

abstract of this thesis.

PART I contains a description of the writer's further work on the absolute measurement of electrical resistance by the new rotating coil method of Hettlein and Helmholtz. The preliminary investigation was published in Verh. Phys. Ges. Zurich, 1907.

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is the fundamental equation, $Z = K\omega$, K , a constant, depends upon the mutual inductance between the coils.

THESES ON "The Absolute Measurement of Electrical Resistance by New Rotating Coil Methods."

It is shown to be accurately determined at $\theta = 19.5^\circ$ and with the E.G. Balls in the Helmholtz position. To satisfy these conditions and to render M/C constant at the rotor edges, it is necessary to construct a differential rotor consisting of two coplanar coils in series and opposition.

The theory of this rotor necessitates the calculation of mutual inductances and Tables of Functions, given in Appendix II. With such a rotor, resistances of the order of one ohm are measured absolutely.

ω , the angular velocity of revolution, is measured by the method described in Appendix I.

Part II is a preliminary account of the writer's investigation, suggested by Dr. Hettlein, on the absolute measurement of resistance by the method, proposed by him, of the rotating coil method of Hettlein and Helmholtz, applied to the absolute measurement of resistance by the method, proposed by him, of the rotating coil method of Hettlein and Helmholtz.

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Abstract of this thesis.

PART I contains a description of the writer's further work on the absolute measurement of electrical resistance by the new rotating coil method of Nettleton and Balls. The preliminary investigation was published in 1934.

In the fundamental equation, $R = K\omega$, K , a constant, depends upon M/θ , where M is the mutual inductance between rotor and stator at angle θ of rotation from the conjugate position where the brush is at the commutator sector's edge. K is shown to be most accurately determined at $\theta = 19.5^\circ$ and with the field coils in the Helmholtz position. To satisfy these conditions and to render M/θ constant at the sector edges, it is necessary to construct a differential rotor consisting of two coplanar coils in series and opposition.

The theory of this rotor necessitated the calculation of mutual inductances and Tables of Functions, given in Appendix II. With such a rotor, resistances of the order of one ohm are measured absolutely.

ω above, is the angular velocity of revolution and is measured by the method described in Appendix I.

Part II is a preliminary account of the writer's investigation, suggested by Dr Nettleton, on the absolute measurement of resistance by the method, proposed by Rosa in 1909, using the e.m.f. of a commutating generator. No data

PREFACE.

by any form of this method have ever been published.

The basic equation of this method, $R=4.n.M$, has the great advantage that a frequency and the maximum mutual inductance only are required.

A remarkable flat of maximum mutual inductance between rotor and stator was obtained, permitting commutation without loss of generated e.m.f.

Rosa's difficulties, due to self-inductance and to break at commutation in the steady e.m.f. drawn from the resistance to be measured, are overcome by a short-circuiting device covering each commutation.

This method in which resistance is measured directly in terms of a mutual inductance and a period; this method contains certain features in common with the British Association, the Carey Foster, and the Lippmann methods.

This thesis contains an account of further unpublished work, carried out by the writer, on the suggestion of Dr. Nettleton.

Part I contains an account of the various improvements which have been made subsequent to the publication of the preliminary account in 1935, in the original rotating coil method of Nettleton and Halls.

PREFACE.

In recent years at Birkbeck College, under the direction of Dr.H.R.Nettleton, attention has been given to various methods of measuring the ohm absolutely.

Nettleton and Llewellyn (Proc.Phys.Soc.44.195.1932.) have completely transformed Weber's method of damping and Nettleton and Balls (Proc.Phys.Soc.45.545.1933.) have described a simple laboratory modification of Campbell's two phase alternating current method and further have introduced a simple air transformer method.

Again, Nettleton and Balls (Proc.Phys.Soc.47.54.1935.) have given a preliminary account of a new rotating coil method in which resistance is measured directly in terms of a mutual inductance and a period; this method contains certain features in common with the British Association, the Carey Foster, and the Lippmann methods.

This thesis contains an account of further unpublished work, carried out by the writer, on the suggestion of Dr. Nettleton.

Part I contains an account of the various improvements which have been made subsequent to the publication of the preliminary account in 1935, in the original rotating coil method of Nettleton and Balls.

PART I.

A Further Investigation of the Absolute Measurement of
Electrical Resistance by a New Rotating Coil Method.

§ 1. Introduction.

In the preliminary account of this method (Proc. Phys. Soc. 47.54.1935.), it is shown that a coil, spinning about a horizontal diameter, may be so placed between two larger fixed field coils, having their planes horizontal, that the quotient M/θ , of the mutual inductance between the rotor and the fixed coils in series and in conjunction, and the angle of displacement of the rotor from the position of zero inductance, may be rendered sensibly constant over a considerable range in the neighbourhood of a desired angle of displacement. This constancy of M/θ in the neighbourhood of an angle θ enables resistance to be measured absolutely in the following way.

