<tr>
<td>$1542.405 + 620.371 = 2162.776$</td>
<td>2162.731</td>
</tr>
<tr>
<td>$657.741 + 818.015 + 686.967 = 2162.723$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 1504.953 = 2162.694$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 1421.054 + 116.147 = 2194.942$</td>
<td>2194.942</td>
</tr>
<tr>
<td>$1542.405 + 677.603 = 2220.008$</td>
<td>2219.981</td>
</tr>
<tr>
<td>$657.741 + 818.015 + 744.266 = 2220.022$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 1562.173 = 2219.914$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 1125.726 + 467.123 = 2250.59$</td>
<td>2250.634</td>
</tr>
<tr>
<td>$657.741 + 818.015 + 774.83 = 2250.586$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 1592.985 = 2250.726$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 1629.518 = 2287.259$</td>
<td>2287.259</td>
</tr>
<tr>
<td>$657.741 + 1698.3 = 2356.041$</td>
<td>2356.041</td>
</tr>
<tr>
<td>$657.741 + 818.015 + 957.4 = 2433.156$</td>
<td>2432.939</td>
</tr>
<tr>
<td>$657.741 + 1774.981 = 2432.722$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 1592.985 + 229.465 = 2480.191$</td>
<td>2480.162</td>
</tr>
<tr>
<td>$657.741 + 1467.0 + 355.67 = 2480.411$</td>
<td></td>
</tr>
<tr>
<td>$657.741 + 884.664 + 937.48 = 2479.885$</td>
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</tr>
<tr>
<td>$657.741 + 884.664 + 997.187 = 2539.592$</td>
<td>2539.592</td>
</tr>
<tr>
<td>$657.741 + 884.664 + 677.603 + 304.612 = 2560.62$</td>
<td>2560.02</td>
</tr>
<tr>
<td>$657.741 + 884.664 + 1018.925 = 2561.33$</td>
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</tr>
<tr>
<td>$657.741 + 818.015 + 1085.396 = 2561.152$</td>
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</tr>
<tr>
<td>$657.741 + 1903.237 = 2560.978$</td>
<td></td>
</tr>
</tbody>
</table>

continued ......
<table>
<thead>
<tr>
<th>Energy sum (keV)</th>
<th>Mean (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>657.741 + 884.664 + 997.187 + 120.125 = 2659.717</td>
<td>2659.911</td>
</tr>
<tr>
<td>657.741 + 1592.985 + 409.701 = 2660.427</td>
<td></td>
</tr>
<tr>
<td>657.741 + 884.664 + 1117.185 = 2659.59</td>
<td></td>
</tr>
<tr>
<td>657.741 + 818.015 + 957.4 + 229.465 = 2662.621</td>
<td>2662.316</td>
</tr>
<tr>
<td>657.741 + 2004.5 = 2662.241</td>
<td></td>
</tr>
<tr>
<td>657.741 + 884.664 + 1163.338 = 2705.743</td>
<td>2705.743</td>
</tr>
<tr>
<td>657.741 + 1504.953 + 544.526 = 2707.22</td>
<td>2707.116</td>
</tr>
<tr>
<td>657.741 + 884.664 + 1164.607 = 2707.012</td>
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</tr>
<tr>
<td>657.741 + 1774.981 + 360.684 = 2793.406</td>
<td>2793.406</td>
</tr>
<tr>
<td>657.741 + 1562.173 + 573.038 = 2792.952</td>
<td></td>
</tr>
<tr>
<td>657.741 + 1421.054 + 116.147 + 598.426 = 2793.368</td>
<td>2793.314</td>
</tr>
<tr>
<td>657.741 + 1504.953 + 630.62 = 2793.525</td>
<td></td>
</tr>
<tr>
<td>657.741 + 884.664 + 1251.12 = 2793.525</td>
<td></td>
</tr>
<tr>
<td>657.741 + 1774.981 + 409.701 = 2842.423</td>
<td>2842.915</td>
</tr>
<tr>
<td>657.741 + 884.664 + 1301.003 = 2843.408</td>
<td></td>
</tr>
<tr>
<td>657.741 + 1504.953 + 626.322 + 2877.048 = 2876.868</td>
<td>2876.692</td>
</tr>
<tr>
<td>657.741 + 884.664 + 1333.756 = 2876.161</td>
<td></td>
</tr>
<tr>
<td>657.741 + 884.664 + 1251.12 + 133.348 = 2926.873</td>
<td>2926.639</td>
</tr>
<tr>
<td>657.741 + 884.664 + 1164.607 + 219.568 = 2926.58</td>
<td></td>
</tr>
<tr>
<td>657.741 + 884.664 + 1163.338 + 220.419 = 2926.162</td>
<td></td>
</tr>
<tr>
<td>657.741 + 2004.5 + 264.32 = 2926.561</td>
<td></td>
</tr>
<tr>
<td>657.741 + 884.664 + 1117.185 + 267.203 = 2926.793</td>
<td></td>
</tr>
<tr>
<td>657.741 + 1903.128 + 365.825 = 2926.694</td>
<td></td>
</tr>
<tr>
<td>657.741 + 884.664 + 997.187 + 387.528 = 2927.12</td>
<td></td>
</tr>
<tr>
<td>657.741 + 884.664 + 937.48 + 447.11 = 2926.995</td>
<td></td>
</tr>
<tr>
<td>657.741 + 1774.981 + 493.286 = 2926.008</td>
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</tr>
<tr>
<td>657.741 + 1562.173 + 706.666 = 2926.58</td>
<td></td>
</tr>
<tr>
<td>657.741 + 1504.953 + 763.925 = 2926.619</td>
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</tr>
<tr>
<td>657.741 + 884.664 + 1384.276 = 2926.681</td>
<td></td>
</tr>
</tbody>
</table>
Fig. (4.14). Proposed energy level scheme for $^{110}\text{Cd}$. Newly observed levels and transitions are denoted by dashed lines. The energies in the level are in keV.
(E2+M1) multipolarity while the second has a pure E2 character, so $I^\pi=2^+$ is assigned to this level.

Similar arguments can be given to the level at 1542.40 keV with $4^+$ assignment resulting from the E2 nature of the 884.66 keV gated transition which is seen to decay from this level.

The 1783.55 keV level

This level is suggested by the detection for the first time of the transition at 241.84 keV both in singles and coincidence measurements. Two other transitions at 1125.73 and 1782.93 keV are seen to be consistent with the depopulation of this level to the $2_1^+$ and $0_1^+$ levels, respectively. The $\log f t = 6.9$ taken from the $\beta^-$ decay of $^{110}\text{Ag}$ ($t_1=24.6$ sec)$^{69}$ indicates a $2^+$ assignment for this level.

The 2078.98 keV level

This level is established on the basis of the very strong coincidence seen between the gamma ray transition at 603.42 keV and the 818 keV gate. However, there seems to be a confusion concerning the $I^\pi=3^-$ previously assigned to this level$^{69,73}$. If considered on the basis of the $\beta^-$ feeding to this level as seen by ref. 73, this assignment contradicts the $\beta^-$ selection rules which forbid a transition from the $6^+$ level of $^{110}\text{Ag}$ to the $3^-$ level of $^{110}\text{Cd}$. In the present work, a $\beta^-$ feeding is observed ($\log f t = 12.04$) but by considering the 69.1 mn decay of $^{110}\text{In}$, the $\log f t = 7.29$ given to the 2078.98 keV allows the $3^-$ assignment$^{69}$. Thus, this assignment is favoured in this work due to the fact that there could be background contributions to the present relative intensities of the transitions which de-excite this level.

The 2124.53 keV level

This level is suggested by the observation of the 648.57 and 1467 keV transitions. The former is in coincidence with the 818 keV gate, which suggests that it acts as a transition between this level and the level at 1475.73 keV.

Once again, the $4^+$ assignment to this level from previous works was not made on the basis of the $\log f t$ value. Furthermore, in ref. 73 there appears to be a clear indication for a $\beta^-$ feeding to this level but was not taken into account. The detailed intensity balance, in the present work, reveals a $\beta^-$ feeding ($\log f t = 12.19$) so that the possible value of spin/parity is $4^+$.

The 2162.73 keV level

This level is observed to de-excite through three transitions (620.37,
Table (4.4). K-shell internal-conversion coefficients for $^{110}$Cd

<table>
<thead>
<tr>
<th>Energy  (keV)</th>
<th>$I^+_1$</th>
<th>$I^+_2$</th>
<th>Experiment $a(K) \times 10^3$</th>
<th>Theoretical $a(K) \times 10^3$</th>
<th>Adopted multipolarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>E1</td>
<td>E2</td>
<td>M1</td>
</tr>
<tr>
<td>447.11</td>
<td>$5^+$</td>
<td>$6^+$</td>
<td>6.47(71)</td>
<td>2.98</td>
<td>8.10</td>
</tr>
<tr>
<td>620.37</td>
<td>$3^+$</td>
<td>$4^+$</td>
<td>2.99(28)</td>
<td>1.21</td>
<td>3.26</td>
</tr>
<tr>
<td>657.74</td>
<td>$2^+$</td>
<td>$0^+$</td>
<td>2.75</td>
<td>0.98</td>
<td>2.75</td>
</tr>
<tr>
<td>677.60</td>
<td>$4^+$</td>
<td>$4^+$</td>
<td>2.67(19)</td>
<td>0.92</td>
<td>2.53</td>
</tr>
<tr>
<td>686.97</td>
<td>$3^+$</td>
<td>$2^+$</td>
<td>2.36(19)</td>
<td>0.90</td>
<td>2.48</td>
</tr>
<tr>
<td>706.67</td>
<td>$5^+$</td>
<td>$4^+$</td>
<td>2.46(14)</td>
<td>0.85</td>
<td>2.29</td>
</tr>
<tr>
<td>744.27</td>
<td>$4^+$</td>
<td>$2^+$</td>
<td>1.99(17)</td>
<td>0.75</td>
<td>2.20</td>
</tr>
<tr>
<td>763.93</td>
<td>$5^+$</td>
<td>$3^+$</td>
<td>1.80(12)</td>
<td>0.74</td>
<td>1.90</td>
</tr>
<tr>
<td>818.02</td>
<td>$2^+$</td>
<td>$2^+$</td>
<td>1.64(11)</td>
<td>0.65</td>
<td>1.60</td>
</tr>
<tr>
<td>884.66</td>
<td>$4^+$</td>
<td>$2^+$</td>
<td>1.32(4)</td>
<td>0.54</td>
<td>1.31</td>
</tr>
<tr>
<td>937.48</td>
<td>$6^+$</td>
<td>$4^+$</td>
<td>1.15(8)</td>
<td>0.48</td>
<td>1.17</td>
</tr>
<tr>
<td>1384.27</td>
<td>$5^+$</td>
<td>$4^+$</td>
<td>0.57(4)</td>
<td>0.24</td>
<td>0.52</td>
</tr>
<tr>
<td>1475.71</td>
<td>$2^+$</td>
<td>$0^+$</td>
<td>0.43(5)</td>
<td>0.21</td>
<td>0.44</td>
</tr>
<tr>
<td>1504.95</td>
<td>$3^+$</td>
<td>$2^+$</td>
<td>0.462(9)</td>
<td>0.20</td>
<td>0.43</td>
</tr>
<tr>
<td>1562.17</td>
<td>$4^+$</td>
<td>$2^+$</td>
<td>0.47(6)</td>
<td>0.19</td>
<td>0.40</td>
</tr>
</tbody>
</table>
686.97 and 1504.95 keV) to the $4^+_1$, $2^+_2$ and $2^+_1$ levels, respectively. Each of these transitions has an M1+E2 indication resulting in a unique $I^\pi=3^+$ for this level considered as a member of the 3-phonon quintet (see Table (4.5) for all members at 0, 2, 3, 4 and 6).

The 2194.94 keV level

This new level is proposed on the basis of energy sum considerations resulting from the detection in singles measurements of the 116.15 and 598.44 keV transitions. Considerations of the spins of the levels giving rise to the transitions feeding and depopulating the above state would allow a $3^+,4^+$ assignment.

The 2219.98 keV level

The existence of this level is confirmed by the observation of three transitions by which it could depopulate: to the level at 1542.40 keV by the 677.60 keV seen in strong coincidence with the 885 keV gate, to the 1475.73 keV level by the 744.27 keV seen in very strong coincidence with the 818 keV gate and to the level at 657.74 keV by the 1562.17 keV transition seen in strong coincidence with the 658 keV gate. The M1+E2 multipolarities of each of these transitions agree with an assignment of $4^+$.

The 2250.63 keV level

The present work provides evidence for the establishment of this level mainly on the basis of the strong coincidence observed between the gamma ray transition at 774.83 keV and the gate at 818 keV. The logft > 11.1 taken from ref. 11 leads to an assignment of $4^+$. 

The 2287.26 keV level

This level is suggested as a consequence of the strong coincidence observed between the 1629.52 keV gamma ray transition and the gate at 658 keV. A firm $2^+$ spin and parity assignment can be made to this level taking into account:

(i) the logft = 6.68 quoted from the $^{110}$Ag decay 

(ii) the level is a member of the 3-phonon quintet.

One notes that the logft = 12.7 as given by ref. 73 would not allow the $(0,1,2)^+$ assignment as in this case the $\beta^-$ selection rules are violated.

The 2356.04 keV level

The only evidence for the existence of this level is provided by the weak coincidence of the 1698.3 keV transition with the 658 keV gate.
The logft = 7.1 taken from the decay of $^{110}$In leads to an assignment of 3$^+$ (see ref. 11). This assignment is consistent with the logft = 13.15 found in this work and which denotes a second forbidden unique transition.

**The 2432.94 keV level**

This level is suggested by the observation of the 957.4 keV transition seen in strong coincidence with the 818 keV gate. Further evidence is provided by the weak coincidence of the 1774.98 keV transition with the 658 keV gate. Four other transitions at 229.47, 360.68, 409.70 and 493.29 are seen to feed this level from higher energy states. The above considerations thus indicate an uncertain $I^\pi$ assignment of (3$^+$,$4^+$).

**The 2480.16 keV level**

This level is established mainly from the very strong coincidence of the pure E2 transition at 937.48 keV with the 885 keV gate. It is seen to depopulate by two other transitions at 229.47 and 355.67 keV, each in weak coincidence with the 658 keV gate.

A firm 6$^+$ spin and parity assignment can be given to this level taking into account:

(i) the logft = 8.31
(ii) the E2 nature of the 937.48 keV transition
(iii) the 6$^+$ is the last member of the 3-phonon quintet.

**The 2539.59 keV level**

The existence of this level is confirmed by the strong coincidence observed between the 997.19 keV transition and the gate at 885 keV. The logft = 10.8, which denotes a first-forbidden non-unique 5$^-$ transition, together with the considerations of the spin and parity of the levels giving rise to the transitions at 120.13 and 387.53 indicate a definite assignment of 5$^-$.

**The 2561.02 keV level**

This level is established from the observation in singles and coincidence measurements of the transitions at 340.61, 1085.39 and 1903.13 keV. These transitions, together with the uncertain (E2) multipolarity of the 1018.93 keV gamma ray, cause feedings to the levels at 2219.98, 1475.73, 657.74 and 1542.40 keV, respectively, so the spin and parity assignment of the 2561.02 keV level is taken to be (4$^+$).

**The 2659.91 keV level**

The existence of this level is based on the observation of three transitions at 120.13, 409.70 and 1117.19 keV in coincidence with the
658 keV gate. The latter carries a pure $E1$ multipolarity, which together with the $\log ft = 10.77$ suggest a spin/parity assignment of $5^-$.  

**The 2662.31 keV level**

This level is established from the coincidence between the 1186.33 keV and the 658 keV gamma rays and from another coincidence between the 229.47 keV and the 818 keV gamma rays. The $\log ft = 11.29$ which denotes a second-forbidden non-unique $\beta^-$ transition favours a spin/parity assignment of $4^+$.  

It should be mentioned, however, that the $0^+$ assignment to this level from the work of G.Mallet et al.\textsuperscript{73} is inconsistent with their observation of a $3^-$ feeding as deduced from the $\log ft = 12.1$. In such a case, the $\beta^-$ selection rules are violated.

**The 2705.74 and 2707.12 keV levels**

The two gamma ray transitions at 1163.34 and 1164.61 keV are observed to be in coincidence with the gate at 885 keV. They accordingly considered to come from the decay of the levels at 2705.74 ($\log ft = 10.38$) and 2707.12 ($\log ft = 10.79$) to the level at 1542.40 keV. The above $\log ft$ allow the $5^+,4^+,5^-$ assignments for each level.

**The 2793.37 keV level**

This level is established from the strong coincidence between the 630.62 keV gamma ray and the gate at 620 keV. It is also seen to de-excite by a number of transitions at 360.68, 573.04, 714.85, 1251.12 and 598.44 keV (new).

A doubt is cast on the $3^+,4^+$ assignments as reported by ref. 73 because this level is observed, in the present work, to have a $\log ft = 9.7$ which implies a first-forbidden non-unique $\beta^-$ transition with $\Delta I=0,1$ and $\Delta n=-$ for $Z<80$. Furthermore, this level gives rise to transitions that feed levels with $4^+,3^+$ and $3^-$ assignments; it is also fed from the $5^+$ state by the 133.35 keV transition. This study thus suggests that the spins and parities of $4^+,5^-$ are the most probable for the 2793.37 keV level. It may be noted, however, that in ref. 11 no $I^\pi$ values were assigned to this level, while in ref. 69 an uncertain ($4^-,5$) assignment was reported.

**The 2842.91 keV level**

The present work provides evidence for the establishment of this level on the basis of the weak coincidence observed between the 409.70 and 657.74 keV gamma rays and of another coincidence between the 1301 and 884.66 keV gamma rays. The $\log ft = 9.66$ found for this level indicates
(4^−, 5, 6^+) spins and parities.

**The 2876.69 keV level**

The logft = 8.20 favours I values of 5^+, 6^+ for this level. These values are consistent with the considerations of the spins and parities of the levels giving rise to the transitions at 396.98, 626.32 and 1333.76, that depopulate the above state.

**The 2926.64 keV level**

This level is well established and all data indicate a unique spin/parity value of 5^+.

### 4.4 Discussion

To begin this section the question "what is a spherical nucleus?" is asked. The answer seems to be trivial: the potential energy of a spherical nucleus has a deep minimum at β=0 (β is the deformation parameter). However, this definition is insufficient as pointed out by D. Janssen at his Conference lecture in Yugoslavia. He defined a spherical nucleus in the following way: "either the potential energy of a spherical nucleus has a deep minimum at β=0, like in the case of magic or semi-magic nucleus, or the potential energy has two minima at β≠0 and one maximum at β=0 where the depth of the minima, contrary to the deformed nuclei, is smaller or equal to the energy of the first 2^+ state E_{2^+} ≥ E_{deff}.

The calculations performed by Gneuss and Greiner and by Meyer et al. for the even Cd nuclei predict that, as neutron pairs are added beyond the N=50 neutron shell closure (98^+_{50}), the minimum in the potential energy surface (PES) shifts to a slightly deformed position until, at approximately 110^+_{50}, a second minimum appears (see Fig. 4.15). In this case the coupling to non-collective degrees of freedom is smaller than that when the potential energy has a deep minimum at β=0, but the quadrupole vibration is, in general, strongly anharmonical and the wave function is a superposition of many-phonon states.

Ever since the original observation by Scharff-Goldhaber and Weneser that vibrational states might exist in nuclei and particularly after the failure of the simple Bohr model to explain the quadrupole moments of the first 2^+ excited states, many phenomenological models were proposed. One notes that, within the framework of the harmonic vibrational model, pure vibrational (pure phonon) states have static quadrupole moments that are zero, because the quadrupole-moment operator is a linear combination of a creation and annihilation operator of a
Fig (4.15). The predicted trend in the PES, $\beta_1$, $\beta_2$ as neutron pairs are added from $^{110}$Cd to $^{114}$Cd.
phonon, and thus the diagonal matrix elements vanish with respect to states that have definite number of phonons. These phenomenological models will not be taken into account here, the regularities in levels spacings and transition rates in $^{110}$Cd expected on the basis of the SU(5) limit of the (IBM) will be discussed. This is because this model provides an elegant and powerful tool for the description of collective properties of nuclei.

4.4a The SU(5) limit results

The reader is referred to section 2.4a, Chapter II, for a proper understanding of the results given here. Moreover, the abbreviation (IBA) which denotes interacting boson approximation is used, instead of the (IBM). The reason is that the (IBM) is an approximation to the shell model.

In the unperturbed SU(5) limit, the calculations of the low-lying states in $^{110}$Cd were carried out using the (IBA-1) computer code PHINT. The parameters in Eq. (2.4.7) were determined by normalizing theory to four known experimental levels. These levels are: the 1542.40 ($4^+$), 2480.16 ($6^+$) of the Y-band, the 2162.73 ($3^+$) of the Z-band and the 2287.26 keV ($2^+$) of the $g$-band. The parameters of Eq. (2.4.7) are:

$$
\begin{align*}
\epsilon &= 715.87 \text{ keV} \\
\alpha &= 26.17 \text{ keV} \\
\beta &= 13.42 \text{ keV} \\
\gamma &= 10.56 \text{ keV}
\end{align*}
$$

The four parameters which were fed into PHINT in addition to the total number of bosons $N=7$ in $^{110}$Cd can be deduced from the above parameters by employing Eq. (2.4.8). Thus:

$$
\begin{align*}
\hbar &= \epsilon = 715.87 \text{ keV} \\
C_0 &= 33.55 \text{ keV} \\
C_2 &= -37.21 \text{ keV} \\
C_4 &= 110.68 \text{ keV}
\end{align*}
$$

Fig. (4.16) gives the comparison between experimental and resulting theoretical (IBA-1) spectrum for $^{110}$Cd. The bands in the theoretical spectrum are labelled by Y, X, Z, $g$ and $\Delta$ (see Eq. (2.4.15)). This figure presents a number of interesting features. For instance, because of the boson-boson interaction, states belonging to the same phonon number are no longer degenerate. On the other hand, although many models can predict rather well the positions of the levels within the ground state band (Y-band), presumably only few, if any, can at the same time
Fig. (4.16). Comparison between experimental (EXPT.) and theoretical (IBA-1) spectrum for $^{116}\text{Cd}$. The parameters of the (IBA-1) are $\epsilon=715.87$ keV, $C_0=33.55$ keV, $C_2=-37.21$ keV, $C_4=110.68$ keV.
describe the other levels. The experimental identification of the triplet for n=2 phonons precisely determines whether a particular model correctly describes the system. In fact, the (IBA-1) is destined to do such a systematic description since even with a relatively small number of parameters it provides a consistent description of very different nuclei with complex decay schemes. Returning to Fig. (4.16), it can thus be observed that the three experimental members of the two-phonon triplet (n=2) at 1473.09 (2^+) taken from ref. 69, 1475.73 (2^+) and 1542.4 keV (4^+) taken from the present measurements, are clearly identified with the corresponding (IBA-1) predictions at 1465.3, 1394.5 and 1542.4 keV.

Furthermore, a good agreement between theory and experiment based on the figures of Table (4.5) is obtained.

Results for B(E2) ratios in the d-boson model (or unperturbed SU(5) limit) are presented in Table (4.6) where a comparison is drawn between these values and those obtained from the perturbed SU(5) limit (see below). Here one only notes that all forbidden Δn=2 transitions, where n_d is the number of d-bosons, are zero in the d-boson limit. The B(E2) ratios in this limit were easily deduced from the analytical B(E2) expressions given in ref. 35.

In what follows, the treatment of the energy spectrum and transition rates in ^{110}Cd in terms of the perturbed SU(5) limit is considered. In such a case, the full Hamiltonian of Eq. (2.4.6) was considered with the "n_d changing terms" being treated as the perturbation. This treatment led Arima and Iachello to deduce a set of analytical expressions both for energies and transition rates and are given in ref. 35. The parameters that were introduced in the energy expressions are ξ and ξ' and those in the T(E2) operator expressions are η, η', q_2 and q'_2. ξ and ξ' can be adjusted to give good fits to the B(E2) values by appropriately choosing q_2 and q'_2. The resulting values for the above parameters were determined in the following way:

(i) q_2 value: by fitting the experimentally deduced B(E2) value of the first excited 2^+ state, B(E2) = (869.32 ± 6.51) e^2 F^2 (see Eq. (1.4.5), Chapter I) to the corresponding B(E2) value calculated from the expression in the first order perturbation theory^{35}.

(ii) q'_2 value: by taking q'_2 = - 0.7 \sqrt{2} q_2 (ref. 35)

(iii) ξ and ξ' values: by choosing ξ = 0.33, ξ' = -0.22, deduced from an adjustment procedure to give good fits to experimental energies and B(E2) values, knowing that 0.1 < ξ, -ξ' < 0.3.
Table (4.5). Comparison between experimental (Exp) and theoretical (IBA) energy levels in $^{110}$Cd. The parameters used in the calculations of $E_{\text{IBA}}$ are the same as in Fig. (4.16), whereas those used in the perturbed $E_{\text{IBA}}$ are $\varepsilon=715.87$ keV, $\xi=0.33$ and $\xi'=0.22$. $n$ and $v$ denote the phonon number and phonon seniority respectively.

<table>
<thead>
<tr>
<th>L or I (n,v)</th>
<th>$E_{\text{Exp}}$ (keV)</th>
<th>$E_{\text{IBA}}$ (keV)</th>
<th>$E_{\text{IBA}}$ (keV)</th>
<th>$E_{\text{IBA}}$ (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1st order pert.</td>
<td>1st order pert.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>normalized to</td>
<td>normalized to</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2162.7 keV</td>
<td>2162.7 keV</td>
</tr>
<tr>
<td>0</td>
<td>(0,0)</td>
<td>0</td>
<td>0</td>
<td>-334.1</td>
</tr>
<tr>
<td>2</td>
<td>(1,1)</td>
<td>657.7</td>
<td>715.9</td>
<td>686.1</td>
</tr>
<tr>
<td>0</td>
<td>(2,0)</td>
<td>1473.1$^*$</td>
<td>1465.3</td>
<td>1539.5</td>
</tr>
<tr>
<td>2</td>
<td>(2,2)</td>
<td>1475.7</td>
<td>1394.5</td>
<td>1443.6</td>
</tr>
<tr>
<td>4</td>
<td>(2,2)</td>
<td>1542.4</td>
<td>1542.4</td>
<td>1512.3</td>
</tr>
<tr>
<td>0</td>
<td>(3,0)</td>
<td>2078.9$^*$</td>
<td>2036.0</td>
<td>2240.7</td>
</tr>
<tr>
<td>3</td>
<td>(3,3)</td>
<td>2162.7</td>
<td>2162.7</td>
<td>2286.8</td>
</tr>
<tr>
<td>4</td>
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<td>2219.9</td>
<td>2247.3</td>
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</tr>
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<td>(3,1)</td>
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<td>2287.3</td>
<td>2569.3</td>
</tr>
<tr>
<td>6</td>
<td>(3,3)</td>
<td>2480.2</td>
<td>2479.6</td>
<td>2476.6</td>
</tr>
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<td>5</td>
<td>(4,4)</td>
<td>2926.6</td>
<td>3083.9</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>(4,4)</td>
<td>3121.6$^*$</td>
<td>3210.7</td>
<td></td>
</tr>
</tbody>
</table>

*Values taken from ref. 69
Tables (4.5) and (4.6) summarize the results, obtained for energies and B(E2) ratios, respectively. One can observe no such improvement in the calculated energies by introducing the symmetry breaking terms as they act in second order in the energies while the boson interaction acts in first order. However, one can see from Table (4.6) a substantial improvement in the calculated B(E2) ratios in first order perturbation over those in the d-boson model, by comparing them with experimental values. The experimental B(E2) ratios in the last column of this Table are those given in ref. 73, and were obtained on the basis that the magnitude of the mixing ratio $\delta^2 = E2/M1$ of the transitions involved were not taken into account. As a result, some B(E2) values were found to be larger than those obtained in the present work where mixing ratios were introduced and their values were taken from ref. 65. The expression employed in the calculation is:

$$\frac{B(E2, I_i \rightarrow I_f)}{B(E2, I_i \rightarrow I_f')} = \frac{(E_{v_f})^5 \times N_{v_f} \times \delta^2 \times (1 + \delta^2)}{(E_{v_f'})^5 \times N_{v_f'} \times \delta'^2}$$

(4.1)

and is resulted from the combination of Eqs. (1.4.5), (1.4.6) and (1.4.7), and from the fact that the partial $\gamma$-ray transition probability $\lambda_{\gamma}$ is, in this case (where a mixture of multipolarities occurs), equal to the intensity sum $\lambda_{\gamma} = \lambda(M1) + \lambda(E2)$.

In Eq. (4.1), $E_{v_f}$, $N_{v_f}$ and $\delta^2$ are, respectively, the energy, relative intensity (both taken from Table (4.1)) and the magnitude of the mixing ratio (ref. 65), that characterize the $I_i \rightarrow I_f$ transition. $E_{v_f'}$, $N_{v_f'}$ and $\delta'^2$ are those characterizing the $I_i \rightarrow I_f'$ transition.

4.4b The negative parity levels

Three negative parity states are observed in the level scheme of $^{110}$Cd. These are the 2078.98 (3^−), 2539.59 (5^−) and 2659.91 keV (5^−). In general, the collective $3^-$ mode is expected to play an important role in the description of the nuclear collective motion. In the present scheme, the limited number of the negative parity states makes such a description insufficient. Thus, some lights shall be shed on the collective nature of these states. It is noted that the coupling between a quadrupole phonon and an octupole one should lead to a collective doublet. Thus, the coupling between the $2^+_1$ state and the $3^+_1$ state leads to the 2736.73 keV doublet. The level at 2539.59 keV (5^−) seems to be a candidate for this doublet.

The level at 2659.91 (5^−) can be understood in terms of excitation of pairs from the ground band shell configuration $\pi(g_{9/2})^{-2}$ to the quasi
Table (4.6). Comparison between experimental and calculated B(E2)_{IBA-1} values in ^{110}\text{Cd}$. The parameters used in the first order perturbation theory are $\xi = 0.33$, $\xi' = -0.22$, $q_2 = 23$ e\text{f}^2$, $q_3 = -7.2$ e\text{f}^2$. B(E2) values in the simple d-boson model are also shown. Last column shows the experimental B(E2) values from the work by Mallet et al.*

<table>
<thead>
<tr>
<th>Transition From</th>
<th>Levels From</th>
<th>Levels To</th>
<th>B(E2) calc. (e^2\text{f}^4)</th>
<th>Ratio to</th>
<th>Ratio calc. 1st order pert.</th>
<th>Ratio calc. d-boson model</th>
<th>Ratio exp. (present)</th>
<th>Ratio exp. from Mallet et al.*</th>
</tr>
</thead>
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<tr>
<td>$2^+_1$</td>
<td>$0^+_1$</td>
<td>657.5</td>
<td>0</td>
<td>869.32</td>
<td>$2^+_1 \rightarrow 0^+_1$</td>
<td>1.67</td>
<td>2</td>
<td>1.53(19)**</td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>$2^+_1$</td>
<td>1542.40</td>
<td>657.5</td>
<td>1447.83</td>
<td>$2^+_1 \rightarrow 0^+_1$</td>
<td>1.42</td>
<td>2</td>
<td>1.01(12)</td>
</tr>
<tr>
<td>$2^+_2$</td>
<td>$2^+_1$</td>
<td>1475.73</td>
<td>657.5</td>
<td>1233.20</td>
<td>$2^+_2 \rightarrow 2^+_1$</td>
<td>0.06</td>
<td>0</td>
<td>0.072(14)</td>
</tr>
<tr>
<td>$3^+_1$</td>
<td>$2^+_2$</td>
<td>2162.73</td>
<td>1475.73</td>
<td>1039.22</td>
<td>$3^+_1 \rightarrow 2^+_2$</td>
<td>0.33</td>
<td>0.4</td>
<td>0.49*0.34</td>
</tr>
<tr>
<td>$3^+_1$</td>
<td>$4^+_1$</td>
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<td>347.29</td>
<td>$3^+_1 \rightarrow 2^+_2$</td>
<td>0.004</td>
<td>0</td>
<td>0.02(1)</td>
</tr>
<tr>
<td>$3^+_1$</td>
<td>$2^+_1$</td>
<td>2162.73</td>
<td>657.5</td>
<td>3.79</td>
<td>$3^+_1 \rightarrow 2^+_1$</td>
<td>0.004</td>
<td>0</td>
<td>0.0066(2)</td>
</tr>
<tr>
<td>$4^+_2$</td>
<td>$2^+_2$</td>
<td>2219.98</td>
<td>1475.73</td>
<td>938.82</td>
<td>$4^+_2 \rightarrow 2^+_2$</td>
<td>0.68</td>
<td>0.91</td>
<td>0.21(13)</td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>$2^+_1$</td>
<td>2219.98</td>
<td>1542.40</td>
<td>642.71</td>
<td>$4^+_2 \rightarrow 2^+_2$</td>
<td>0.003</td>
<td>0</td>
<td>0.0066(2)</td>
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<td>$5^+_1$</td>
<td>$3^+_1$</td>
<td>2926.64</td>
<td>2162.73</td>
<td>827.05</td>
<td>$5^+_1 \rightarrow 3^+_1$</td>
<td>0.3</td>
<td>0.45</td>
<td>0.40(8)</td>
</tr>
<tr>
<td>$5^+_1$</td>
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<td>2926.64</td>
<td>2480.16</td>
<td>248.63</td>
<td>$5^+_1 \rightarrow 3^+_1$</td>
<td>0.3</td>
<td>0.45</td>
<td>0.55(5)</td>
</tr>
<tr>
<td>$5^+_1$</td>
<td>$4^+_2$</td>
<td>2926.64</td>
<td>2219.98</td>
<td>525.09</td>
<td>$5^+_1 \rightarrow 3^+_1$</td>
<td>0.64</td>
<td>0.45</td>
<td>0.0070(1)</td>
</tr>
<tr>
<td>$5^+_1$</td>
<td>$4^+_1$</td>
<td>2926.64</td>
<td>1542.40</td>
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<td>$5^+_1 \rightarrow 3^+_1$</td>
<td>0.04</td>
<td>0</td>
<td>0.0070(1)</td>
</tr>
</tbody>
</table>


\( \beta \) band configuration \( \pi(1g_9/2)^{-4}(2d_5/2)^2 \) built on the \( 0^+_2 \) state at 1473.09 keV.

It is interesting to note that such an excitation is equivalent to shape coexistence between states of different deformation. Recently, V.Paar and R.A.Meyer\(^7\) treated the \( ^{110}\text{Cd} \) nucleus as a two-protons hole cluster coupled to quadrupole vibrations. They showed that the quadrupole moments of the ground state band are negative, while those of the quasi \( \beta \) band are positive. This indicates that the cluster–vibration interaction has created an effective prolate deformation for the ground state band and an oblate deformation for the quasi \( \beta \) band.

Before coming to the end of this section, it is worthwhile to consider the quadrupole moment of the first excited \( 2^+_1 \) state. In order, however, to account for the observed deviations in quadrupole moments\(^91-92\) (which are in contrast to the predictions of the vibrational model) in the \( A=110 \) region, one may consider the effects of anharmonic terms in the vibrational model \(^81-82,93-95\). The problem of harmonicity was studied from different viewpoints based on a microscopic description. Bes, Dussel and Gratton\(^96\) treated some of the important particle degrees of freedom and Sorensen (see ref. 96, p. 580) took into account that the quasi bosons formed by the combination of fermion operators do not posses the properties of ideal bosons. The interpretation by Tamura and Udagawa\(^84\) of the first excited state in terms of a superposition of the one- and two-phonon harmonic vibrational \( 2^+ \) states corresponds, in essence, to an anharmonic oscillator model. This interpretation is followed here in order to calculate the quadrupole moment \( Q_2 \) of the first excited state in \( ^{110}\text{Cd} \).