The axis of rotation is arranged to lie in the magnetic meridian and the earth's vertical flux through the rotor is neutralised by a small current passing through large compensating coils in the Helmholtz position. A steady current C of about 1 ampere is passed through the twin field coils and through a variable manganin resistance which is adjusted until the e.m.f. across its potential leads is balanced by that across the commutating sectors of the rotating coil when spinning with a constant angular

An important feature of the original method is the determination of an angle θ from the relationship $\text{Sine } \theta = M_{\theta} / M_{\text{max}}$ by means of an "angle coil" which obeys a Sine Law strictly when M_{θ} and M_{max} are mutual inductances between this angle coil and the twin field coils. M_{θ} in the neighbourhood of the sector edges is arranged to differ by only a few micro-henries from M , the corresponding inductance between the rotor and the field coils. Under such circumstances, any errors of the order possible in the calibration of the mutual inductometer used for the measurements are rendered quite negligible as regards the determination of the ratio M/M_{θ} and the accuracy of measurement of M/θ and the constant K is solely dependent on the sensibility and on the accuracy of the inductometer at the angle coil reading M_{max} which, in the preliminary test, was some 7000 μH .

§ 2. Outline of new investigation.

In designing a new rotor, preliminary consideration has been given to the accuracy with which the constant K can be measured with the aid of an angle coil. It is shown in § 3 below that the Sine Law is obeyed most closely when the angle coil, wound on a conical former, is used with the twin field coils in the Helmholtz position and when the angle θ is about 19.5° -- a result which fixes the best angle for the commutating sectors at 39° .

The fixing of the twin field coils in the Helmholtz

of the system... resolution.
Special system... for the attainment of
constant... is controlled by
taking... the measurement of
these... coefficient.

The single layer

The neutral
radius, a , super
layered solenoid
from the coil

Figure 1. overleaf

the expression

$$N_0 = G_1 \sum_{n=1}^{\infty} \dots$$

where Z_n is the
 a ; R_n is its
 ψ is the galvanic
origin of symmet
and ψ are with

circles A and B of
(Fig. 1.) and a circle
length, $2l$, rotated
 ψ , θ , is given by

$$T_n(S, \theta) \dots (2)$$

is first kind of order
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 n together at the
of S and s , ψ , θ ,

1. single

If the angle... small radius, the
series converges... only the first few terms are
of importance. They lead to the expressions:

$$N_0/N_1 = S_0 + K_1 R(S_0) + K_2 R(S_0) \dots (3)$$

$$N_0/N_2 = 1 + K_0 + K_1 \dots (4)$$

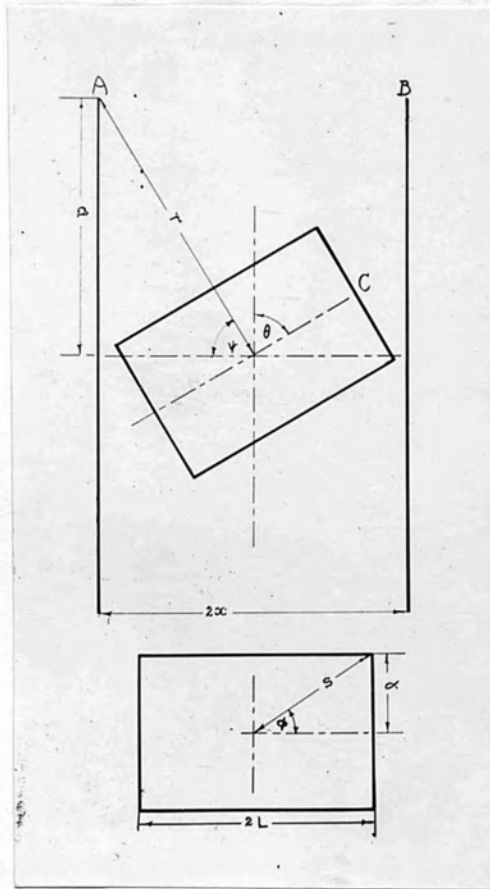


Figure 1.

by the rotor over some 80° in a complete revolution.

Special provision has been made for the attainment of constant speeds by a synchronised motor controlled by tuning fork and bar frequencies and for the measurement of these frequencies at the time of the experiment.

§ 3. Theory of the angle coil.

The single layered solenoid:

The mutual inductance between twin circles A and B of radius, a, separated by a distance, 2x, (Fig.1.) and a single layered solenoid, C, of radius, α, and length, 2L, rotated from the conjugate position by an angle, θ, is given by the expression:-

$$M_{\theta} = Gq \sum_{n=1}^{\infty} \frac{2}{\pi(n+1)(n+2)} \left(\frac{s}{r}\right)^{n-1} P'_n(\cos \psi) \frac{P'_{n+1}(\cos \phi)}{\cos \phi} P_n(\sin \theta) \dots \dots \dots (2)$$

where P_n is the Legendre function of the first kind of order n : P'_n is its differential coefficient : n is an odd integer ; G is the galvanometer constant of A and B together at the origin of symmetry : q is the total area of C and s, r, φ, and ψ are sufficiently defined in the figure.

If the angle solenoid, C, is of small radius, the series converges rapidly and only the first few terms are of importance. They lead to the expressions:-

$$M_{\theta} / Gq = \sin \theta + K_1 P_3(\sin \theta) + K_2 P_5(\sin \theta) + \dots \dots \dots (3)$$

and $M_{max} / Gq = 1 + K_1 + K_2 + \dots \dots \dots (4)$

where

$$K_1 = \frac{3.4}{2} \cdot \frac{1}{r^2} \left[x^2 - \frac{a^2}{4} \right] \left[\frac{L^2}{3} - \frac{\alpha^2}{4} \right] \text{-----(5)}$$

and

$$K_2 = \frac{5.6}{2} \cdot \frac{1}{r^2} \left[x^4 - \frac{3}{2} x^2 a^2 + \frac{a^4}{8} \right] \left[\frac{L^4}{5} - \frac{L^2 \alpha^2}{2} + \frac{\alpha^4}{8} \right] \text{-----(6)}$$

It is evident from the equations that the first step which must be taken towards the attainment of the relationship $\text{Sine } \theta = M_\theta / M_{\max}$ is to ensure that K_1 is zero. This is done in the first place by making L^2 / α^2 equal, as nearly as possible, to $3/4$, thus fixing $\text{Cos}^2 \phi = 3/7$; and in the second place by making $x^2 = a^2 / 4$, thus choosing the Helmholtz position, the criterion of which is very sharp. Having thus made K_1 doubly vanish, the values of K_2 and K_3 , etc. may be estimated and the deviations from the Sine Law examined, when at once it becomes apparent that, if θ is such that $P_5(\text{Sin } \theta) = \text{Sin } \theta$, the effect of K_2 vanishes. This occurs when $\text{Sine } \theta = 1/3$ and thus, as the effect of higher terms is small, in the neighbourhood of $\theta = 19^\circ 28'$, we may expect the Sine Law to be very closely followed and a change of sign in the discrepancy between M_θ / M_{\max} and $\text{Sine } \theta$.

Taking a specific example for an angle coil C, in which $\alpha/a = 1/5$; $L^2 / \alpha^2 = 3/4$; $x^2 / a^2 = 1/4$, we have $s^2 / r^2 = 7/125$; $\text{Cos}^2 \phi = 3/7$; $\text{Cos}^2 \psi = 1/5$. By using the values of $P'_n(\text{Cos } \psi)$ and $P'_n(\text{Cos } \phi) / \text{Cos } \phi$ in appendix 2, we find:-

$$K_1 = 0.0$$

$$K_2 = 0.000253440$$

$$K_3 = 0.000002343$$

$$K_4 = 0.000000104$$

Further by using Tallqvist's values of $P'_n(\text{Sin } \theta)$

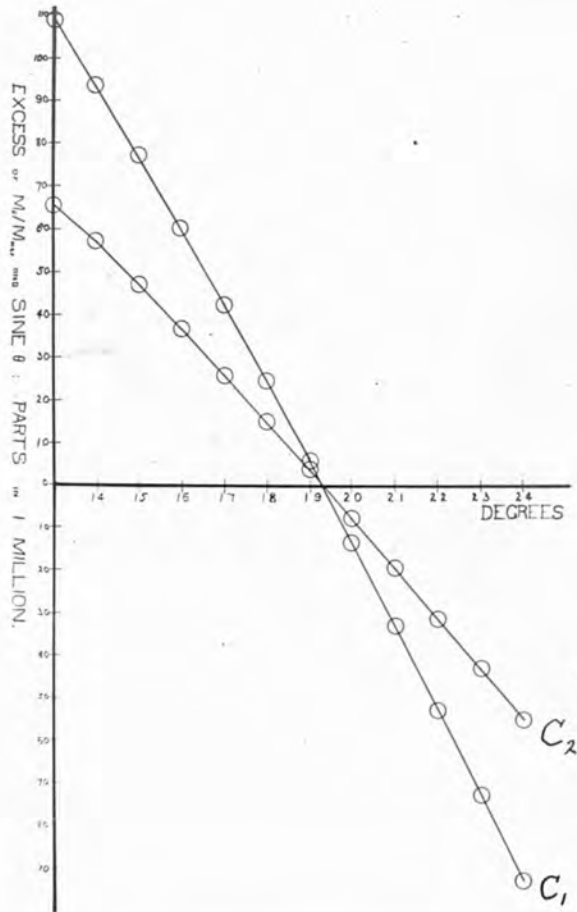


Figure II relating to Table I.

up to $n=7$ and Hayashi's values of $F_n(\sin \theta)$, the ratio M_θ/M_{\max} has been calculated for each degree from $\theta=13^\circ$ to $\theta=24^\circ$ inclusive. The difference between M_θ/M_{\max} and $\sin \theta$ is shown in Table I and graphically in Fig. II: the difference vanishes at $\theta=19.29^\circ$.

Table I. Differences in parts in a million between M_θ/M_{\max} and $\sin \theta$ for angle coils C, and C₂ as specified below with the twin field coils A and B in the Helmholtz position.