In such a case, the wave functions of the first and second excited \( 2^+ \) states can be written as\(^84\):

\[
\psi(2^+_1) = -(1-a^2)^{1/2} |1> + a|2>
\]

\[
\psi(2^+_2) = a|1> + (1-a^2)^{1/2} |2>
\]

where the parameter \( a \) can be determined from the ratio (see Table (4.6)):

\[
R = \frac{B(E2, 2^+_2 \to 2^+_1)}{B(E2, 2^+_1 \to 0^+_1)} = \frac{2(2a^2-1)^2}{a^2}
\]

and the \( Q_2 \) can be obtained from the expression (see ref. 84):

\[
Q_2 = \frac{12}{5} (7\pi)^{-1/2} a(1-a^2)^{1/4} \beta Z e R^2
\]

In this expression, \( \beta=0.19 \) is the deformation parameter\(^97\), \( Z=48 \), and the mean nuclear radius is defined by \( R=1.2A^{1/3} \). The resulting \( |Q_2|=0.47 \) \( \text{eb} \) is in fair agreement with the values \( Q_2=-0.55\pm0.08 \) and \(-0.31\pm0.07 \) \( \text{eb} \) as given by Harper et al.\(^98\), but it is about twice the value of ref. 73.
CHAPTER V

STUDIES OF THE LOW-LYING LEVELS IN $^{192}$Pt AND $^{192}$Os

5.1 Introduction

The Pt-Os nuclei, lying in a well known transitional region between the deformed earth nuclei and the doubly closed shell $^{208}$Pb, have recently been the subject of many experimental and theoretical studies$^{40,99-102}$. The main purpose of these studies was to determine the applicability of the (IBA-1) to the nuclear structure of these nuclei. This region, however, has shed considerable light on the $O(6)$ limit$^{37}$ of the (IBA-1) with a tendency towards the SU(3) limit$^{36}$ when going to the lightest isotopes.

On the other hand, by considering the $^{192}$Pt and $^{192}$Os level schemes following the 74 d decay of $^{192}$Ir, some discrepancies were noticed as a result of the singles measurements. Moreover, some of the $\gamma$-ray transitions were not firmly placed in the level schemes$^{104-106}$. For example, Gehrke$^{104}$ studied the decay of $^{192}$Ir with Ge(Li) detectors with no mention of any $\gamma$-$\gamma$ coincidence measurements being undertaken. He identified the existence of the transitions at 329, 420, 593 and 703 keV and was able to establish the 909 keV level in $^{192}$Os on the basis of the energy sum considerations. His relative intensity results differ, in most cases, from those of Plaska et al.$^{105}$ by more than the combined uncertainty.

The most recent study of the decay of $^{192}$Ir was taken by Prasad et al.$^{106}$ using a Ge(Li) detector and a NaI(Tl)-NaI(Tl) sum-coincidence spectrometer in conjunction with a fast-slow coincidence circuit. They proposed the establishment of the 1118 keV level in $^{192}$Os and confirmed the existence of the $\gamma$-ray transitions at 177.6, 457.6, 594 and 1201 keV in the level scheme of $^{192}$Pt. Furthermore, the transitions at 784.5, 921 and 1201 keV were seen by ref. 106 to, respectively, depopulate levels at, $4^+$, $3^+$ and $4^+$, to the ground state. In this situation, the angular momentum carried away by each of these transitions is very large, and thus more reinvestigations are needed.

The work of Yoshizawa et al.$^{107}$ mainly provided the relative intensities of the strongest peaks of $^{192}$Ir because they are very useful for efficiency calibrations of Ge(Li) detectors in the region 300-1400 keV.
Based on the above considerations, and in order to resolve the discrepancies and differences from previous studies, regarding gamma transition intensities and thus energy level placements and spin/parity assignments, in the level schemes of $^{192}\text{Pt}$ and $^{192}\text{Os}$, the study of $^{192}\text{Ir}$ decay has been undertaken. For this purpose, an intrinsic germanium detector, a 12\% efficient Ge(Li) detector and a Compton suppression system have been used for singles measurements (see Table (3.1)), and a (DPDCS) for $\gamma-\gamma$ coincidence studies (see section 3.5). The results were compared with the (IBA-1) predictions by employing the code programs PHINT for energies and eigenvectors determinations, and FBEM for electromagnetic transitions calculations. It has been shown that this model provides an interpretation for the $^{192}\text{Pt-Os}$ nuclei in terms of the Hamiltonian in the perturbed $O(6)$ limit: from $O(6)$ to $SU(3)$, as well as in terms of the parameters of the $T(E2)$ operator.

5.2 Experimental procedure and results

5.2a Source preparation

The source of $^{192}\text{Ir}$ was prepared using the University of London Reactor irradiation facilities, by irradiation by thermal neutron capture $(n,\gamma)$ reaction of 3 mg of Iridium Trichloride enriched with $^{191}\text{Ir}$ to 99\%. After 15 min of irradiation, the product, placed in a standard polythene capsule, was left for about 10 days for the short half-life activities to die away and to provide a source strength of about 10 $\mu$Ci.

5.2b Singles spectra

Singles spectra of $^{192}\text{Ir}$ were taken by the above-mentioned detectors. The source was placed at 25 cm from the Ge(Li) detector to reduce the summing-up effects and the coincidence summing corrections. The energy and efficiency calibrations were carried out using the method described in section 3.2. The energies and relative intensities of all transitions observed in the present work are listed in Table (5.1) together with the relative intensities reported by other workers $^{104,105-107}$. Among the forty-nine transitions of this Table, thirteen at, 176.27, 213.17, 214.95, 267.04, 325.38, 362.32, 452.68, 624.88, 773.1, 904.52, 1147.05, 1237.51 and 1413.59 keV, are observed for the first time. The main components of the present relative intensity errors are that of the efficiency curve for the strongest gamma rays and the statistical errors of the peak areas for the weak gamma rays.

It can be seen from Table (5.1) that our values are in good agreement with those of ref. 107, whose corresponding transitions were measured
<table>
<thead>
<tr>
<th>Energy E(keV)</th>
<th>Present work</th>
<th>Ref. 107</th>
<th>Ref. 106</th>
<th>Ref. 104</th>
</tr>
</thead>
<tbody>
<tr>
<td>110.665(Os)</td>
<td>0.011(4)</td>
<td></td>
<td></td>
<td>0.0028</td>
</tr>
<tr>
<td>136.507(Pt)</td>
<td>0.209(8)</td>
<td>0.19(6)</td>
<td>0.218(10)</td>
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<tr>
<td>156.946*</td>
<td>0.015(4)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>176.266(Pt)</td>
<td>0.006(2)</td>
<td></td>
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<tr>
<td>201.426(Os)</td>
<td>0.62(2)</td>
<td>0.56(5)</td>
<td>0.551(12)</td>
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<tr>
<td>205.943(Os)</td>
<td>3.93(7)</td>
<td>3.90(45)</td>
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</tr>
<tr>
<td>213.168(Pt)</td>
<td>0.014(4)</td>
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<td></td>
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</tr>
<tr>
<td>214.945(Os)</td>
<td>0.0014(10)</td>
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</tr>
<tr>
<td>267.041(Pt)</td>
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<tr>
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<td>&lt;0.0041</td>
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<tr>
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<tr>
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<td>35.6(1.3)</td>
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</tr>
<tr>
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<td>35.84(18)</td>
<td>37.10(8)</td>
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<tr>
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<tr>
<td>325.384(Os)</td>
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<td>0.019(3)</td>
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<tr>
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<td>0.79(3)</td>
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<td>0.078(9)</td>
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<td>468.056(Pt)</td>
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<tr>
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<td>4.10(21)</td>
<td>3.81(5)</td>
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<tr>
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<td>588.573(Pt)</td>
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</tr>
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<td>6.55(13)</td>
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<tr>
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</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>703.591(Os)</td>
<td>0.006(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>738.194*</td>
<td>0.0015(1)</td>
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</tr>
<tr>
<td>766.53(Pt)</td>
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<td>0.031(1)</td>
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<tr>
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<td>0.0031(2)</td>
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</tr>
<tr>
<td>784.235(Pt)</td>
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</tr>
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<td>0.3420(24)</td>
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<tr>
<td>911.89(Os)</td>
<td>0.0064(33)</td>
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<td></td>
</tr>
<tr>
<td>920.763(Pt)</td>
<td>0.0095(9)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1056.159*</td>
<td>0.0021(2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1061.607(Pt)</td>
<td>0.063(2)</td>
<td>0.0631(11)</td>
<td>0.070(4)</td>
<td>0.067(3)</td>
</tr>
<tr>
<td>1090.308(Pt)</td>
<td>0.0018(5)</td>
<td>0.0010(5)</td>
<td>0.0030(2)</td>
<td>0.0020(7)</td>
</tr>
<tr>
<td>1147.046(Pt)</td>
<td>0.003(2)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1200.723(Pt)</td>
<td>0.0022(1)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1237.51(Pt)</td>
<td>0.009(1)</td>
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<td></td>
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</tr>
<tr>
<td>1378.778(Pt)</td>
<td>0.0010</td>
<td>0.0016</td>
<td>0.0020</td>
<td>0.0015</td>
</tr>
<tr>
<td>1413.585(Pt)</td>
<td>0.0013(1)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Not placed in the decay schemes
with a 57 cm$^3$ ORTEC true coaxial Ge(Li) detector and at a source-to-detector distance of 20 cm. The values of Gehrke$^{104}$ agree with our values over the whole region, except for a few weaker gamma rays at 110.67, 328.89, 478.44, 594.03, 738.19 and 1056.16 keV whose intensity values largely deviate from ours. Large deviations are also seen between the present values and those of Prasad et al.$^{106}$, especially in the region above 416.47 keV.

Fig. (5.1) illustrates the $^{192}\text{Ir}$ spectrum of the Compton suppression measurement, which was proved to be very useful for the detection of those energies lying on the huge Compton background (see section 3.3).

### 5.2c Coincidence spectra

The $\gamma-\gamma$ coincidence measurements were similar to those taken for $^{110}\text{mAg}$ (see section 4.2c). Here, one notes that the effective resolution time of the coincidence circuit was 50 nsec, and the data were recorded on four magnetic tapes.

A total of eight coincidence separate gated peaks were taken for the authentication of the decay schemes of $^{192}\text{Pt}$ and $^{192}\text{Os}$, and are shown in Table (5.2). The first five are transitions in the $^{192}\text{Pt}$ scheme, while the remaining three belong to that of $^{192}\text{Os}$. The coincidence spectra are illustrated in Figs. (5.2 to 5.10).

### 5.3 Decay schemes

Based on the $\gamma-\gamma$ coincidence results and on the energy sum relations, the decay schemes of $^{192}\text{Pt}$ and $^{192}\text{Os}$ were deduced and are shown in Fig. (5.11).

In the $\beta^-$ decay of $^{192}\text{Ir}$ to $^{192}\text{Pt}$ and in the electron capture (E.C.) to $^{192}\text{Os}$, the log$ft$ values were computed from the branching ratios obtained from a detailed intensity balance for each level, and by taking for $Q_{\beta^-}$ and $Q_{\text{E.C.}}$ the values of 1453 and 1468 keV, respectively$^{108}$. These log$ft$ values together with the experimentally available K internal conversion coefficients $\alpha(K)$ were considered for the assignments of the spins and parities to the levels. The electron conversion intensities and the relative intensities, that are needed for the calculations of $\alpha(K)$, were taken from ref. 108 and from the present work, respectively. Normalization was made to the theoretical $\alpha(K)$ for the E2 character of the 612.46 keV transition. A summary of the measured $\alpha(K)$ values for some transitions in $^{192}\text{Pt}$ and Os together with the theoretical $\alpha(K)$ corresponding to E2, E1, M2, M1 and E3 multipolarities$^{23}$ is shown in Table (5.3).
Fig. (5,2). Total spectrum of Ir-192.
Fig. 5.3. Spectrum of Ir-192 in coincidence with 296 keV.
Fig. (5.5). Spectrum of Ir-192 in coincidence with 316.5 keV.
Fig. (5.6). Spectrum of Ir-192 in coincidence with 468 keV.
Fig. (5.10). Spectrum of Ir-192 in coincidence with 374 keV.
Table (5.2). $\gamma$-$\gamma$ coincidence results from the decay of $^{192}$Ir.
Unlabelled transitions from $^{192}$Pt.

<table>
<thead>
<tr>
<th>Transition (keV)</th>
<th>Gate(keV) for $^{192}$Pt</th>
<th>Gate(keV) for $^{192}$Os</th>
</tr>
</thead>
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<tr>
<td></td>
<td>296 308 316.5 468 588</td>
<td>205 283 374</td>
</tr>
<tr>
<td>136.507</td>
<td>S VS</td>
<td></td>
</tr>
<tr>
<td>176.266</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>201.426(0s)</td>
<td>W VS</td>
<td>VS VS</td>
</tr>
<tr>
<td>205.943(0s)</td>
<td>VS</td>
<td>VS</td>
</tr>
<tr>
<td>213.168</td>
<td>VS VW</td>
<td></td>
</tr>
<tr>
<td>214.945</td>
<td>VS W VS W VS</td>
<td></td>
</tr>
<tr>
<td>279.304</td>
<td>W VS W VW</td>
<td></td>
</tr>
<tr>
<td>283.2(0s)</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>295.949</td>
<td>VS VS VS VS</td>
<td></td>
</tr>
<tr>
<td>308.449</td>
<td>VS S</td>
<td></td>
</tr>
<tr>
<td>316.497</td>
<td>VS VS VS VS</td>
<td></td>
</tr>
<tr>
<td>325.384(0s)</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>328.878(0s)</td>
<td>S VS</td>
<td></td>
</tr>
<tr>
<td>362.32</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>374.43(0s)</td>
<td>VS S</td>
<td></td>
</tr>
<tr>
<td>416.47</td>
<td>VS S S</td>
<td></td>
</tr>
<tr>
<td>420.565(0s)</td>
<td>W VS VW VW VW</td>
<td></td>
</tr>
<tr>
<td>457.754</td>
<td>VS W</td>
<td></td>
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<tr>
<td>468.056</td>
<td>VS VS</td>
<td></td>
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<tr>
<td>478.438</td>
<td>VS W</td>
<td></td>
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<tr>
<td>484.57(0s)</td>
<td>S VS</td>
<td></td>
</tr>
<tr>
<td>588.573</td>
<td>S S</td>
<td></td>
</tr>
<tr>
<td>594.03</td>
<td>VW W</td>
<td></td>
</tr>
<tr>
<td>604.425</td>
<td>VS VS VS</td>
<td></td>
</tr>
<tr>
<td>612.463</td>
<td>VS VS VS</td>
<td></td>
</tr>
<tr>
<td>624.876</td>
<td>VS</td>
<td></td>
</tr>
<tr>
<td>629.651(0s)</td>
<td>VW W</td>
<td></td>
</tr>
<tr>
<td>703.591(0s)</td>
<td>VW</td>
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</tr>
<tr>
<td>766.53</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>773.1</td>
<td>VS</td>
<td></td>
</tr>
<tr>
<td>884.323</td>
<td>W</td>
<td></td>
</tr>
<tr>
<td>911.89(0s)</td>
<td>VW</td>
<td></td>
</tr>
<tr>
<td>1061.607</td>
<td>VW</td>
<td></td>
</tr>
<tr>
<td>1090.308</td>
<td>VW W</td>
<td></td>
</tr>
</tbody>
</table>
5.3a Decay scheme of $^{192}\text{Pt}$

The aim, in this sub-section, is to comment on the newly observed transitions. However, the case of the 784.24 and 920.76 keV crossover transitions, as stated in the Introduction, has been clarified in the present work. These transitions have been seen in the Compton suppression measurements (see Fig. (5.1)). It should be noted that the 784.24 keV peak could be a pure sum peak of the 316.49 and 468.06 keV transitions, but there is no doubt about the firmness of the $4^+$ value assigned to the 784.39 keV level (logft=8.51) which de-excites by a pure E2 transition at 468.06 keV to the well known $2^+$ level at 316.5 keV.

The previously assigned $4^+$ spin and parity to the 1200.8 keV level$^{10,106,11}$ is ruled out from this work. Instead, a firm $2^+$ value can be made taking into account:

(i) the logft = 8.18
(ii) this level decays by an E2+M1 radiation at 416.47 keV to the $4^+$ state at 784.39 keV. It is also observed to decay to the ground state via the 1200.72 keV transition.

The new transition at 773.1 keV is seen for the first time in the Compton suppression and the 12% efficient Ge(Li) measurements, and found in strong coincidence with the 316.5 keV gate. It is considered to arise from the decay of the 1090.26 keV level ($I^\pi=2^+$) to that of the 316.5 keV ($I^\pi=2^+$).

The newly observed level at 1146.95 keV is suggested from the strong coincidence between the new transition at 362.32 keV, seen from the measurements performed by the Ge and the 12% Ge(Li) detectors, and the pure E2 transition gate at 468 keV. The logft = 11.13 suggests a spin/parity assignment of $2^+$. Further support for this assignment is provided by the (IBA-1) calculations (see Discussion) which indicate that this level is the second member of the sequence $0^+-2^+-2^+$, characterizing the slightly perturbed $0(6)$ limit in the $^{192}\text{Pt}$ nucleus, while the second $2^+$ of this sequence is assigned to the newly observed level at 1413.76 keV.