Angle in degrees	Excess for C,	Excess for C ₂
13	109.3	66.0
14	93.7	56.7
15	77.4	46.9
16	60.3	36.6
17	42.6	26.0
18	24.4	15.0
19	5.6	3.7
20	-13.6	-7.8
21	-33.1	-19.5
22	-52.8	-31.3
23	-72.7	-43.3
24	-92.7	-55.3

Effect of coils possessing multiple layers:

Both the twin coils A and B and the angle solenoid C

will in practice be multiple layered. The effect on the large field coils is best examined by a theorem due to Maxwell (cf. Andrew Gray, Absolute Measurements in Electricity and Magnetism, pp. 219, 220). If a coil of rectangular cross section has axial breadth, $2b$, and radial depth, $2d$, and P_0 is any term in the expression of action of any kind at a point, due to the central circular filament of the coil, then, to a first order, the corresponding term for the action of the entire coil is given by :-

$$\bar{P} = P_0 + \frac{b^2}{6} \cdot \frac{\partial^2 P_0}{\partial x^2} + \frac{d^2}{6} \cdot \frac{\partial^2 P_0}{\partial a^2} \quad \text{-----}(7)$$

The various terms of the series Eq. 2 can thus be corrected for multiple layers. Any correction of the fundamental term in $Gq \cdot \sin \theta$ is without effect on the Sine Law and the only consideration of any importance is the effect on the second term $GqK, P_0(\sin \theta)$. Applying Eq. 7 to this term, it can readily be shown that in the Helmholtz position for the central circular filaments, K , for the multiple layered combination no longer vanishes but has a residual value, K_1^m , given by :-

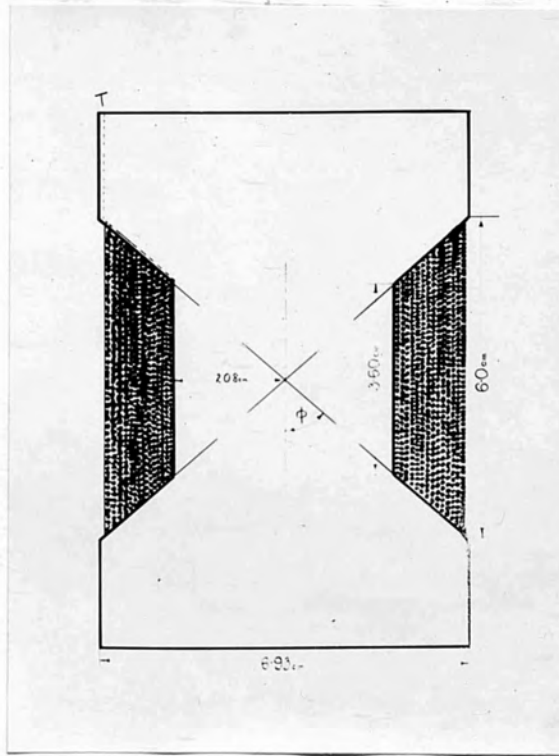
$$K_1^m = \frac{10}{256} \cdot \frac{a^2}{r^2} \cdot \frac{\alpha^2}{a^2} \left\{ 1 - \frac{4}{3} \cdot \frac{L^2}{\alpha^2} \right\} \left\{ \frac{36b^2 - 31d^2}{a^2} \right\} \quad \text{-----}(8)$$

which doubly vanishes when $L^2/\alpha^2 = 3/4$ and $b/d = 0.928$.

Taking $a^2/r^2 = 4/5$; $\alpha/a = 1/5$; $b = d = 1.8$; $a = 16.4$; and

assuming that compensation is sufficiently imperfect for

$\left(1 - \frac{4}{3} \frac{L^2}{\alpha^2} \right) = \frac{1}{10}$, we have that $K_1^m = 3.8 \times 10^{-6}$, corresponding



Figure

Figure III. The Conical Angle Coil.

some 8 parts in a million in the Sine Law at 19.5° . The correction in the third term in K_2 can be shown to vanish when $b/d = 0.957$ in the Helmholtz position and is always a very small percentage. Further, as the influence of K_2 is itself minute in the neighbourhood of $\sin^{-1} \frac{1}{3}$, this correction need not be considered.

The influence of multiple layers on the angle coil C is a totally different matter. This coil of relatively small mean radius must be wound with many layers in order that $M_{p_0} = Gq$ may attain 10,000 μH . By far the best solution is to use a conical former so that L^2/α^2 shall equal $3/4$ very closely for every layer.

The multiple layered conical angle coil.

The boxwood former of the actual conical angle coil used is shown in Fig. III. It was wound with 22 layers of d.s.c. copper wire of s.w.g. 24, the number of turns per layer rising from 60 to 90 and the total number of turns amounting to 1616. The radius of the first layer is $\alpha_1 = 2.1$ cms. and of the last layer, $\alpha_2 = 3.3$ cms. The leads emerge side by side at T. The theoretical value of the angle ϕ is given by $\cos^2 \phi = 3/7$ and the semi angle of the cone by $90 - \phi = 40^\circ 53' 36''$. Thus the semi length L of a layer divided by its radius α is everywhere given by $L/\alpha = \cot \phi = \sqrt{3}/2$.