The 452.68, 624.88 and 1237.51 keV transitions are new and are seen in the Compton suppression and the 12% efficient Ge(Li) measurements. They are considered to depopulate the newly suggested level at 1237.35 keV. The very strong coincidence between the 624.88 keV and the gate at 296 keV provides a strong support for this suggestion. The logft = 9.94 allows a spin/parity assignment of $2^+$ for this level.
Fig. (5.11). Proposed energy level schemes for $^{192}$Os and $^{192}$Pt. Newly observed levels and transitions are presented by dashed lines. The energies in the level are in keV. Uncertain spin and parity quantum numbers $I^\pi$ are given in parenthesis.
Table (5.3). K-shell internal-conversion coefficients for
(a) $^{192}\text{Pt}$

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>$I_i^\pi$</th>
<th>$I_f^\pi$</th>
<th>Experiment $\alpha(K)$</th>
<th>Theoretical $\alpha(K)$</th>
<th>Adopted multipolarity</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>E2</td>
<td>E1</td>
</tr>
<tr>
<td>136.51</td>
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<td>$4^+_1$</td>
<td>0.885(9)</td>
<td>0.420</td>
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<tr>
<td>295.95</td>
<td>$2^+_2$</td>
<td>$2^+_1$</td>
<td>0.068(3)</td>
<td>0.065</td>
<td>0.022</td>
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<tr>
<td>308.45</td>
<td>$3^+_1$</td>
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<td>0.062(3)</td>
<td>0.057</td>
<td>0.020</td>
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<tr>
<td>316.49</td>
<td>$2^+_1$</td>
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<td>0.054(2)</td>
<td>0.054</td>
<td>0.019</td>
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<tr>
<td>416.47</td>
<td>$2^+_5$</td>
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<td>0.034(2)</td>
<td>0.027</td>
<td>0.010</td>
</tr>
<tr>
<td>468.06</td>
<td>$4^+_1$</td>
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<tr>
<td>1090.31</td>
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<tr>
<td>1378.78</td>
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<td>0.0045(10)</td>
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</table>

(b) $^{192}\text{Os}$

<table>
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<th>$I_f^\pi$</th>
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<th>Theoretical $\alpha(K)$</th>
<th>Adopted multipolarity</th>
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</thead>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>E2</td>
<td>E1</td>
</tr>
<tr>
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<td>$2^+_2$</td>
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<td>0.146(5)</td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>283.20</td>
<td>$2^+_5$</td>
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<td>0.082(4)</td>
<td>0.076</td>
<td>0.025</td>
</tr>
<tr>
<td>374.43*</td>
<td>$4^+_1$</td>
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<td>0.039(4)</td>
<td>0.036</td>
<td>0.013</td>
</tr>
<tr>
<td>484.57</td>
<td>$3^+_1$</td>
<td></td>
<td>0.0206(6)</td>
<td>0.0192</td>
<td>0.0072</td>
</tr>
<tr>
<td>489.06</td>
<td>$2^+_2$</td>
<td></td>
<td>0.021(1)</td>
<td>0.019</td>
<td>0.007</td>
</tr>
</tbody>
</table>

*Relative electron intensity for this transition was taken from ref. 109.
The level at 1378.62 keV (logft = 8.05) is observed to decay by a pure E3 transition to the ground state; thus resulting in an unique $I^\pi$ value of $3^-$.  

The level at 1406.8 keV, appearing in the Table of Isotopes, is placed in the present level scheme due to the weak coincidence observed between the 1090.30 keV and the gate at 316.5 keV. The logft = 8.84 indicates a spin/parity assignment of $(3,4)^+$.  

The new level at 1413.76 keV is suggested by the observation of four new transitions by which it could depopulate: to the new level at 1237.35 keV by the 176.27 keV, seen in weak coincidence with the 296 keV gate, to the 1200.86 keV by the 213.17 keV seen in strong coincidence with the 308 keV gate, to the new level at 1146.95 keV by the 267.04 keV seen in the Ge detector singles measurements, and to the ground state by the direct 1413.59 keV. From the logft = 7.91 and the (IBA-1) calculations, a spin/parity assignment of $2^+$ is the most probable for this level.

5.3b Decay scheme of $^{192}$Os

The three gated energies at 205, 283 and 374 keV were considered for the authentication of the decay scheme of $^{192}$Os.

Even in such a small scheme, one can throw light on previously established levels as well as give interpretation to newly observed transitions. In all, eight levels are built up in this scheme. The first five were well established from previous works and further confirmed from the present.

The level at 905.23 keV, appearing in ref. 109 with an uncertain $(4^+)$ assignment is suggested from the present work but with $I^\pi=2^+$. This level de-excites to the $3^+$ state at 690.7 keV via the 214.95 keV transition, and to the 580.37 keV and the ground states via two new transitions at 325.38, observed in coincidence with the 374 keV gate, and at 904.52 keV, respectively. Considerations of the levels giving rise to the transitions depopulating from the 905.23 keV level together with the logft = 10.16 ensure the unique $I^\pi$ value of $2^+$ for the level under consideration. It should be noted that, in the works by refs. 104 and 106, no evidences were found for the existence of this level.

The level at 909.49 keV (logft = 9.41) is established from the strong coincidence between the transition at 328.88 keV and the gate at 374 keV, and between the 420.57 keV and the gate at 283 keV. The above logft value indicates a spin/parity assignment of $4^+$. 


The level at 1118.31 keV observed by ref. 106 is also seen in the present work due to the detection in singles and coincidence measurements of two gamma ray transitions at 629.46 and 911.89 keV. The log\(f_t\) = 6.91 limits the spin/parity assignment for this level to \((3,4)^+\). One notes that, in Gehrd's singles spectrum\(^{104}\), there appears to be a clear indication of the 628.51 keV \(\gamma\)-ray but did not take it into account. Moreover, the theoretical calculations of Kumar and Baranger\(^{110}\) predict a level at 1118 keV. It is also interesting to note that, in the recent low-lying scheme of \(^{192}\)Os deduced from the \(^{192}\)Os\((n,n'\gamma)\) reaction study by Kleepinger et al.\(^{102}\), the 1118 keV was not reported. In fact, if the electron capture decay energy of \(^{192}\)Ir is only \(Q_{E.C.} = (1049\pm5)\) keV (see ref. 102), this level could not have been observed in decay measurements.

5.4 Discussion

As was mentioned in the Introduction the Pt-Os nuclei lie in a region of transition: a region of onset or disappearance of nuclear deformation and provide a crucial testing ground of nuclear structure models. The Bohr-Mottelson idea\(^2,^{111}\) of collective quadrupole motion, that is the cooperative movement \((\lambda=2^+)\) of many nucleons, appears to be valid for low-lying levels of such nuclei, as evidenced by the B\((E2, 2^+_1 \rightarrow 0^+_1)\) values which are 60-100 single particle units\(^{112}\). However, the energy levels and electromagnetic moments of the Pt-Os nuclei show large deviations\(^{81,^{113}\)} from both the rotational model\(^2,^{111}\) and the vibrational model\(^{111,^{81}}\).

The recent evidence\(^{113}\) that the \(^{196}\)Pt nucleus corresponds to a new symmetry, the O(6) symmetry\(^{37}\) of the (IBA-1) model, offers the possibility for a new understanding of the Pt-Os regions in terms of small and progressive departures from that limiting symmetry. The symmetry triangle in Fig. (5.12) shows in a symbolic way the three limiting symmetries SU(5)\(^{35}\), SU(3)\(^{36}\) and O(6)\(^{37}\) and all possible transitions between the limiting cases in such a way that all complex transition regions are contained within the area. The Pt-Os region has been found to be a good example for the transition between O(6) and SU(3).

In the following discussion, the (IBA-1) calculations of the excitation states and E2 transition probabilities of \(^{192}\)Pt and \(^{192}\)Os going from the O(6) to the SU(3) limit, is presented. The O(6) limit was treated in Chapter II, section 2.4c.
Fig. (5.12). Symmetry triangle illustrating the three limiting symmetries of the IBA-1 and the three direct transitions between them.
5.4a The $O(6) \rightarrow SU(3)$ transitional region

Returning to Eq. (2.4.2), which was written as:

$$H = \varepsilon \sum_m d_m^+ d_m + k'' P^+ P^+ - k' L^+ L^+ - k Q^+ Q^+ + k_3 T^3_3 T^3_3 + k_4 T^4_4 T^4_4$$

one can remark that, in the three limiting cases, different terms of the Hamiltonian $H$ are used. In particular, the spectrum of $O(6)$ nuclei corresponds to the vanishing of the coefficients $k$ and $k_4$ and to $k'' \gg \varepsilon$, whereas $SU(3)$ nuclei are characterized by $k'' = 0$ and correspond to the case in which the $Q^+ Q^+$ and $L^+ L^+$ terms are dominant over $\varepsilon$.

In the $O(6) \rightarrow SU(3)$ transition, the strength of the quadrupole-quadrupole interaction increases from $k=0$ (in the $O(6)$ limit) and, for larger values, one approaches the $SU(3)$ limit. This interaction changes the energies of the levels that will become the band head of intrinsic $\beta, \gamma$ excitations in the rotational spectra. It also adds an effective contribution to the $L(L+1)$ term characterizing the level energies in the ground state band in the $SU(3)$ limit and implies a ratio of $E_{1^+} / E_{1^+} = 10/3$ in this limit. It is noted that in the $^{192}\text{Pt}$ nucleus this ratio is near 2.5 (and it is practically the same for all the Pt nuclei) indicating a very small $Q^+ Q^+$ interaction and a dominant $B$ term in Eq. (2.4.35). In the case of $^{192}\text{Os}$ this ratio is 2.8, and varies to about 3.2 by $^{186}\text{Os}$ which indicates a considerably larger $k$ than in Pt nuclei.

5.4b Results

The present (IBA-1) calculations were performed using the computer codes PHINT and FEEM. The interaction parameters of Eq. (2.4.2) were adjusted to give good fits to 10 energy levels in $^{192}\text{Pt}$ and 12 in $^{192}\text{Os}$. The two parameters $\varepsilon$ and $k_4$ are negligible in the $O(6) \rightarrow SU(3)$; the remaining ones are related to the Variable Names in PHINT by (see Eq. (2.4.40)): $\text{PAIR}=k'', \text{ELL}/2=k', \text{QQ}/4=k$, $\text{OCT} \sqrt{5} k_3$. These Variable Names with the exception of $\text{QQ}$ were first calculated from the expressions $A=2 \times \text{PAIR}$, $B=15 \times \text{OCT}$ and $C=(\text{ELL}-\text{OCT})/2$ (see ref. 43), where $A$, $B$ and $C$ are the quantities deduced from Eq. (2.4.35) of the $O(6)$ limit. Then, these parameters along with $\text{QQ}$ were gradually varied to produce the desired fit.

In the calculations of the absolute $B(E2)$ values, the two parameters $\alpha_2$ and $\beta_2$ of Eq. (2.4.4) (their equivalent parameters in PHINT are $E2SD=\alpha_2$, $E2DD=\sqrt{5} \beta_2$ of Eq. (2.4.41)) were adjusted to approximately reproduce the experimental $B(E2, 2^+_1 \rightarrow 0^+_1)$ and $B(E2, 2^+_2 \rightarrow 0^+_1)$ values for
the first two $2^+$ levels of the ground and the gamma vibrational bands. Table (5.4) gives the values of the parameters used in the calculations, whereas in Table (5.5) a comparison is given between the experimental and (IBA-1) calculated $B(E2)$ values. The last column shows the theoretical $B(E2)$ values quoted in ref. 110.

Table (5.4). Values of the parameters corresponding to the Variable Names in Programs PHINT and FBEM. (N=8)

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>PAIR (MeV)</th>
<th>ELL (MeV)</th>
<th>QQ (MeV)</th>
<th>OCT (MeV)</th>
<th>E2SD (eb)</th>
<th>E2DD (eb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{192}$Pt</td>
<td>0.047</td>
<td>0.0388</td>
<td>0.0001</td>
<td>0.023</td>
<td>0.1488</td>
<td>-0.105</td>
</tr>
<tr>
<td>$^{192}$Os</td>
<td>0.0489</td>
<td>0.0389</td>
<td>0.0085</td>
<td>0.0152</td>
<td>0.162</td>
<td>-0.0045</td>
</tr>
</tbody>
</table>

Fig. (5.13) illustrates the (IBA-1) calculations for the low-lying excited states of $^{192}$Pt and $^{192}$Os in the $O(6) \rightarrow SU(3)$ transitional case. The experimental values at $0^+_2$ (1195.2 keV), $2^+_8$ (1576.3 keV) and $2^+_9$ (2200.3 keV) in $^{192}$Pt were obtained from the Table of Isotopes\textsuperscript{11}, and those at $0^+_3$ (956.5 keV), $6^+_1$ (1089.2 keV), $0^+_5$ (1205.9 keV), $5^+_1$ (1362 keV), $2^+_4$ (1409.9 keV) and $4^+_3$ (1456.6 keV) from the decay scheme of ref. 102. A clarified version of this figure is presented in Table (5.6) where each level is specified by the quantum numbers, $L$, $(\sigma, \tau, \nu, \lambda)$ necessary to describe the $O(6)$ limit (see section 2.4c).

An inspection of Table (5.5) reveals a rather remarkable agreement between the experimental and the (IBA-1) $B(E2)$ values. These experimental $B(E2)$ values were calculated from the experimentally available lifetime values\textsuperscript{11} for levels in $^{192}$Pt and $^{192}$Os by making use of Eqs. (1.4.5) and (1.4.6) of section 1.4, Chapter I.

The (IBA-1) $B(E2)$ values thus deduced from the breaking of the $O(6)$ symmetry by the QQ and E2SD parameters of Table (5.4) played a significant role in asserting the spins for the levels. In the case of $^{192}$Pt, the effects of these parameters on the energy spectrum and $B(E2)$ values are negligible. This implies that the $^{192}$Pt nucleus exhibits most of the characteristics of the $O(6)$ limit. Thus, in Fig. (5.13), the experimental ratio $B(E2, 2^+_7 \rightarrow 2^+_4) / B(E2, 2^+_7 \rightarrow 0^+_1) = 2539 \pm 421$ (where $2^+_7$ and $2^+_4$ are the $I^=$ values assigned to the newly observed levels at 1413.76 and 1146.95 keV, respectively) is in a rather close agreement with the corresponding (IBA-1) ratio of 3482. This
Table (5.5). Experimental $B(E2)$ values ($e^2b^2$) compared with theoretical (IBA-1) $B(E2)$ in (a) $^{192}$Pt, (b) $^{192}$Os. Last column shows ref. 110 predictions.

(a) $^{192}$Pt

<table>
<thead>
<tr>
<th>Energy $E(\text{keV})$</th>
<th>Transition $I_i \rightarrow I_f$</th>
<th>$B(E2)$ values</th>
<th>Experiment present work</th>
<th>(IBA-1)</th>
<th>Ref. 110</th>
</tr>
</thead>
<tbody>
<tr>
<td>316.49</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>0.42(2)</td>
<td>0.42</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>295.95</td>
<td>$2^+ \rightarrow 2^+$</td>
<td>0.46(5)</td>
<td>0.58</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>612.46</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>0.0044(5)</td>
<td>0.0036</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>468.06</td>
<td>$4^+ \rightarrow 2^+$</td>
<td>0.62(3)</td>
<td>0.58</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>136.51</td>
<td>$3^+ \rightarrow 4^+$</td>
<td>0.21(3)</td>
<td>0.18</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>308.45</td>
<td>$3^+ \rightarrow 2^+$</td>
<td>0.43(6)</td>
<td>0.44</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td>604.43</td>
<td>$3^+ \rightarrow 2^+$</td>
<td>0.0046(6)</td>
<td>0.0038</td>
<td>0.0021</td>
<td></td>
</tr>
</tbody>
</table>

(b) $^{192}$Os

<table>
<thead>
<tr>
<th>Energy $E(\text{keV})$</th>
<th>Transition $I_i \rightarrow I_f$</th>
<th>$B(E2)$ values</th>
<th>Experiment present work</th>
<th>(IBA-1)</th>
<th>Ref. 110</th>
</tr>
</thead>
<tbody>
<tr>
<td>205.94</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>0.46(1)</td>
<td>0.46</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>283.20</td>
<td>$2^+ \rightarrow 2^+$</td>
<td>0.36(4)</td>
<td>0.33</td>
<td>0.74</td>
<td></td>
</tr>
<tr>
<td>489.06</td>
<td>$2^+ \rightarrow 0^+$</td>
<td>0.038(6)</td>
<td>0.035</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>374.43</td>
<td>$4^+ \rightarrow 2^+$</td>
<td>0.55(6)</td>
<td>0.64</td>
<td>0.78</td>
<td></td>
</tr>
<tr>
<td>420.57</td>
<td>$4^+ \rightarrow 2^+$</td>
<td>0.18(7)</td>
<td>0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>328.88</td>
<td>$4^+ \rightarrow 4^+$</td>
<td>0.18(7)</td>
<td>0.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>703.59</td>
<td>$4^+ \rightarrow 2^+$</td>
<td>0.0010(5)</td>
<td>0.0026</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
indicates that the $I^+$ assignments for these levels are quite justified. In addition, the corresponding theoretical (IBA-1) levels (see Fig. (5.13)) allow the formation of a triad of states built on the $0^+$ excitation (844.3 keV) whose corresponding experimental value has yet to be observed.

Returning to Table (5.4), it can be seen that a principal parameter change in going from the Pt to the Os is a reduction in the OCT parameter. This serves to increase sharply the $k_{Q,Q}$ term in $^{192}$Os and to lower those $0^+$ states which de-excite to the $2_1^+$ level. In the $O(6)$ limit, the predominant decay mode of the low-lying $0^+$ states is to the $2_2^+$ state. This can be seen, in the case of $^{192}$Pt, from the $0_2^+ (1195.2 \text{ keV}) \rightarrow 2_2^+$ transition which is dominated by an intensity value large compared to that from $0_2^+ \rightarrow 2_1^+$ (see ref. 11). On the other hand, the sharp increase of the $k_{Q,Q}$ term would contribute to a mixture of $0^+$ decay mode in $^{192}$Os, that is to $2_1^+$ and $2_2^+$ with preference for de-excitation to the $2_2^+$. This is evidenced from the experimental ratio $B(E2, 0_2^+ (956.5 \text{ keV}) \rightarrow 2_2^+)/B(E2, 0_2^+ \rightarrow 2_1^+)$ = 52.6 ± 30 which is found to be consistent to within the experimental accuracy with that of the (IBA-1) prediction (=71).

5.5 Concluding remarks

Extensive gamma rays singles and $\gamma-\gamma$ coincidence measurements enable the construction of comprehensive decay schemes of $^{192}$Pt and $^{192}$Os nuclei following the 74 d decay of $^{192}$Ir. The use of the Compton suppression system allows for those transitions lying in the Compton regions to be revealed.