Let there be n_1 turns per cm. of axial length and n_2 turns per cm. of radial depth; then the number of turns

in the windings having radii lying between α and $\alpha + d\alpha$ is:-

$$dn = 2 n_1 n_2 \cot \phi \cdot \alpha \cdot d\alpha \quad \text{-----}(9)$$

and the area of whose turns between α and $\alpha + d\alpha$ is:-

$$dq = 2\pi n_1 n_2 \cot \phi \cdot \alpha \cdot d\alpha \quad \text{-----}(10)$$

Thus the total turns on the former having radii ranging from the extreme values α_1 and α_2 is:-

$$N = n_1 n_2 \cot \phi (\alpha_2^2 - \alpha_1^2) \quad \text{-----}(11)$$

and the total area of all these turns is:-

$$q = \pi n_1 n_2 \cot \phi (\alpha_2^4 - \alpha_1^4) / 2 \quad \text{-----}(12)$$

the effective mean radius for the area being $\sqrt{(\alpha_2^2 - \alpha_1^2) / 2}$

Whence applying Eq.2. and writing $\alpha = s \cdot \sin \phi$ and $a = r \cdot \sin \psi$,

we have for the mutual inductance of all turns of the angle coil lying between radii, α and $\alpha + d\alpha$, and the field coils:-

$$dM_\theta = G \cdot 2\pi n_1 n_2 \cot \phi \cdot \alpha^3 \cdot d\alpha \sum_{n=1}^{\infty} k_{\frac{n-1}{2}} \left(\frac{\alpha}{a}\right)^{n-1} P_n(\sin \theta) \quad \text{-----}(13)$$

where n is an odd integer and

$$k_{\frac{n-1}{2}} = \frac{2}{n(n+1)(n+2)} \left(\frac{\sin \psi}{\sin \phi}\right)^{n-1} P_n'(\cos \psi) \frac{P_n'(\cos \phi)}{\cos \phi} \quad \text{-----}(14)$$

and is independent of α .

Thus for the mutual inductance between the whole conical coil and the field coils we have;-

$$\begin{aligned} M_\theta &= G q \cdot \pi n_1 n_2 \cot \phi \int_{\alpha_1}^{\alpha_2} \sum_{n=1}^{\infty} k_{\frac{n-1}{2}} \frac{1}{a^{n-1}} P_n(\sin \theta) \cdot \alpha^{n+2} d\alpha \\ &= G q \sum_{n=1}^{\infty} k_{\frac{n-1}{2}} \left(\frac{\alpha_2}{a}\right)^{n-1} P_n(\sin \theta) \left[\frac{4}{n+3} \cdot \frac{1 - \left(\frac{\alpha_1}{\alpha_2}\right)^{n+3}}{1 - \left(\frac{\alpha_1}{\alpha_2}\right)^4} \right] \quad \text{-----}(15) \end{aligned}$$

and putting $\alpha = \alpha_2$ in the values of K_n of Eq.3. we have:-

$$K_{\frac{n-1}{2}} = \left(\frac{\alpha_2}{\alpha}\right)^{n-1} k_{\frac{n-1}{2}} \quad \text{-----}(16)$$

Thus we obtain the series:-

$$\begin{aligned}
M_{\theta}/Gq &= \sin \theta + K_1 \left[\frac{2}{3} \cdot \frac{1 - (\alpha_1/\alpha_2)^6}{1 - (\alpha_1/\alpha_2)^4} \right] P_3(\sin \theta) & 13. \\
&+ K_3 \left[\frac{2}{4} \cdot \frac{1 - (\alpha_1/\alpha_2)^8}{1 - (\alpha_1/\alpha_2)^4} \right] P_5(\sin \theta) \\
&+ K_5 \left[\frac{2}{5} \cdot \frac{1 - (\alpha_1/\alpha_2)^{10}}{1 - (\alpha_1/\alpha_2)^4} \right] P_7(\sin \theta) + \dots \quad (17)
\end{aligned}$$

The fundamental term of the series 3 has thus no correction. The second term doubly vanishes as both $P_3'(\cos \psi)$ and $P_4'(\cos \phi)/\cos \phi$ in K, equal zero. The most important correcting factor is that for K_3 , viz. $\frac{1}{2} [1 + (\alpha_1/\alpha_2)^4]$.

Taking a specific example for a conical angle coil C_2 , to which the actual angle solenoid of Fig. III approximates closely, we put $\alpha_2/a = 1/5$; $\alpha_1/\alpha_2 = 2/3$; $\cos^2 \phi = 3/7$; $\cos^2 \psi = 1/5$, giving the previous values of K_2, K_3 , and K_4 deduced with the aid of Appendix 2. The correcting factors are $97/162$, $4642/9477$, and $8113/19683$ respectively. The differences between the quotient M_{θ}/M_{\max} and $\sin \theta$ have been calculated for every degree between $\theta = 13^\circ$ and $\theta = 24^\circ$ inclusive and are given in Table I and shown graphically in Fig. II. The difference is seen to vanish at $\theta = 19^\circ 32'$.

The impossibility of rigorous test of the Sine Law.

It should be pointed out that it is not possible by direct measurement of θ and of the quotient M_{θ}/M_{\max} with an inductometer to verify the Sine Law to the accuracy anything like that to which the constant K of Eq. 1. may be measured by the artifice described in § 1 which eliminates errors in the calibration. It is just as legitimate, however, to rely on a calculated law of mutual inductance as it is to accept the calculated values of the primary standards at

the national laboratories.

Accuracy of measurement of θ and of M/θ .