Together with the results reported here, good agreement between experimental and (IBA-1) $B(E2)$ values has been obtained when also taking into account other experimentally observed levels in both nuclei (see ref. 102 and 11).

The Pt-Os nuclei have been treated as initiating an $O(6) \rightarrow$ rotor transition in the framework of the (IBA-1). However, it is clear from the resulting PAIR values of Table (5.4), which lie around PAIR=0.05 for both $^{192}$Pt and $^{192}$Os nuclei, that the Hamiltonian presented here did not account for a dramatic change for the observed properties obtained on the basis of the rigorous $O(6)$ Hamiltonian. For the $E2$-transition probabilities, the situation appears to be less complex. Some transitions probabilities have changed characteristically over the transition region, between the $O(6)$ and the SU(3), to produce a good agreement with experiments as shown in Table (5.5). This change can thus be understood
Fig. (5.13). Experimental levels in (a) $^{192}$Pt, (b) $^{192}$Os compared with (IBA-1) predictions.
Table (5.6). Comparison between experimental and theoretical (IBA-1) energy levels in (a) $^{192}$Pt, (b) $^{192}$Os

<table>
<thead>
<tr>
<th>L or I</th>
<th>$(\sigma, \tau, \nu_\Delta)$</th>
<th>Energy level (keV)</th>
<th>Energy level (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Experiment</td>
<td>(IBA-1)</td>
</tr>
<tr>
<td>0</td>
<td>$(8,0,0)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$(8,1,0)$</td>
<td>316.5</td>
<td>277.2</td>
</tr>
<tr>
<td>2</td>
<td>$(8,2,0)$</td>
<td>612.5</td>
<td>621.8</td>
</tr>
<tr>
<td>4</td>
<td>$(8,2,0)$</td>
<td>784.4</td>
<td>732.5</td>
</tr>
<tr>
<td>3</td>
<td>$(8,3,0)$</td>
<td>920.9</td>
<td>1128.8</td>
</tr>
<tr>
<td>2</td>
<td>$(6,1,0)$</td>
<td>1146.9</td>
<td>1121.5</td>
</tr>
<tr>
<td>0</td>
<td>$(8,3,1)$</td>
<td>1195.2</td>
<td>1033.9</td>
</tr>
<tr>
<td>2</td>
<td>$(6,2,0)$</td>
<td>1413.8</td>
<td>1466.2</td>
</tr>
<tr>
<td>2</td>
<td>$(8,4,1)$</td>
<td>1576.3</td>
<td>1778.3</td>
</tr>
<tr>
<td>2</td>
<td>$(8,5,1)$</td>
<td>2200.3</td>
<td>2123.1</td>
</tr>
<tr>
<td></td>
<td>$(8,4,0)$</td>
<td>1362.0</td>
<td>1328.7</td>
</tr>
<tr>
<td>2</td>
<td>$(8,5,1)$</td>
<td>1409.9</td>
<td>1475.6</td>
</tr>
</tbody>
</table>
in terms of the two E2SD values of Table (5.4) which are zero in the SU(3) limit. The introduction of proton and neutron bosons into the Hamiltonian of Eq. (2.4.2) results in the (IBA-2) model which has the effect of changing considerably the energy levels in the Pt-Os region but not greatly altering the E2 branching ratios. The most recent study by ref. 102 of the level scheme of $^{192}$Os, deduced from the $^{192}$Os(n,n'γ) reaction, resulted in a reasonable accord with the prediction of the (IBA-1 and -2).
CHAPTER VI

EXCITED STATES OF $^{166}$Er

6.1 Introduction

Nuclear spectra showing rotational-like structure are characterized by sequences of levels with energies which behave approximately as $I(I+1)$, beginning with some values $I_0$. In even-even nuclei, the ground state band has always $I_0=0$, while the next excited bands have usually $I_0=0$ ($\beta$-band) or $I_0=2$ ($\gamma$-band). Although the properties of the gamma-vibrational bands and their coupling to other bands in the deformed even nuclei have been recognized for many years, they are still of interest in the effort to expand the current idea of nuclear structure. The extensive studies by A. Bohr and B.R. Mottelson$^{33}$, L.L. Riedinger et al.$^{115}$, and I.A. Fraser et al.$^{116}$ support the conclusion that improvement in the predictions of the geometrical model for $\gamma$-g transitions is obtained by the introduction of two bands $\gamma$-g mixing parameter $Z\gamma$ (the ratio of intraband to interband $E2$ transition strength). Mottelson$^{118}$ discussed the mixing of ground state-gamma vibrational bands with special reference to the case of $^{166}$Er. More studies of the mixing parameter $Z\gamma$ were taken by refs. 119-123 employing the techniques of $\beta$- and $\gamma$-ray spectroscopy. The work by C.W. Reich and J.E. Cline$^{124}$ considered the effect of the mixing on the interband $E2$ transition probabilities of $^{166}$Er from the point of view of the $K=2$ band-mixing picture, and the geometrical approach by Mikhailov$^{34}$. The most recent study by D.D. Warner and R.F. Casten$^{125}$ of the collective states of $^{168}$Er provided an exacting test for the theoretical descriptions close to the $SU(3)$ limit$^{36}$ of the (IBA-1).

The most crucial result of their study was the prediction of the dominance of the gamma decay branch from the $\beta$ to the $\gamma$ band over that to the ground band. They further showed that such a dominance can be reproduced in the Bohr-Mottelson description by the explicit introduction of $\beta$-$\gamma$ band mixing.

In this respect, a well deformed nucleus far from the transitional regions present an ideal choice for the study of the low-lying excitations and should provide a considerable insight into the nature of the excited collective bands. The information given by ($n,\gamma$) reactions coupled with that from other experimental studies provide a detailed picture of nuclear spectra, since the radioactive product, obtained from this reaction, ensures the population of broad classes of states, irresp-
ective of their structure. The $^{166}$Er nucleus is a good experimental choice for such a study. It satisfies some of the above nuclear structure criteria. In addition, the advantageously high spin $7^+$ of the $^{166m}$Ho parent, and as a result of gamma-ray cascade process, levels with spins as high as 8 can be populated.

The level scheme of $^{166}$Er has been a subject of study by many workers using different techniques. Some of these studies had come from the radioactive decays of $^{166}$Ho (>30 y) and $^{166}$Tm (7.7 h), the $(n,\gamma)$ spectroscopy, and the $(n,\gamma)$ and $(p,2n)$ spectroscopies. The most comprehensive study was that of Reich and Cline who measured gamma-transition energies from the 1200 y decay of $^{166}$Ho, the 27 h decay of $^{165}$Ho and the 7.7 h decay of $^{165}$Tm using Ge(Li) detectors for singles measurements and 3 in x 3 in NaI(Tl) detectors for $\gamma-\gamma$ coincidence studies. The results were used to investigate the mixing between the ground-state band and the gamma-vibrational band of $^{166}$Er. However, it became evident that if $\gamma-\gamma$ coincidence study was to be of more general utility and significance, the severe limitation imposed on the data by the use of NaI(Tl) would have to be removed—principally because of the poor NaI(Tl) resolution and the complexity of the low-energy gamma-ray spectrum.

The present study of the collective states of $^{166}$Er resulted from the 1200 y decay of $^{166}$Ho. These states were interpreted in terms of the rotational energy expressed as a function of $I(I+1)$ (see Eq. (2.3.5)), and of the E2 transitions as given by Eq. (2.3.9), Chapter II. In a boson framework, theoretical descriptions close to the SU(3) limit of the IBA-1 were applied to the level scheme of $^{166}$Er. Experimental measurements were performed using high resolution Ge(Li) detectors, both for singles and $\gamma-\gamma$ coincidences. It is concluded that the feasibility and practicality of the use of these coincidence techniques provide a significant contribution to the understanding of the level structure studied in the $^{166}$Ho decay. Moreover, the nanosecond technique has allowed the measurement of the lifetime of the first excited state of $^{166}$Er using a fast plastic-NaI(Tl) detector combination.

6.2 Experimental procedure and results

6.2a Source preparation

The 10 μCi $^{165}$Ho activity was produced by a 5-day irradiation of 25 mg of holmium metal, enriched to 99.99% in $^{165}$Ho, in the DIDO Reactor of the Isotope Production Unit, Harwell, with a thermal neutron flux of
2×10^{14} \text{ n cm}^{-2} \text{ sec}^{-1}. After irradiation, the sample, sealed in a thin Lucite disk, was removed and allowed to decay for about one month to reduce the 27 h \(^{166}\text{Ho}\) activity (=18 Ci) to a negligible level. During the singles measurements, trace contaminants of \(^{46}\text{Sc}, \(^{182}\text{Ta}, \(^{56}\text{Fe}\) and mainly \(^{160}\text{Tb}\) were found in the source. A previous irradiation using 19 mg of \(\text{Ho}_2\text{O}_3\) powder enriched to 99.9\% of \(^{166}\text{Ho}\) was unsuccessful owing to the presence of Terbium \(^{160}\text{Tb}\) impurities at a level of about 1 at 1% which dominated the gamma-ray spectrum.

6.2b Singles spectra

For the gamma-ray energy and intensity measurements of \(^{166}\text{mHo}\), three sufficiently long run spectra from the 12\% efficient Ge(Li) detector, two long run spectra from the Compton suppression system and three run spectra from the Ge detector (spanning the region 50-350 keV) were analysed (see Table (3.1) for the specifications of the detectors). For the final results an weighted average was taken. The data for efficiency calibration were analysed using the method described in section 3.2. Fig. (6.1) shows a typical gamma-ray single spectrum of \(^{166}\text{mHo}\) obtained with the 12\% efficient Ge(Li) detector. The spectrum includes a number of additional peaks due to trace contaminations of \(^{160}\text{Tb}, \(^{182}\text{Ta}, \(^{46}\text{Sc}, \(^{166}\text{Ho}\) and \(^{56}\text{Fe}\) which were found along with the \(^{166}\text{mHo}\) source. Also included are background peaks of \(^{192}\text{Ir}, \(^{60}\text{Co}, \(^{40}\text{K}\) and \(^{154}\text{Eu}\). A special care was taken to correct for the 1121 keV line of \(^{182}\text{Ta}\) which is superimposed on that of \(^{166}\text{mHo}\).

The energies and intensities of fifty-one gamma-rays from the present work are listed in Table (6.1). Of these six at, 97.7, 351.95, 373.69, 476.29, 895.59 and 1303.09 keV, are new. The intensity results of Sooch et al.\(^{128}\), Sampson\(^{129}\) and Reich et al.\(^{124}\) are also given in this Table for comparison.

6.2c Coincidence spectra

\(\gamma-\gamma\) coincidence spectra were written on four magnetic tapes and eight prominent gamma-rays at, 81, 184, 280, 366, 411, 671, 712 and 810 keV, were taken to establish the decay scheme on the basis of the coincidence between these gates and the rest of the spectrum. In each gate, the background and chance coincidence were subtracted.

Figs. (6.3 to 10) illustrate the corrected coincidence spectra, while Fig. (6.2) shows the total spectrum of \(^{166}\text{mHo}\). A summary of the coincidence results is given in Table (6.2) with transitions not specified by the usual entries VS, S, W and VW are placed in the decay scheme on the basis of energy sum considerations.
Table (6.1). Relative intensities of γ-rays emitted from the decay of $^{166}$Ho

<table>
<thead>
<tr>
<th>Energy $E$(keV)</th>
<th>Intensity related to $I(184)=100$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Present work</td>
</tr>
<tr>
<td>80.598</td>
<td>17.59(2)</td>
</tr>
<tr>
<td>94.842</td>
<td>0.22(1)</td>
</tr>
<tr>
<td>97.7</td>
<td>0.052(9)</td>
</tr>
<tr>
<td>119.34</td>
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</tr>
<tr>
<td>121.308</td>
<td>0.45(1)</td>
</tr>
<tr>
<td>135.51</td>
<td>0.13(1)</td>
</tr>
<tr>
<td>160.41</td>
<td>0.11(2)</td>
</tr>
<tr>
<td>161.663</td>
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</tr>
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<tr>
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<td>0.32(3)</td>
</tr>
<tr>
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<td>0.29(1)</td>
</tr>
<tr>
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<td>1.28(4)</td>
</tr>
<tr>
<td>1282.194</td>
<td>0.253(6)</td>
</tr>
<tr>
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<td>1400.738</td>
<td>0.75(3)</td>
</tr>
<tr>
<td>1427.227</td>
<td>0.69(3)</td>
</tr>
</tbody>
</table>
6.3 Decay scheme

The energy sum relations and the extensive $\gamma-\gamma$ coincidence studies have proved to be a very firm basis for the establishment of the decay scheme of $^{166}$Er resulting from the $\beta^-$-decay of $^{166m}$Ho (see Fig. (6.11)). The $Q_{\beta^-}$ value for the decay is $(1861.5 \pm 2.6)$ keV$^\ast$, and as a result many excited states are populated. It is interesting to note that, in previous works (see, for example, refs. 119, 124, 127, 130), there are no indications for more than two $\beta^-$ decaying states at 1827.57 and 1787.0 keV. However, from the present measured branching ratios, a number of logft values are assigned to previously well known levels and are presented in Table (6.3). In this context, it should be emphasized that the spin and parity assignments for the $^{166}$Er states are made on the basis of relative gamma-ray intensities, logft values and collective structure studies. The reason for the above considerations is the lack of information on the internal-conversion electron intensities which are needed to deduce the multipolarities of the transitions involved.

In the decay scheme, the half life of the first excited state at 81 keV was measured using the experimental arrangement shown in Fig.(3.9). In the slow channel of this figure, the window of the TSCA (ORTEC 551) was set as closely as possible on the 81 keV line which depopulates the 81 keV level to the ground state. The lifetime spectrum, accumulated for over one month period is shown in Fig. (6.12). For the half life calculation, the slope method as described by Meiling and Stray$^57$, and the least squares fitting procedure were employed. They yielded a value of $(1.87 \pm 0.06)$ nsec which is in excellent agreement with the average value of $(1.87 \pm 0.03)$ nsec$^{130}$ deduced from several experimental results (refs. 131-137).

Since the first excited state connects via a strong E2 transition provided by the measured half life value, a firm $I^\pi$ value of $2^+$ results for this state.

The 265 keV level

It can be seen from Table (6.2) that most of the gamma-rays are in coincidence with the 184 keV gate which de-excites this level with the strongest intensity among the transitions of $^{166m}$Ho. Using the rotation-vibration formula of Eq. (2.3.5) (see Discussion) and the position of the $2^+$ of the previous state, an unique $I^\pi$ value of $4^+$ is given to this level.

The 545.46 keV level

This level is a member of the rotation-vibration group originating
Fig. 6.4. Spectrum of Ho-166m in coincidence with 184 keV.
Fig. (6.7). Spectrum of Ho-166m in coincidence with 411 keV.
Fig. (6.8). Spectrum of Ho-166m in coincidence with 671 keV.
Table (6.2). Summary of the $\gamma\gamma$ coincidence results from the decay of $^{166}$Ho.

<table>
<thead>
<tr>
<th>Transition (keV)</th>
<th>Gate (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>81 184 280 366 411 671 712 810</td>
</tr>
<tr>
<td>80.598</td>
<td>S   S   S   S   W   W   S   VS</td>
</tr>
<tr>
<td>119.34</td>
<td>W   VS</td>
</tr>
<tr>
<td>121.308</td>
<td>S   VS</td>
</tr>
<tr>
<td>135.51</td>
<td>VS</td>
</tr>
<tr>
<td>141.03</td>
<td>VS</td>
</tr>
<tr>
<td>160.41</td>
<td>VS   VS   VS   VS   W   VS</td>
</tr>
<tr>
<td>184.403</td>
<td>VS   VS   VS   VS   W   VS</td>
</tr>
<tr>
<td>215.743</td>
<td>S   W   W   W   S   VS</td>
</tr>
<tr>
<td>259.828</td>
<td>S   S   VW   W   VS</td>
</tr>
<tr>
<td>280.455</td>
<td>S   S   VS   VS   VS</td>
</tr>
<tr>
<td>300.741</td>
<td>S   S   VS   VS   VS</td>
</tr>
<tr>
<td>351.947</td>
<td>S   VS</td>
</tr>
<tr>
<td>339.743</td>
<td>VS   S   VS</td>
</tr>
<tr>
<td>365.77</td>
<td>S   S   VS</td>
</tr>
<tr>
<td>410.964</td>
<td>S   S   VS   S   VW   W   VS</td>
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<tr>
<td>451.537</td>
<td>W   S   VS   VS   VS</td>
</tr>
<tr>
<td>464.836</td>
<td>S   S   VS   VS   VS</td>
</tr>
<tr>
<td>496.98</td>
<td>VS   VS</td>
</tr>
<tr>
<td>521.069</td>
<td>VS   VS</td>
</tr>
<tr>
<td>529.856</td>
<td>S   S   VS   W   S   VS</td>
</tr>
<tr>
<td>571.028</td>
<td>W   S   VS   S   VS   VW</td>
</tr>
<tr>
<td>594.402</td>
<td>W   VS</td>
</tr>
<tr>
<td>611.632</td>
<td>VS   VS</td>
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<td>640.132</td>
<td>S   VS</td>
</tr>
<tr>
<td>644.605</td>
<td>VS   VS</td>
</tr>
<tr>
<td>670.518</td>
<td>W   VS   VS   W   VS</td>
</tr>
<tr>
<td>691.247</td>
<td>S   S   VS</td>
</tr>
<tr>
<td>711.693</td>
<td>S   VS   S   VS</td>
</tr>
<tr>
<td>752.288</td>
<td>S   VS   S   VS</td>
</tr>
<tr>
<td>778.845</td>
<td>VS   VS   S   S   VS</td>
</tr>
<tr>
<td>810.309</td>
<td>S   VS   VS   VS</td>
</tr>
<tr>
<td>830.607</td>
<td>W   VS   VS   VS</td>
</tr>
<tr>
<td>951.065</td>
<td>VS   W   VS</td>
</tr>
<tr>
<td>1120.656</td>
<td>VW   W   VS</td>
</tr>
<tr>
<td>1146.974</td>
<td>S   VS</td>
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</tr>
<tr>
<td>1400.738</td>
<td>W   VS</td>
</tr>
<tr>
<td>1427.227</td>
<td>VS   VS</td>
</tr>
</tbody>
</table>
from the $0^+$ state of the ground band and is therefore assigned a spin/parity of $6^+$.  