The angle coil which is sensitive to two seconds of arc may be used as a goniometer and the accuracy with which $\sin \theta = M_\theta / M_{\max}$ may be measured is dependent on the accuracy of the calibration of the inductometer in nominal microhenries. At 19.5° , this ratio may be measured to about one part in 4000. The accuracy of measurement of M/θ is much greater. If $\sin \theta = x$, we have:-

$$M/\theta = M/\sin^{-1}x = M/x \times x/\sin^{-1}x = M/M_\theta \cdot M_{\max} \cdot x/\sin^{-1}x \quad \text{---(18)}$$

and if $y = x/\sin^{-1}x$

$$dy/y = dx/x \cdot (1 - \tan \theta/\theta) \quad \text{-----(19)}$$

Thus at 19.5° , $dy/y = 0.04049 \cdot dx/x$ and an error of 1 part in 4000 due to imperfect calibration in the measurement of $\sin \theta$ produces an error of 1 part in 98800 in the measurement of M/θ and the constant K is dependent in practice on the accuracy of the inductometer at the reading M_{\max} .

Symmetry of setting of the angle coil.

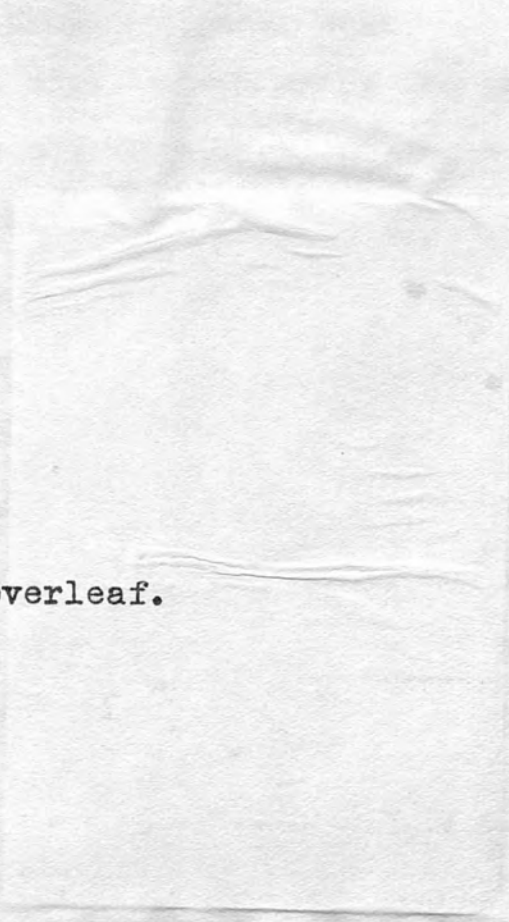
Advantage is taken of the use of the twin field coils to set both the angle coil and the rotor symmetrically about the origin. With the twin coils in opposition, the mutual inductance must be zero at all angles. A technique is readily developed by which displacement of the angle coil about the axis of rotation or of the twin field coils may be detected and remedied. Slight residual effects are

eliminated to ... the method used of
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Criterion of the

If the ... of one coil
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Figure IV overleaf.



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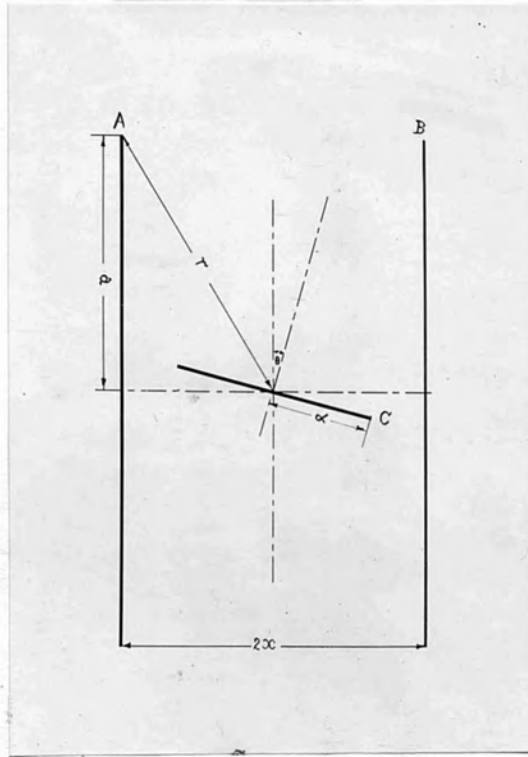


Figure IV.

eliminated in the determination of K by the method used of taking readings of M and θ in all four quadrants.

Criterion of the Helmholtz position.

If the angle coil is placed at the centre of one coil when the other is out of action, and G_s be the mean value of the maximum inductance for the single coils while G_t is the corresponding inductance when the angle coil is at the origin of symmetry of the combination, the separation $2x$ is given by the relation:-

$$x^2/a^2 = (2G_s/G_t)^{2/3} - 1 \quad \text{-----(20)}$$

which affords a very simple method of determining x/a .

Since in the Helmholtz position, $x^2/a^2 = 1/4$, this position will be attained when the separation is such that $G_t = 1.431 G_s$.

§ 4. Theory of the Differential Rotor.

The mutual inductance between a simple rotor C Fig.IV. of radius α and the twin field coils, A and B, of radius a , is given by:-

$$M/G_q = \sum_{n=1}^{\infty} (-z)^{\frac{n-1}{2}} \frac{2}{n(n+1)} \frac{1.3.5 \dots n}{2.4.6 \dots (n-1)} P'_n(\cos \psi) \cdot P_n(\sin \theta) \quad \text{-----(21)}$$

when n is an odd positive integer, $z = \alpha^2/r^2$, G and q have their previous meanings of Eq.2. and r , ψ , and θ are sufficiently defined by the figure.

A useful expansion of this expression has been given by Nettleton and Llewellyn (Proc. Phys. Soc. 44, 200, 1932.) viz.

$$\begin{aligned} \frac{M}{Gq} = & \theta \sum_{n=1}^{\infty} (z)^{\frac{n-1}{2}} P'_n(\cos \psi) \frac{2}{n(n+1)} \cdot \frac{1^2 \cdot 3^2 \cdot 5^2 \dots n^2}{2^2 \cdot 4^2 \cdot 6^2 \dots (n-1)^2} \\ & - \frac{\theta^3}{6} \sum_{n=1}^{\infty} (z)^{\frac{n-1}{2}} P'_n(\cos \psi) \frac{2}{n(n+1)} \cdot \frac{1^2 \cdot 3^2 \cdot 5^2 \dots n^2}{2^2 \cdot 4^2 \cdot 6^2 \dots (n-1)^2} [1 + (n+2)(n-1)] \\ & + \frac{\theta^5}{120} \sum_{n=0}^{\infty} (z)^{\frac{n-1}{2}} P'_n(\cos \psi) \frac{2}{n(n+1)} \cdot \frac{1^2 \cdot 3^2 \cdot 5^2 \dots n^2}{2^2 \cdot 4^2 \cdot 6^2 \dots (n-1)^2} [1 + 10(n+2)(n-1) + (n+4)(n+2)(n-1)(n-3)] \\ & - \frac{\theta^7}{5040} \sum_{n=1}^{\infty} (z)^{\frac{n-1}{2}} P'_n(\cos \psi) \cdot \frac{2}{n(n+1)} \cdot \frac{1^2 \cdot 3^2 \cdot 5^2 \dots n^2}{2^2 \cdot 4^2 \cdot 6^2 \dots (n-1)^2} \times \\ & [1 + 91(n+2)(n-1) + 35(n+4)(n+2)(n-1)(n-3) + (n+6)(n+4)(n+2)(n-1)(n-3)(n-5)] \end{aligned} \quad (22)$$

and these authors have tabulated the logarithms of the various functions of n involved and of $P'_n(\cos \psi)$ up to $n=31$, and are thus able to show, when $\alpha/a = 0.5461$ and $x/a = 0.19438$, the term in θ^3 vanishes rendering M/θ remarkably constant in the neighbourhood of $\theta = 0$. Further, when the ratio α/a has a fixed value lying between the limits $0.587 > \alpha/a > 0.52$, the term in θ^3 may be made to vanish by adjustment of the distance x , defining the separation of the twin field coils. A slight increase in α or a , and a small decrease in x will result in the term θ^3 acquiring a small positive coefficient while the terms in θ^5 and θ^7 remain negative.

Utilising these facts, Nettleton and Balls have attained a law of inductance for their first rotor represented by the equation:-

$$M = K \cdot \theta + A \cdot \theta^3 - B \cdot \theta^5 \quad (23)$$

which gives constancy of M/θ at positions given by $\theta^2 = A/2B$. Though this constancy is all to be desired, the twin field coils were in the position $x/a = 0.212$ which is not satisfactory from the standpoint of the accuracy of the angle coil.

Accordingly, the writer has made some calculations on the mutual inductance between a rotor of radius α , placed between field coils in the Helmholtz position. This has necessitated the evaluation of $P_n'(\cos \psi)$ up to an order $n = 41$ for the Helmholtz angle ψ given by $\cos^2 \psi = 1/5$, as well as an extension of the tables of Nettleton and Llewellyn from $n = 31$ to $n = 41$ of various functions of n . The necessary constants are given in Appendix 2. Using Eq. 22. (and Eq. 21. for the two cases where the maximum mutual inductance has been evaluated) the results obtained are given in Table 2.

Table 2: Mutual inductance between a rotor of radius α and twin field coils of radius a , in the Helmholtz position for different values of $z = \alpha^2/r^2$. The corresponding values of α/a are given. θ is the displacement in radians from the position of zero inductance.

$z = 0 : \alpha/a = 0$: limiting Sine Law for comparison.