The 787.07 keV level

This is a well known level from previous investigations with $I^\pi=2^+$ (see ref. 130 and refs. therein), and where it was considered as the band head of the gamma-vibrational band. Evidence for the existence of this level has come from the decay of $^{166}$Tm and $^{166}$Ho, but not from $^{166}$mHo$^{130}$. The present work, while it did not reveal the 73.5, 170.35, 705.31 and 785 keV, that give rise to this level$^{130}$, did however reveal the 521.07 keV gamma-ray in VS coincidence with the 81 keV gate.

The 859.42 keV level

The existence of this level is mainly confirmed by the VS coincidence of the 778.85 keV with the 81 keV gate. Further evidence is provided by the strong coincidence of the 215.74 keV with the 712 keV gate. Collective structure analyses (see Discussion) together with the considerations of the spin and parity of the levels, giving rise to the transitions at 736.62 and 215.24 keV that feed the 859.42 keV level, are consistent only with a $3^+$ assignment.

The 911.23 keV level

This level de-excites via the 365.77 keV transition to the $6^+$ state at 545.46 keV. Two other transitions at 464.84 and 644.61 keV are seen to feed this level from higher states at 1376.16 ($7^+$) and 1555.81 keV ($8^+$), respectively. The logft = 14.28, not reported before, together with the above considerations clearly indicates a spin/parity of $8^+$.

The 956.30 keV level

This level is established from the observation of the 691.25 and 259.83 keV transitions, each in strong coincidence with the 184 keV gate. Further support is provided by the VS coincidence of the 119.34 keV with the 712 keV gate. The grouping of this level into the gamma-band can be inferred from the excellent agreement with the theoretical prediction of Eq. (2.3.5), thus resulting in an unique $4^+$ assignment.

The 1075.38 keV level

This level is suggested by the observation of four transitions by which it could depopulate: to the level at 956.3 keV ($4^+$) by the 119.34 keV transition seen in VS coincidence with the 712 keV gate; to the 859.42 keV level ($3^+$) by the 215.74 keV transition seen in S coincidence with the 81 and 712 keV gates, and in W coincidence with the 184 keV gate; to
Fig. (6.11). Proposed energy level scheme for $^{166}$Er. New transitions and levels are presented by dashed lines. The energies in the level are in keV.
Table (6.3). Deduced logft values for some levels in $^{166}$Er.

<table>
<thead>
<tr>
<th>Energy level (keV)</th>
<th>$E_{\gamma}$ (keV)</th>
<th>$\Sigma I_{\gamma}$</th>
<th>$\Sigma I_{\gamma}$ feed</th>
<th>B.R.%</th>
<th>logft</th>
<th>I $^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>911.23</td>
<td>950.27</td>
<td>3.43</td>
<td>1.91</td>
<td>1.13</td>
<td>14.28</td>
<td>8$^+$</td>
</tr>
<tr>
<td>1216.14</td>
<td>645.36</td>
<td>13.11</td>
<td>10.24</td>
<td>2.13</td>
<td>13.36</td>
<td>6$^+$</td>
</tr>
<tr>
<td>1555.82</td>
<td>305.68</td>
<td>0.51</td>
<td></td>
<td>0.38</td>
<td>13.15</td>
<td>8$^+$</td>
</tr>
<tr>
<td>1568.05</td>
<td>293.45</td>
<td>0.07</td>
<td>0.05</td>
<td>0.01</td>
<td>14.66</td>
<td>4$^-$</td>
</tr>
<tr>
<td>1572.29</td>
<td>289.21</td>
<td>0.18</td>
<td></td>
<td>0.14</td>
<td>13.56</td>
<td>4$^-$</td>
</tr>
<tr>
<td>1665.88</td>
<td>195.62</td>
<td>1.12</td>
<td>0.59</td>
<td>0.39</td>
<td>12.47</td>
<td>5$^-$</td>
</tr>
<tr>
<td>1692.34</td>
<td>169.16</td>
<td>1.05</td>
<td>0.35</td>
<td>0.52</td>
<td>12.18</td>
<td>5$^-$</td>
</tr>
<tr>
<td>1787.0</td>
<td>74.49</td>
<td>104.24</td>
<td></td>
<td>77.13</td>
<td>8.81</td>
<td>6$^-$</td>
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<tr>
<td>1827.57</td>
<td>33.93</td>
<td>24.57</td>
<td></td>
<td>18.18</td>
<td>8.44</td>
<td>6$^-$</td>
</tr>
</tbody>
</table>

This level is observed in de-excitations from transitions at 31.65, 64.58, 305.68, 169.16 keV, to the $\gamma$, $\delta$, $\gamma$ and $\delta$ levels, respectively. The above information agrees with the spin/parity assignment of $2^+$.

The 1555.82 keV level.

This level is established from the observation of three de-excitation transitions at 159.74, 163.51 and 160.30 keV. The level is seen in the activity coincident with the 289.21 keV gate. The entire sequence of levels in the K3, $\gamma$ or gamma-band (see discussion) indicates that this level is the last member of the band and is therefore assigned a spin/parity of $6^+$. This assignment is consistent with the fully observed $\delta^-$ feeding (logft=12.15).

The 1565.88 keV level.

This new level is established from the observation of the newly observed 319.85 keV transition with the 410 keV gate. Three other transitions at 190.07 and 303.2 keV are placed between this level and the D65 and T6$\delta$80 keV levels, respectively. The transition 319.85 keV does not follow the K2, $\gamma$ or gamma-band (see discussion) sequence of $5^-$.

The 1727.20 keV level.

Evidence for the establishment of this level are given from the US coinidence between the 410.07 keV transition and the 650 keV gate.

The 1827.57 keV level.
the 545.46 keV level (6^+) by the 529.86 keV seen in S coincidence with the 81, 184 and 712 keV gates and in VS coincidence with the 280 keV gate; to the 265 keV level (4^+) by the 810.31 keV seen in VS coincidence with the 184 and 712 keV gates, and in S coincidence with the 81 and 411 keV gates. These information about depopulating transitions indicate that this level has 5^+ as spin and parity.

The 1216.14 keV level

A level at this energy is established from the γ-γ coincidence results. In particular, the very strong coincidence of the two depopulating transitions at 951.07 and 670.52 keV with the 184 keV gate. This level is also populated by six transitions including the newly observed at 351.95 and 476.29 keV. Our results for the first time give a log ft value of 13.36, which together with the above information agree with a 6^+ assignment.

The 1376.16 keV level

This level is observed to de-excite through four transitions (830.61, 464.84, 300.74, 160.41 keV) to the 6^+, 8^2, 5^+ and 6^2 levels, respectively. The above information agree with a spin/parity assignment of 2^+.

The 1555.81 keV level

This level is established from the observation of three depopulating transitions at 339.74, 644.61 and 1010.36 keV. The second is seen in VS coincidence with the 280 keV gate. The entire sequence of levels in the K^=2^+ or gamma-band (see Discussion) indicates that this level is the last member of the band and is therefore assigned a spin/parity of 8^+. This assignment is consistent with the newly observed β^- feeding (log ft= 13.15).

The 1568.05 keV level

This new level is established from the VS coincidence of the newly observed 351.95 keV transition with the 671 keV gate. Two other new transitions at 1303.09 and 97.7 keV are placed between this level and the 265 and 1665.88 keV levels, respectively. The log ft = 14.69 which, denotes a second forbidden unique with ΔI=3 and Δn=±1, suggests a spin/parity assignment of 4^-.

The 1572.29 keV level

Evidence for the establishment of this level has come from the VS coincidence between the 496.98 keV transition and the 810 keV gate. The
Fig. (6.12). Lifetime spectrum of the first excited state of $^{166}$Er.
possibility for the existence of a $\beta^-$ feeding to this level could be inferred from our logft = 13.56 which is consistent with a spin/parity assignment of $4^-$. It may be noted that this level had been previously reported from the decay of $^{166}$Tm but with an uncertain spin of 4 (see ref. 130).

The 1596.28 keV level

A level at this energy is indicated by the existence of the 640.13 keV transition in strong coincidence with the 184 keV gate. Taking into account the depopulating transitions and the rotational model calculations (see Discussion), a spin/parity assignment of $4^-$ is possible for this level.

The 1665.88 keV level

A firm $5^-$ assignment can be made to this level taking into account:

(i) the logft = 12.47, newly reported, which indicates that the $\beta^-$ transition feeding this level is second-order non unique with $\Delta I=2$ and $\Delta \pi=\pm$.

(ii) the level is found to agree remarkably well with the theoretical predictions based on the rotation-vibration formula (see later).

(iii) the depopulating transitions.

The 1692.34 keV level

This level (logft = 12.18, newly reported) is established from the $S$ coincidence between the 1146.97 keV and the 280 keV gamma-rays and from another coincidence between the 1427.23 and the 184 keV gamma-rays. These evidences consequently point to an unique $I^\pi$ of $5^-$ for this level.

The 1787 and 1827.57 keV levels

These levels are well established and all data indicate an unique $I^\pi$ assignment of $6^-$ for each of them.

6.4 Discussion

The proposed decay scheme of $^{166}$Er shown in Fig. (6.11) offers a very good opportunity to investigate the applicability of the (IBA-1) to the deformed $^{166}$Er nucleus, and hence to study the specific characteristics of the SU(3) limit of the model. Moreover, the occurrence of the high-spin isomeric state in $^{166}$Ho ($I^\pi=7^-$) provides the opportunity for a detailed study of long sequences of rotational states in $^{166}$Er as expressed by the two-terms expansion of Eq. (2.3.5). As a result, this study
has been viewed in terms of the admixing of the ground-state band and the gamma-vibrational band of $^{166}$Er.

6.4a (IBA-1) calculations and results

The calculations were done using the (IBA-1) computer codes PHINT for energies and FBEM for $B(E2)$ values. The number of bosons implied by the number of valence neutrons and protons in $^{166}$Er is 15 and the resulting number of basis states involved is thus large. The inclusion of an $f$ boson, to generate negative parity states, then produce a severe computational problem in terms of the dimensions of the matrices which must be diagonalized. For this reason, the calculations have been limited to the positive parity bands. Furthermore, the pairing interaction of the (IBA-1) Hamiltonian removes the degeneracy which exists in an energy level spectrum with SU(3) symmetry, that is, the degeneracy of the states with a given $I$ in the gamma and beta-vibrational like bands.

In keeping this approach, a truncated multipole expansion of the (IBA-1) Hamiltonian was used in the calculations, namely (see Eqs. (2.4.2) and (2.4.19)):

$$H = -kQ \cdot Q - k'L \cdot L + k''P \cdot P$$

(6.4.1)

where, once again, the parameters $k$, $k'$ and $k''$ denote the strength of the quadrupole, angular momentum and pairing interactions between bosons, respectively. The completely equivalent form to Eq. (6.4.1) is given, in PHINT, as:

$$H = \frac{QQ}{4} \cdot Q + \frac{E_{LL}}{2} \cdot L + PAIR \cdot P$$

(6.4.2)

where $QQ = -4k$, $E_{LL} = 2k'$ (see Eq. (2.4.40)) and the parameter PAIR was varied to obtain the final calculated sequence of levels. The quantities $k$ and $k'$ were extracted from Eq. (2.4.22), Chapter II.

In the calculations of the $B(E2)$ values, the two parameters $a_2$ and $b_2$ of Eq. (2.4.4) (see Eq. (2.4.42) for their equivalent values in FBEM) were adjusted to approximately reproduce the measured $B(E2, 2 \rightarrow 0)$ for excitations of $2^+$ members of the ground state and gamma-vibrational bands, respectively. The representations in the SU(3) limit to describe these bands are $(\lambda, \mu)$ and $(\lambda-4, \mu)$ with $\lambda$ is the number of valence particles in the nucleus ($\lambda = 30$ in $^{166}$Er), and $\mu = 0$ for the ground-state band and 2 for the gamma-band.

Thus, the above information characterize a perturbed SU(3) which, in the case of $^{166}$Er, lies in the direction of the O(6) limit.

The results of the calculations for the energy levels are shown in
Fig. (6.13). Experimental levels in $^{166}$Er compared with the results of the IBA-1 calculations. The parameters used are the first three ones of Table (6.4).
Fig. (6.13) and compared with experiment. The values of the parameters used are the first three of Table (6.4), whereas the remaining two were needed for the calculations of the $B(E2)$ values presented in Table (6.5) and compared with the present data. All the experimental transitions shown in Table (6.5) are assumed to be pure E2 and the corresponding $B(E2)$ values, except for the $B(E2, 2^+_1 \rightarrow 0^+_1)$ (see below), were deduced from Eqs. (1.4.5) and (1.4.6). The relative intensity values of Eq. (1.4.6) were taken from the present measurements, except for those transitions depopulating the $2^+_2$ level at 786.07 keV, whose relative intensities are quoted in ref. 130. Also taken from this reference are the half life values of the levels in order to compute Eq. (1.4.5).

For the $B(E2, 2^+_1 \rightarrow 0^+_1)$ calculation, the expression$^{138}$:

$$B(E2) = \frac{56.57}{E_\gamma^2 \tau_1^{(\text{exp})}(1+\alpha_T)} (\xi^2 \lambda^2)$$

was used, where $E_\gamma$ is the gamma-ray transition in keV ($=80.59$ keV), $\tau_1^{(\text{exp})}$ is the half life of the $2^+_1$ level, whose value was taken from the present measurement, and $\alpha_T$ is the total conversion coefficient and is given by $\alpha_T=6.9$ (see ref. 138).

It can be seen from Fig. (6.13) that the entire theoretical sequence of states has been well reproduced and a remarkable agreement is found with the experimental results. In the theoretical (IBA-1) spectrum, the bands are labelled by K quantum numbers, $K^\pi=0^+$ for the ground-band and $K^\pi=2^+$ for the gamma-band. The non-inclusion of the $K^\pi=0^+$, beta-band, is due to the fact that no observed levels were detected as members of this band, whose displacement in the SU(3) symmetry above the gamma-band increases with increasing pairing interaction.

Inspection of Table (6.5) shows that the agreement between theory and experiment for transitions originating within the ground-band as well as gamma-ground bands is excellent. On the other hand, although the breaking of the SU(3) symmetry by the pairing term of Eq. (6.4.1) has played a crucial role in giving a much better account of the $B(E2)$ values, so that, the gamma and beta bands are no longer degenerate, the wave functions are not greatly different from those of the rigorous SU(3) limit. Thus, the calculated relative $B(E2)$ values between intraband transitions or between interband ones alone follow closely the predictions of the Alaga rules in a geometrical description$^{139}$. 
Table (6.4). Values of the parameters corresponding to the Variable Names in Programs PHINT and FBEM; these are applicable to the case of $^{165}$Er with $N=15$.

<table>
<thead>
<tr>
<th>QQ (MeV)</th>
<th>ELL (MeV)</th>
<th>PAIR (MeV)</th>
<th>E2SD (eb)</th>
<th>E2DD (eb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.01652</td>
<td>0.01965</td>
<td>0.01562</td>
<td>0.1397</td>
<td>-0.1012</td>
</tr>
</tbody>
</table>

Table (6.5). Comparison between experimental (Exp.) and theoretical $B(E2)_{IBA-1}$ values in $^{166}$Er.

| Transition $I_f^+ ightarrow I_i^+$ | $B(E2)$ Exp. ($e^2b^2$) | $B(E2)_{IBA-1}$ ($e^2b^2$) | $B(E2)_{IBA-1}/B(E2)_W$ | Ratio To | Ratio Exp. (present) | Ratio calc. (IBA-1) |
|-----------------------------------|--------------------------|-----------------------------|--------------------------|----------|----------------------|----------------------|
| $2^+ ightarrow 0^+$              | 1.13±0.04                | 1.16                        | 214.44                   |          |                      |                      |
| $4^+_1 ightarrow 2^+_1$          | 1.68±0.07                | 1.64                        | 311.85                   |          |                      |                      |
| $6^+_1 ightarrow 4^+_1$          | 1.57±0.14                | 1.77                        | 327.41                   |          |                      |                      |
| $8^+_1 ightarrow 6^+_1$          | 1.70±0.17                | 1.89                        | 332.96                   |          |                      |                      |
| $2^+_2 ightarrow 0^+_2$          | 0.028±0.002              | 0.028                       | 5.19                     |          |                      |                      |
| $2^+_2 ightarrow 2^+_2$          | 0.052±0.003              | 0.042                       | 7.78                     |          |                      |                      |
| $2^+_2 ightarrow 4^+_1$          | 0.0050±0.0003            | 0.0025                      | 0.46                     |          |                      |                      |
| $3^-_1 ightarrow 2^-_1$          |                          |                             |                          |          |                      |                      |
| $4^+_2 ightarrow 4^+_1$          |                          |                             |                          |          |                      |                      |
| $5^-_1 ightarrow 6^-_1$          |                          |                             |                          |          |                      |                      |
| $6^+_2 ightarrow 6^+_1$          |                          |                             |                          |          |                      |                      |
| $7^-_1 ightarrow 8^-_1$          |                          |                             |                          |          |                      |                      |

* $B(E2)_W$ in column 4 is the Weisskopf single particle estimate (see Eq. (1.4.12), Chapter I)
6.4b Rotational model calculations and results

(a) Energies

It is seen from Fig. (6.14) that the observed levels in $^{166}$Er can be arranged into rotational sequences characterized by the quantum numbers $K$ and $\pi$. The states presented above each $K^\pi$ are the predicted energies obtained by using the observed energies of the lowest members of the band and applying the two terms expansion of Eq. (2.3.5). The coefficients $A$ and $B$ of the ground-state rotational band $K^\pi=0^+$ have been obtained by fitting the energies of the two lowest excited states at 80.59 and 265 keV by Eq. (2.3.5). For the remaining bands, an alternative form of Eq. (2.3.5) has been employed by replacing $1(1+1)$ by $1(1+1)-K^2$. Such an expansion has proved to be a somewhat more natural one for the treatment of the rotational bands.

The $\gamma$-band with $K^\pi=2^+$ is built on the band-head energy at 786.07 keV with $2^+, I^i$ assignment. The values of the $A=12.54$ keV and $B=-13$ eV coefficients were deduced by fitting Eq. (2.3.5), using the alternative form, to the states at $3^+_1$ (859.42 keV) and $5^+_1$ (1075.38 keV). An excellent agreement between the predicted and observed data is obtained. It is interesting to note that a number of $\gamma$-ray intensities have been measured for transitions between the $\gamma$-g bands (see later) and have been found to be in agreement with the generalized intensity relation of Eq. (2.3.9).

The $K^\pi=2^-$ band is based on the experimental data at 1458 keV (2$^-$) and 1514 keV (3$^-$), summarized in ref. 130. The remaining energies follow very closely the simple formula of Eq. (2.3.5). The coefficients $A$ and $B$ are given for this band as determined from the (3$^-$) state and 5$^-$ state at 1692.34 keV. In the decay scheme of Fig. (6.11), the spin/parity assignments of 4$^- at 1596.28$ keV and 6$^- at 1827.57$ keV were supported by the theoretical predictions of the rotational model as evidenced by the remarkable agreement between theory and experiment achieved in this band.

The newly observed level at 1568.05 keV with 4$^-$ assignment is considered as the band-head of the $K^i=4^-$ rotational band. The $A$ coefficient is the same as deduced for the $K^i=2^-$ band (this is attributed to the value of the moment of inertia for the $K^i=4^-$ which is similar in magnitude to that of the previous band). The value of $B$ is extracted from the simple fit of the experimental level at 1665.88 keV, $I^i=5^-$, to Eq. (2.3.5). The remaining level of this band at 1787 keV is shown to be in excellent agreement with the predicted value at 1780.29 keV. Thus, once again, one stresses the remarkable support for the spin/parity assignments of the observed levels of this band as provided by the very good agreement of
Fig (6.14). Low-lying rotational $K^\pi$ bands in $^{166}$Er compared with experiment. The experimental levels at (2)$^-$ and (3)$^-$ were taken from A. Buryn, Nucl. Data Sheets 14 (1975) 471. The $A$ and $B$ coefficients given in the figure have been determined from the energies of the lowest excited states in each band (see the discussion of the spectrum in the Rotational Model on p. 162).
these levels with the predictions of Eq. (2.3.5).

(b) Analysis of E2 transitions involving the ground-state $K^\pi=0^+$ band and the gamma-vibration band (see Fig. (6.14))

The gamma transitions between the $K^\pi=2^+$ and $K^\pi=0^+$ bands in $^{166}\text{Er}$ are assumed to be predominantly E2 radiation, as suggested by K-selection rule ($\Delta K=2$). The M1 admixtures are, as evidenced by the angular correlation measurements$^{139}$, at most a few percent in amplitude; therefore, no corrections for these admixtures have been made in the calculations of $B(E2)$ values.

The predictions of the geometrical model for relative interband transitions strengths (i.e. Alaga rules$^{139}$) are not precisely followed, and in a number of cases (see, for example, refs. 115, 140, and particularly for the $^{166}\text{Er}$, refs. 33 and 124), the E2 amplitudes show systematic deviations from these rules. These references, as mentioned in the Introduction, support the conclusion that significant improvement in the predictions of the geometrical model for $\gamma$-$g$ transitions is obtained by the introduction of two $\gamma$-$g$ mixing, usually specified in terms of a mixing parameter $Z_\gamma$. However, the mixing can be discussed from two viewpoints as pointed out by ref. 124. The first of these is the conventional one in which the mixing was treated as a direct $\Delta K=2$ mixing of the two bands. The second method is the much more general approach to band mixing analysed in a form of a graphical technique known as a Mikhailov plot$^{34}$.

To this end, the band mixing calculations have been performed according to the usual formalism of the first approach. The formulas for the $\gamma$-$g$ transitions are given in details in ref. 124 and the expressions for specific transitions are tabulated. The results of the calculations are summarized in Table (6.6).

Some of the branching ratios needed for the determination of the mixing parameters $Z_\gamma(2)$ and $Z_\gamma(0)$ are given in Table (6.5). The result of the analysis, thus shows that the branching ratios data for the $\gamma$-$g$ E2 transition in $^{166}\text{Er}$ are well described by a single $Z_\gamma$ parameter with (see Table (6.6)) $Z_\gamma(2)=Z_\gamma(0)=Z_\gamma=0.044\pm0.003$. This value is slightly higher than that of ref. 124, $Z_\gamma=0.041\pm0.002$. Furthermore, the very small difference in the values of $Z_\gamma(2)$ and $Z_\gamma(0)$ of Table (6.6) as evidenced from the branching ratio calculations, suggests that the values of the intrinsic quadrupole moments of the two bands are equal. Indeed, by following the method reported in ref. 124, a value of $Q_0(2)=(7.55\pm0.22)\text{b}$, for the intrinsic quadrupole moment of the $\gamma$-band, was obtained. This
Table 6.6. Summary of the mixing parameters values involving the ground-state band and the $K^\pi=2^+$ band ($\gamma$-vibration) in $^{166}$Er.

<table>
<thead>
<tr>
<th>$Z\gamma(2)^*$ average (Present)</th>
<th>$Z\gamma(0)^*$ average (Present)</th>
<th>$Z\gamma(2)^*$ average (C.W. Reich and J.E. Cline)</th>
<th>$Z\gamma(0)^*$ average $^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0450\pm0.0010$</td>
<td>$0.0424\pm0.0027$</td>
<td>$0.0415\pm0.0023$</td>
<td>$0.0411\pm0.0010$</td>
</tr>
</tbody>
</table>

* $Z\gamma(2)$ and $Z\gamma(0)$ are the mixing parameters, respectively determined from the branching ratios of the interband transitions de-exciting the odd-spin and even-spin members of the band. Their weighted average values were calculated by assuming that the two bands have different intrinsic quadrupole moments.

value is in excellent agreement with that of \( Q_o^0 = (7.52 \pm 0.12) \) b, deduced for the ground-state band from Eq. (2.3.7) using our measured \( B(E2, 2^{-\to 0^+}) \) value listed in Table (6.5).

In what follows, the data are treated according to the Mikhailov approach\(^4\). In this case, the transition strengths are given by Eq. (2.3.9) which can be written as:

\[
B(E2, I_i K=2 \to I_f K=0) =
2 <I_{22-2} I_f 0^+>^2 \{M_1 + M_2 (I_f^2 + 1) - I_i (I_i + 1))\}^2
\]

(6.4.3)

where (see ref. 125):

\[
M_1 = <2 | \mu_{(E2)} | 0> - 4M_2
\]

\[
M_2 = \left[ \frac{15}{8\pi} \right] e Q_o c_\gamma
\]

Here \( c_\gamma \) is the spin-independent parameter which depends on the detailed form of the moments of inertia and \( Q_o \) is the intrinsic quadrupole moment assumed to be the same for the two bands as discussed above. Thus, on this basis, the square root of Eq. (6.4.3) can be represented by a straight line by plotting:

\[
\frac{B(E2, I_i K=2 \to I_f K=0)}{<I_{22-2} I_f 0^+>} \quad \text{against} \quad I_f (I_f + 1) - I_i (I_i + 1)
\]

In the present work, using results of seven well known transitions originating from states of the gamma-band, the Mikhailov plot was obtained and is shown in Fig. (6.15). The line represents a linear least squares fit to the experimental data listed in Table (6.7)(see below, for the explanation of this Table). The fit gives:

\[
M_1 \sqrt{2} = (0.420 \pm 0.005) \text{ eb}
\]

\[
M_2 \sqrt{2} = (0.0097 \pm 0.0042) \text{ eb}
\]

For the present case, where \( Q_o^0 = Q_o^2 \), \( Z_\gamma \) can be given by\(^{124}\):

\[
Z_\gamma = \frac{2M_2}{M_1 - 4M_2}
\]

This expression gives \( Z_\gamma = 0.050 \pm 0.002 \). A comparison involving this value and that of ref. 124, \( Z_\gamma = 0.041 \pm 0.002 \), results in a very small difference of 0.009; the errors are the same.
Fig (6.15). Mikhailov plot for E2 transitions between gamma and ground bands in $^{166}$Er. The line represents a linear least square fit to the experimental data listed in Table (6.7).
Table (6.7). \( \frac{1}{2} \left\{ B(E2; \ I_f=2 \rightarrow I_f=0) \right\}^{1/2} \) values for transitions originating from states of the \( \gamma \)-band in \( {}^{166}\text{Er} \). The denominator denotes a Clebsch-Gordan coefficient.

<table>
<thead>
<tr>
<th>Transition</th>
<th>( I_f(I_f + 1) - I_i(I_i + 1) )</th>
<th>( \frac{1}{2} \left{ B(E2; \ I_i=2 \rightarrow I_f=0) \right}^{1/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2\gamma \rightarrow 0g )</td>
<td>-6</td>
<td>0.38 ± 0.01</td>
</tr>
<tr>
<td>( 2\gamma \rightarrow 2g )</td>
<td>0</td>
<td>0.43 ± 0.01</td>
</tr>
<tr>
<td>( 2\gamma \rightarrow 4g )</td>
<td>14</td>
<td>0.59 ± 0.02</td>
</tr>
<tr>
<td>( 4\gamma \rightarrow 2g )</td>
<td>-14</td>
<td>0.29 ± 0.02</td>
</tr>
<tr>
<td>( 5\gamma \rightarrow 4g )</td>
<td>-10</td>
<td>0.32 ± 0.01</td>
</tr>
<tr>
<td>( 6\gamma \rightarrow 4g )</td>
<td>-22</td>
<td>0.19 ± 0.02</td>
</tr>
<tr>
<td>( 7\gamma \rightarrow 6g )</td>
<td>-14</td>
<td>0.290 ± 0.009</td>
</tr>
<tr>
<td>( 8\gamma \rightarrow 8g )</td>
<td>0</td>
<td>0.41 ± 0.04</td>
</tr>
</tbody>
</table>
If \( Q_0(2) \neq Q_0(0) \), additional terms appear in Eq. (6.4.3). Thus, one can write (see ref. 33, p. 161):

\[
B(E2, I_1, K=2 \rightarrow I_f, K=0) = 2^{<I_1,2I-2|I_f0|^2}{M_1 + M_2[I_f(I_f+1) - I_1(I_1+1)]} (6.4.4)
\]

\[
+ M_3[[I_f(I_f+1) - I_1(I_1+1)]^2 - 2[I_f(I_f+1) - I_1(I_1+1)]}]^2
\]

It has shown that the observed value of \( M_3 \) is negligible, of less than 100 order of magnitude smaller than that of \( M_2 \) (ref. 124). In other words, \( M_3=0 \); thus the assumption \( Q_0(2) \neq Q_0(0) \) is invalid.

For more clarity, the data of Table (6.7) need the following explanations: the first three transitions were easily deduced from the data of Table (6.5); the relevant Clebsch-Gordan coefficients were obtained from ref. 141. The remaining transitions were deduced in the following way:

(i) for each transition, the ratio of intraband to interband transitions was calculated using Eqs. (1.4.5) and (1.4.6) of Chapter I. This allowed for the ratio:

\[
R^2 = \frac{|<2| U'(E2;0)|2>|^2}{2|<0| U'(E2;-2)|2>|^2} \tag{6.4.5}
\]

the expression (see ref. 124):

\[
\frac{B(E2, I_1, K = I_f K)}{B(E2, I_1, 2 = I_f' 0)} = \frac{<I_1K20|I_fK>^2}{<I_1,2I-2|I_f0>^2} \times \frac{|<K| U'(E2;0)|K>|^2}{2|<0| U'(E2;-2)|2>|^2} \times \frac{1}{f(ZI,I_1,I_f')}
\]

where the first factor is the square of Clebsch-Gordan coefficients; the second factor, \( R^2 \), contains the quantities \( <K| U'(E2;0)|K> \) with \( K=2 \), and \( <0| U'(E2;-2)|2> \), which are, respectively, the intrinsic intraband and interband E2 matrix elements. The last factor contains the function \( f(ZI,I_1,I_f') \) whose values for the case \( Q_0(2) \neq Q_0(0) \) are tabulated in Table (6.8).

(ii) having computed the \( R^2 \) value, then the weighted average of all \( R^2 \) was taken.

(iii) finally, by employing this average value and the measured \( B(E2, 2^+ \rightarrow 0^+_1) \) (see Table (6.5)), the intrinsic intraband and
Table (6.8). Correction factor $f(Z_Y, I_i', I_f')$ for the transition probability between the $\gamma$-band and the ground-state band (case where $Q_o(2)\neq Q_o(0)$)*.

<table>
<thead>
<tr>
<th>Initial state</th>
<th>Final state</th>
<th>$f(Z_Y, I_i', I_f')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_i = I-2$</td>
<td>I</td>
<td>( {1 + \frac{1}{2}((I+1)(I+2)Z_Y(2) - I(I-1)Z_Y(0))}^2 )</td>
</tr>
<tr>
<td>$I-1$</td>
<td>I</td>
<td>((1 + (I+2)Z_Y(2))^2 )</td>
</tr>
<tr>
<td>I</td>
<td>I</td>
<td>( {1 + \frac{1}{2}((12-I(I+1)Z_Y(2) + I(I+1)Z_Y(0))}^2 )</td>
</tr>
<tr>
<td>$I+2$</td>
<td>I</td>
<td>( {1 + \frac{1}{2}(I(I-1)Z_Y(2) - (I+1)(I+2)Z_Y(0))}^2 )</td>
</tr>
</tbody>
</table>

* For $Q_o(2) = Q_o(0)$, the values of $f(Z_Y, I_i', I_f')$ are tabulated in the article by P.O. Lipas, Nucl. Phys. 39 (1962) 408.
Table (6.9). Experimental $B(E2)$ ratios for some $^{166}\text{Er}$ transitions compared with the Asymmetry-rotator model, the $\text{SL}(3,R)$ symmetry and the Rotation-Vibration model (RVM)

<table>
<thead>
<tr>
<th>Transition</th>
<th>Ratio To</th>
<th>Ratio Exp. (present)</th>
<th>Ratio Calc. Asymmetry-rotator</th>
<th>Ratio Calc. SL(3,R)symmetry</th>
<th>(RVM)**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\gamma + 0g$</td>
<td>$2\gamma + 2g$</td>
<td>$0.54 \pm 0.05$</td>
<td>$0.53$</td>
<td>$0.31$</td>
<td>$0.14$</td>
</tr>
<tr>
<td>$3\gamma + 2g$</td>
<td>$3\gamma + 4g$</td>
<td>$1.37 \pm 0.07$</td>
<td>$1.26$</td>
<td>$1.46$</td>
<td>$1.39$</td>
</tr>
<tr>
<td>$4\gamma + 2g$</td>
<td>$4\gamma + 4g$</td>
<td>$0.154 \pm 0.008$</td>
<td>$0.17$</td>
<td>$0.21$</td>
<td>$0.02$</td>
</tr>
<tr>
<td>$5\gamma + 6g$</td>
<td>$5\gamma + 4g$</td>
<td>$1.37 \pm 0.04$</td>
<td>$1.7$</td>
<td>$1.33$</td>
<td>$0.82$</td>
</tr>
<tr>
<td>$6\gamma + 6g$</td>
<td>$6\gamma + 4g$</td>
<td>$11.7 \pm 0.4$</td>
<td>$16$</td>
<td>$7.47$</td>
<td>$21.78$</td>
</tr>
<tr>
<td>$5\gamma + 8g$</td>
<td>$5\gamma + 6g$</td>
<td>$2.22 \pm 0.08$</td>
<td>$3$</td>
<td>$2.13$</td>
<td>$0.87$</td>
</tr>
<tr>
<td>$8\gamma + 8g$</td>
<td>$8\gamma + 6g$</td>
<td>$18.65 \pm 2.64$</td>
<td>$-$</td>
<td>$23.47$</td>
<td>$54.43$</td>
</tr>
</tbody>
</table>


**Values determined from the expression: $R = \frac{B(E2; I_i K = 2 \rightarrow I_f K = 0)}{B(E2; I_i K = 2 \rightarrow I_f K = 0)} \left[ \frac{E_y}{E_{y'}} \right]^{2} \left\langle I_i 2 \ 2-2 | I_f 0 \right|^2$ (L. Weaker and L.C. Biedenharm, Phys. Lett. 32B (1970) 326) resulted from the SL(3,R) symmetry study. This symmetry is the non-compact real form of SU(3). In the above expression, $E_y$ or $E_{y'}$ is the energy of the photon emitted in the transition. This energy is taken from experiment because SL(3,R) is not viewed as a symmetry of the nuclear Hamiltonian, but rather a set of algebraic constraints on the transition operators.

***The $B(E2; I_i \rightarrow I_f)\gamma \rightarrow \gamma s$-bands values are determined from the expression $B(E2; I_i \rightarrow I_f) = A \frac{2I_f + 1}{2I_i + 1} (I_f 2I_i | 022)^2 \chi^2 (1 - 2\alpha)^2$ taken from J.M. Eisenberg and W. Greiner: in Nuclear Models, Nuclear Theory, Vol.1 (North-Holland, Amsterdam, 1970), Chapter 6. In this expression, $A = \frac{32R_0^2}{4\pi}$ ($R_0 = 1.2A^{1/3}$ fm), $\beta_0 = 0.33$, $(I_f 2I_i | 022)$ denotes a Clebsch-Gordan coefficient, $\chi = \frac{E_f}{E_y}$ ($\varepsilon = \frac{\alpha}{3}$ which is taken from the experimental energy of the first rotational band. $\chi$ is the equilibrium moment of inertia. $E_y$ is the $\gamma$-vibrational energy which is fitted to the energy of the second $2^+$ state) and $\alpha = 0.36 \beta_0$. Note that, in the calculations of $B(E2)$ ratios, only the second and fourth factors of the above expression are taken into account, the others are constants for all ratios.
interband matrix elements were extracted. The former enabled the intra-band E2 transition to be calculated from the expression:\(^{124}\):

\[ B(\text{E2}) = \frac{\langle I, 2^+ \rangle}{\langle I, 2^+ \rangle} = \frac{\langle I, 2^+ \rangle}{\langle I, 2^+ \rangle} = \frac{\langle I, 2^+ \rangle}{\langle I, 2^+ \rangle} \]

By using this expression, the value of the interband E2 transition can thus be deduced from (i).

To end this sub-section, a comparison of the measured B(E2) ratios for transitions originating from states of the gamma-band with those ratios predicted from the asymmetric-rotor model, the SL(3,R) symmetry and the rotation-vibration model (RVM), is briefly indicated in Table (6.9).

6.5 Concluding remarks

In the present study, the authentication of the level scheme of \(^{166}\)Er can be attributed to two essential elements in the investigations. These are as follows:

(a) use of high-resolution spectrometers to resolve the gamma-ray spectra observed both in singles and coincidence measurements and give very precise energy and intensity results to allow the level scheme to be established with confidence.

(b) the decay of \(^{166}\)Ho high-spin isomer (I^T=7^+) has made possible studies of transitions concerning a wide range of I values with the results that rotational bands comprising several members emerge (Fig. (6.14)).

Aside from the fact that an impressive agreement of the data with the (IBA-1) model in the region of deformed nuclei is obtained, there seems to be an apparent dominance of the gamma-decay branch from the \(\gamma\) to the \(\gamma\) band over that to the ground. This feature was singled out from the results of the study of the collective states of \(^{168}\)Er within the framework of the SU(3) limit of the (IBA-1) (see ref. 125). Its appearance in the \(^{166}\)Er level scheme is provided by the decay of the \(2^+\) state at 1159 keV which was observed in the \(^{168}\)Er(p,t)\(^{166}\)Er reaction\(^{130}\). Table (6.10) summarizes the results of the (IBA-1) B(E2) predictions for transitions depopulating the 1159 keV level considered as a member of the \(\beta\)-band. The experimental B(E2) values were taken from ref. 142.
Table (6.10). Experimental and IBA-1 predicted \( B(E2) \) values for \( \gamma \)-ray transitions depopulating the 1159\,keV level considered as a member of the \( \beta \)-band in \( ^{166}\text{Er} \) (see ref.142).

<table>
<thead>
<tr>
<th>Initial state ( K_i, I_i )</th>
<th>Final state ( K_f, I_f )</th>
<th>( B(E2) ) values ( (\varepsilon^2 b^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0, 2^+ ) 1159</td>
<td>( 0, 0^+ ) 0</td>
<td>( 0.00084 \pm 0.00012 )</td>
</tr>
<tr>
<td></td>
<td>( 0, 2^+ ) 80.59</td>
<td>( 0.00096 \pm 0.00020 )</td>
</tr>
<tr>
<td></td>
<td>( 0, 4^+ ) 265.00</td>
<td>( 0.0029 )</td>
</tr>
<tr>
<td></td>
<td>( 2, 2^+ ) 786.07</td>
<td>( 0.0045 \pm 0.0009 )</td>
</tr>
</tbody>
</table>

1(a). The \( ^{166}\text{Er} \) Matrix

The \( ^{166}\text{Er} \) nucleus with 68 protons and 98 neutrons has been extensively studied as a nearly spherical, or almost spherical, nucleus. The nuclei with a similar \( N \) = \( Z \) value have been found to exhibit special characteristics. In this case, it is possible to fit a set of \( ^{166}\text{Er} \) levels where the excitation energy provides a measure of the spin of the state. The ground state of \( ^{166}\text{Er} \) is \( 2^+ \) and 1.38\,MeV. The next \( 2^+ \) level is at 1.52\,MeV, followed by a \( 0^+ \) level at 3.67\,MeV. The levels at 5.98 and 8.41\,MeV are assigned as analogues of the 5.97 and 8.41\,MeV levels in \( ^{164}\text{Er} \) and \( ^{166}\text{Er} \), respectively. The 8.41\,MeV level is also considered to be the \( 2^+ \) level at 8.41\,MeV, which is assumed to be the second excited state of the \( 2^+ \)-band. The \( 4^+ \) level at 7.3\,MeV is considered to be the third excited state of the \( 2^+ \)-band. The \( 2^+ \) and \( 4^+ \) levels are considered to be the first and second members of the \( 2^+ \)-band, respectively. Support for this assignment is provided by the fact that the \( E2 \) transitions between the levels are observed in the \( ^{166}\text{Er} \) decay.
CHAPTER VII
SUMMARY AND CONCLUSIONS

In the present work, comprehensive studies of the gamma radiations from the three radioactive isotopes $^{110m}$Ag, $^{192m}$Ir, $^{166m}$Ho were undertaken. Ge(Li) detectors used in conjunction with the powerful (DPDCS) allowed greater numbers of transitions and levels than previously determined to be included in the decay schemes of the daughter nuclei.

These additional levels and transitions led to the construction of new comprehensive schemes of $^{110}$Cd, $^{192}$Pt, $^{192}$Os and $^{166}$Er. An investigation of the nuclear properties of these isotopes on the basis of the IBM enabled new aspects of the collective nuclear dynamics to be revealed.

The most salient and interesting points are summarized below.

(a) The $^{110}$Cd nucleus

The $^{110}$Cd nucleus with 48 protons and 62 neutrons has been characterized as nearly spherical in shape, partially on the basis of a vibrational like spectrum. The whole energy range of the $^{110m}$Ag spectrum (116-2004 keV) was covered; sixty-eight transitions were observed, of which three, at 116.15, 241.84 and 598.44 keV, are new. On the basis of energy sum considerations, a new energy level at 2194.94 keV with $3/4^+$ assignment is placed in the level scheme of $^{110}$Cd as built up with the aid of $\gamma-\gamma$ coincidence measurements. The spin/parity assignments of $0^+, 1^+, 2^+$ for, respectively, the 2287.26, 2662.31 and 2793.37 keV levels, as taken from the work of G.Mallet et al., were ruled out by the present investigations. Instead the following assignments are made:

(i) $2^+$ to the 2287.26 keV level due to the observation of $\beta^-$-feeding as shown by $\log f t = 6.68$ (see ref. 69), and to the fact that this level can be fixed as the $(n,v) = (3,1)$ member of the 3-phonon quintet.

(ii) $4^+$ to the 2662.31 keV level, given on the basis of the $\log f t = 11.29$ which denotes a second-forbidden non-unique $\beta^-$-transition ($\Delta I = 2, \Delta \pi = +$; no change of parity). Also there are $\gamma$-transitions to $2^+$, $(3,4)^+$ levels.

(iii) $4^+/5^-$ to the level at 2793.37 keV, resulting from the $\log f t = 9.7$ which implies a first-forbidden non-unique $\beta^-$-transition with $\Delta I = 0,1$ and $\Delta \pi = -$. Further support for this assignment is provided by the spins of the levels giving rise to the transitions feeding and depopulating the above state.
The anharmonicity of the nearly vibrational energy spectrum of $^{110}$Cd has been viewed in terms of an interaction between pairs of bosons in the IBA-1. Therefore, because of this interaction, states belonging to the same phonon number are no longer degenerate. This is implied, in the case of $^{110}$Cd, by the experimental identification of the triplet for n=2 phonons. However, there is another way of viewing the anharmonicity effects and this is in terms of an expansion of the vibrational Hamiltonian (see Eq. (2.3.11)) in powers of the vibrational amplitudes and their time derivatives. In this case, there are restrictions on the pattern of energy shifts within the n=2 triplet (see ref. 33, p.543),

$$E(n=2,I=0) - 2E(n=1,I=2) = \frac{7}{2}(E(n=2,I=4) - 2E(n=1,I=2))$$

(7.1)

Using the values from the energy level scheme of $^{110}$Cd, Fig. (4.14) (see also Table (4.5), the relations (7.1) are found to be violated.

On the basis of the IBA-1 calculations, the perturbed SU(5) symmetry has accounted rather well for the B(E2) values when 1st order perturbation theory was applied to the Hamiltonian of Eq. (2.4.6). The experimental B(E2) ratios show, in most of the cases, quite large deviations from those of ref. 73 in which they were assumed to arise from pure E2 radiation. The introduction of the mixing parameter $\delta$ due to M1 components Eq. (4.1) into the B(E2) calculations, thus modifies the results of the B(E2) ratios. As a consequence, a good agreement has been found between these ratios and those deduced from the IBA-1 predictions as can be seen from Table (4.6).

Finally, the observation of the static quadrupole moment of the first excited $2^+$ state, as exhibited by the value $|Q_2|=0.47$ eb, is characteristic of a system with anharmonical behaviour as this value differs significantly from that of zero as would be the case in the harmonic approximation.

(b) The $^{192}$Pt and $^{192}$Os nuclei

The collective states of $^{192}$Pt and $^{192}$Os nuclei have played a significant role in exploiting the application of the IBA-1 model in order to further our understanding of the collective behaviour of nuclei in the mass region just below A=200.

In this work, the $^{192}$Pt and $^{192}$Os level schemes are provided by studying the 74 d decay of $^{192}$Ir and by following similar experimental procedures as were applied to the study of $^{110}$Cd. Here the main
results are given.

- The $^{192}$Pt nucleus

Singles measurements of the $^{192}$Ir decay reveal the existence of forty-nine gamma-ray transitions. Of these ten new at 176.27, 213.17, 267.04, 362.32, 452.68, 624.88, 773.1, 1147.05, 1237.51 and 1413.59 keV were shown by $\gamma-\gamma$ coincidence results to belong to the level scheme of $^{192}$Pt. Thus, and a result of the $\gamma-\gamma$ coincidence measurements, three new levels at 1146.95, 1237.35 and 1413.76 keV are established. Furthermore, the present data do not agree with the $4^+$ assignment for the 1200.8 keV level as reported by refs. 104, 106, 11. Instead, a firm $2^+$ assignment is given by taking into account the $\log t=8.18$ and the $E2+M1$ radiation of the 416.47 keV transition which de-excites the above state to the well known $4^+$ state at 784.39 keV.

In the framework of the IBA-1 model, the experimental situation for the $^{192}$Pt nucleus was described by a smooth transition from $O(6)$ to SU(3) symmetry. As can be seen from Table (5.4), Chapter V, the boson-boson interaction expressed by the QQ term, which is dominant in the energy spectra of nuclei with SU(3) symmetry over the pairing-pairing term interaction characterizing spectra of the $O(6)$ symmetry, is not significantly different from zero. This implies that the observed energies of the states in $^{192}$Pt nucleus follow quite closely the predictions of the $O(6)$ limit (see Eq. (2.4.35) and Table (5.6)). One can further see that the $B(E2)$ values (see Table (5.5)) are not strongly influenced by the introduction of the E2DD parameter (-0.105) which is zero in the $O(6)$ limit. In other words, the $^{192}$Pt nucleus preserves most of the characteristics of the rigorous $O(6)$ limit.

A striking point, however, which receives a simple quantitative interpretation in terms of the $B(E2)$ ratios is the case of the newly observed levels at 1146.95 and 1413.76 keV. The presently available data on the experimental ratio $R = B(E2, 2^+_1 1413.76 \rightarrow 2^+_1 1146.95)/B(E2, 2^+_1 1413.76 \rightarrow 0^+_g) = 2539 \pm 421$ comes close to agreeing with the IBA-1 predictions of $R = 3482$, and lends support to the $2^+$ assignment for each of the above level. This, therefore, gives a good evidence for the experimental appearance of the $0^+_g - 2^+_1 1146.95 - 2^+_1 1413.76$ sequence whose counterpart in the IBA-1 is a characteristic feature of the $O(6)$ limit (see Fig. (5.13), part (a)). However, the observation of the above $0^+_g$ state in the level scheme of $^{192}$Pt nucleus remains an open question.
Two newly observed transitions from the $^{192}$Ir decay are placed in the level scheme of $^{192}$Os. These are the 325.38 keV which de-excites the 905.23 keV level ($I^\pi=2^+$) to that at 580.37 keV ($I^\pi=4^+$), and the 904.52 keV transition which decays from the former level to that of the ground. These transitions together with the observation of a $\beta^-$-feeding to the 905.23 keV state ($\log f_t=10.16$) gave a potential support for the unique $2^+$ assignment. This contradicts the previously assigned $4^+$ spin and parity.

A potentially valuable source of evidence on the behaviour of the $^{192}$Os nucleus is provided by the study of the $O(6) \rightarrow SU(3)$ transition in the IBA-1. The decay scheme of $^{192}$Os presented in Fig. (5.13), part (b), reveals the $0^+\rightarrow 2^+\rightarrow 2^+$ sequence (with strong cascading $E2$ transitions within the group, see Table (5.5), part (b)), which characterizes the energy spectrum in the $O(6)$ limit. Furthermore, the rather large QQ value (0.0085) of Table (5.4) compared with that of the $^{192}$Pt case (0.0001) provides an immediate explanation of the fact that there is a transition taking place between the $O(6)$ and $SU(3)$ limits. In such a transition, the newly assigned $2^+$ value to the 905.23 keV state appears to be consistent with the $2^+$ value predicted by the IBA-1 calculations and specified by the quantum numbers $(\sigma,\tau,\nu_\lambda)=(6,1,0)$ as can be seen from Table (5.6).

(c) The $^{166}$Er nucleus

The spectrum of $^{166}$Er shown in Fig. (6.11) has been constructed from the measured gamma-transition energies of $^{166}$Ho following the capture of thermal neutrons in $^{165}$Ho metal. The singles measurements reveal the existence of fifty-one gamma-rays; of these six at 97.7, 351.95, 373.69, 476.29, 895.59 and 1303.09 keV are new. It has thus been possible to establish the new level at 1568.05 keV in the level scheme of $^{166}$Er, and to identify it as the band-head of the $K^\pi=4^-$ rotational band (see Fig. (6.14)).

These gamma decay measurements also provided a large body of data on gamma-ray branching ratios, which was interpreted in terms of the $B(E2)$ values given by the $SU(3)$ limit of the IBA-1 as well as being consistent with the intensity rules implied by the rotational description of Eq. (2.3.9) (see Tables (6.5) and (6.9)). The latter analysis further provided information on the mixing of the $K^\pi=0^+$ and $K^\pi=2^+$ bands. The former study confirmed the significance of the introduction of the pairing-pairing term into the Hamiltonian of the $SU(3)$ limit: this is
revealed from the analysis of Eqs. (6.4.2) and (2.4.42) through the use of the programs PHINT and FBEM. It should be stressed, however, that the one parameter, the strength of the pairing interaction, is seen to account for the complete set of levels and branching ratios; the other four constants in the Hamiltonian and E2 operator having been fixed from simple prescriptions.

Another potentially significant conclusion is the prediction of the dominance of the gamma decay branch from the $\beta$ to the $\gamma$ band, over that to the ground, which is in agreement with the experimental data as can be seen from Table (6.10). The only disagreement is the observed $B(E2)$ value for the transition from the 1159 keV level of the $\beta$-band to that at 786.07 keV of the $\gamma$-band, which is appreciably smaller (by a factor of 6) than the theoretical IBA-1 estimate. This, therefore, might be attributed to additional (as yet unidentified) M1 admixture effects.

The phenomenological rotational Hamiltonian whose associated energy is expressed in a power series in $I(I+1)$ has been shown to give rise to spectra in excellent agreement with data (Fig. (6.14)). What is truly remarkable in this figure is the fact that the rotational model generates an essentially perfect spectrum with an additional explicit term proportional to $I^2(I+1)^2$. The coefficients $A$ and $B$ in the expansion (see Eq. (2.3.5)) have been obtained by requiring a fit to the observed energies of the lowest members of the bands. It is seen that, besides the usual ground-state rotational band of the even-even $^{166}$Er nucleus ($K^\pi=0^+$), three other bands have been identified in the spectrum, having $K^\pi=2^+$, $2^-$ and $4^+$, respectively. The last band is built on the newly observed level at 1568.05 keV.

An analysis of the relative $E2$ transition strengths between the $K^\pi=2^+$ ($\gamma$-band) and $K^\pi=0^+$ ($g$-band) reveals a systematic deviation from the simple Alaga rules\textsuperscript{139}. It is well known, however, that the degree of mixing of the wave functions of the ground-state band causes significant correction to the relative $E2$ transitions. This mixing has been discussed (Chapter VI) from two viewpoints. The first of these is the conventional one which resulted from the introduction of a correction factor $f(Z\gamma,I^f,I^l)$ to the transition probabilities $B(E2)$ derived by Alaga et al.\textsuperscript{139}. It appears, therefore, that the mixing between the two-bands seems to be reasonably well explained by a single value of the mixing parameter $Z\gamma=0.044\pm0.003$ which agrees rather well with the value of ref. 124 (0.041\pm0.002). Furthermore, this value indicates that the intrinsic quadrupole moments of these two bands are equal.

The other viewpoint is the more general approach to the problem
of mixing of γ and g bands and is referred to as the Mikhailov plot\textsuperscript{34}. The results obtained from this method are presented graphically in Fig. (6.15) and are summarized in Table (6.7). They demonstrated that this approach is capable of giving extraordinarily good fits to inter-band E2 transitions for the \(^{166}\text{Er}\) nucleus with a \(Z\gamma=0.050\pm0.002\) which is slightly greater than \(Z\gamma\) of the first approach. It is interesting to note that the \(Z\gamma\), which gives a measure of the rotation-vibration interaction, is equal to zero in the simple Bohr and Mottelson approach, since the rotational and intrinsic motions of the nucleus, considered as an axially symmetric rotor, do not disturb each other.

Despite the success of the simple rotational and the IBA-1 models employed in the case of \(^{166}\text{Er}\), one must emphasize a very important point that requires further study in connection with the experimental deduction of the internal conversion coefficients. In this study, the spin and parity assignments of the \(^{166}\text{Er}\) states have been made on the basis of relative gamma-ray intensities, \(\log f\) values and collective model analyses expressed in terms of the rotation-vibration formula (see Eq. (2.3.5)). It was felt, however, that a more detailed study on the relative intensities of internal conversion electrons may provide additional information on the structure of the different modes involved in the \(^{166}\text{Er}\) level scheme.
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