$$M = Gq(\theta - 0.166,6 \theta^3 + 0.008,333 \theta^5 - 0.000,198,412 \theta^7) \quad \text{-----} (24)$$

$$M_{\max} = Gq.$$

$$z = 0.13 : \alpha/a = 0.4031 :$$

$$M = Gq(0.993,753,446,5 \theta - 0.140,032,567 \theta^3 \\ - 0.018,826,57 \theta^5 + 0.007,287,2 \theta^7) \text{ ----(25)}$$

$$M_{\max} = Gq(0.995,763,299)$$

$$z = 0.2 : \alpha/a = 0.5 :$$

$$M = Gq(0.986,246,092 \theta - 0.112,117,335 \theta^3 \\ - 0.037,790,90 \theta^5 + 0.003,810,9 \theta^7) \text{ ----(26)}$$

$$z = 0.25 : \alpha/a = 0.5590 :$$

$$M = Gq(0.979,591,305 \theta - 0.089,874,90 \theta^3 \\ - 0.047,681,72 \theta^5 - 0.002,530,0 \theta^7) \text{ ----(27)}$$

$$z = 0.35 : \alpha/a = 0.6614 :$$

$$M = Gq(0.963,901,963 \theta - 0.044,481,3 \theta^3 \\ - 0.055,64 \theta^5 - 0.017,8 \theta^7) \text{ -----(28)}$$

$$M_{\max} = Gq(0.964,039,90)$$

This table reveals the slow fall in the coefficient of Gq for M_{\max} with rise of α and of the coefficient of θ in the series M/Gq . The coefficient of θ^3 rises from $-1/6$ to -0.0445 for the highest value of α which will permit a safe clearance for rotation from the field coil formers. Further calculation shows that when $z = 0.40$ and $\alpha/a = 0.7071$, the coefficient of θ is $-0.022,745$ and by extrapolation, it becomes apparent that this term cannot vanish and change sign until z reaches about 0.45 and α/a is 0.75 . The term in θ^5 becomes more strongly negative as α rises but its value for $z = 0.40$ is 0.0542 which shows that its numerical value

has passed a maximum.

The conclusion is thus reached that, with the field coils in the Helmholtz position, a simple rotor is not possible for the term, θ^3 , does not acquire a positive value until $\alpha/a = 3/4$ --- a value far too large to allow clearance for the rotor unless the whole apparatus was greatly enlarged at very considerable expense.

Attainment of the law, M/θ constant at $\sin^{-1} 1/3$ by a differential rotor between field coils in the Helmholtz position.

An examination of Table 2. reveals that a suitable rotor may be obtained by connecting in opposition an outer coil of area q_2 and a concentric inner coil of smaller area q_1 . Writing for the inner coil $M_1/\theta = Gq_1 f_1$, and for the outer coil $M_2/\theta = Gq_2 f_2$, where f_1 and f_2 are functions of θ , and putting $q_2 = n \cdot q_1$, we have for the resulting inductance M:-

$$M/\theta = (M_2 - M_1)/\theta = Gq_1 (n \cdot f_2 - f_1) \text{ ----- (29)}$$

By choice of n , the quotient M/θ may be rendered constant in the neighbourhood of $\theta = 19.5^\circ$ and by choice of q_1 , the resultant inductance in this region may be adjusted to that of the angle coil.

To illustrate this, the values of f_1 and f_2 for various angles θ are given in Table 3. below for the inner and outer coils, having the values of z equal to 0.13 and 0.35 respectively, these values being calculated from the

corresponding equations in Table 2. Values for $n.f_2 - f_1$ can thus be readily compiled. When $n = 2.50$, the desired result is obtained and the figures are given in the last column. The corresponding equation for the differential coil is readily found to be:-

$$M/\theta = Gq_1 (1.416,001,461 + 0.028,829,35 \theta^2 \\ - 0.120,273,4 \theta^4 - 0.051,787,2 \theta^6) \text{ -----(30)}$$

where q_1 is the area of the smaller coil having $z = 0.13$, The area of the larger coil, having $z = 0.35$, is $q_2 = 2.5 q_1$. The values of $n.f_2 - f_1$ in Table 3. have been checked from Eq.30. and additional values have been computed for angles of 0.325 , 0.335 , and 0.345 radians.

Table 3. Values of $f_2 = M_2/Gq_2\theta$ and $f_1 = M_1/Gq_1\theta$ at various angles θ for outer and inner coils of a differential rotor. The last column gives the corresponding values of $n.f_2 - f_1 = M/Gq\theta$ for the combination where $q_2 = n.q_1$ and $n = 2.5$.

θ radians	Outer coil f_2	Inner coil f_1	Differential coil; $2.5f_2 - f_1$
0.0	0.963,901,96	0.993,753,45	1.416,001,5
0.1	0.963,451,57	0.992,351,245	1.416,277,7
0.2	0.962,032,55	0.988,122,49	1.416,958,9
0.3	0.959,434,99	0.981,003,33	1.417,584,1
0.31	0.959,097,665	0.980,128,91	1.417,615,3
0.32	0.958,744,54	0.979,224,52	1.417,636,8
0.325			1.417,643,7
0.33	0.958,375,115	0.978,290,04	1.417,647,7
0.335			1.417,648,9
0.34	0.957,988,89	0.977,325,35	1.417,646,9
0.345			1.417,641,6
0.35	0.957,585,335	0.976,330,33	1.417,633,0
0.36	0.957,163,90	0.975,304,87	1.417,604,9
0.37	0.956,724,02	0.974,248,84	1.417,561,2

This table reveals that over the range $18^\circ 36'$ to $19^\circ 46'$, the extreme variations of M/θ for the differential coil are about 5 parts in a million.

The position of the maximum value of M/θ may be raised or lowered by slightly decreasing or increasing the value of n .

Figures V and VI overleaf.

Figure V. The layout of the apparatus.

Figure VI. The rotor, Commutator, and Brush.

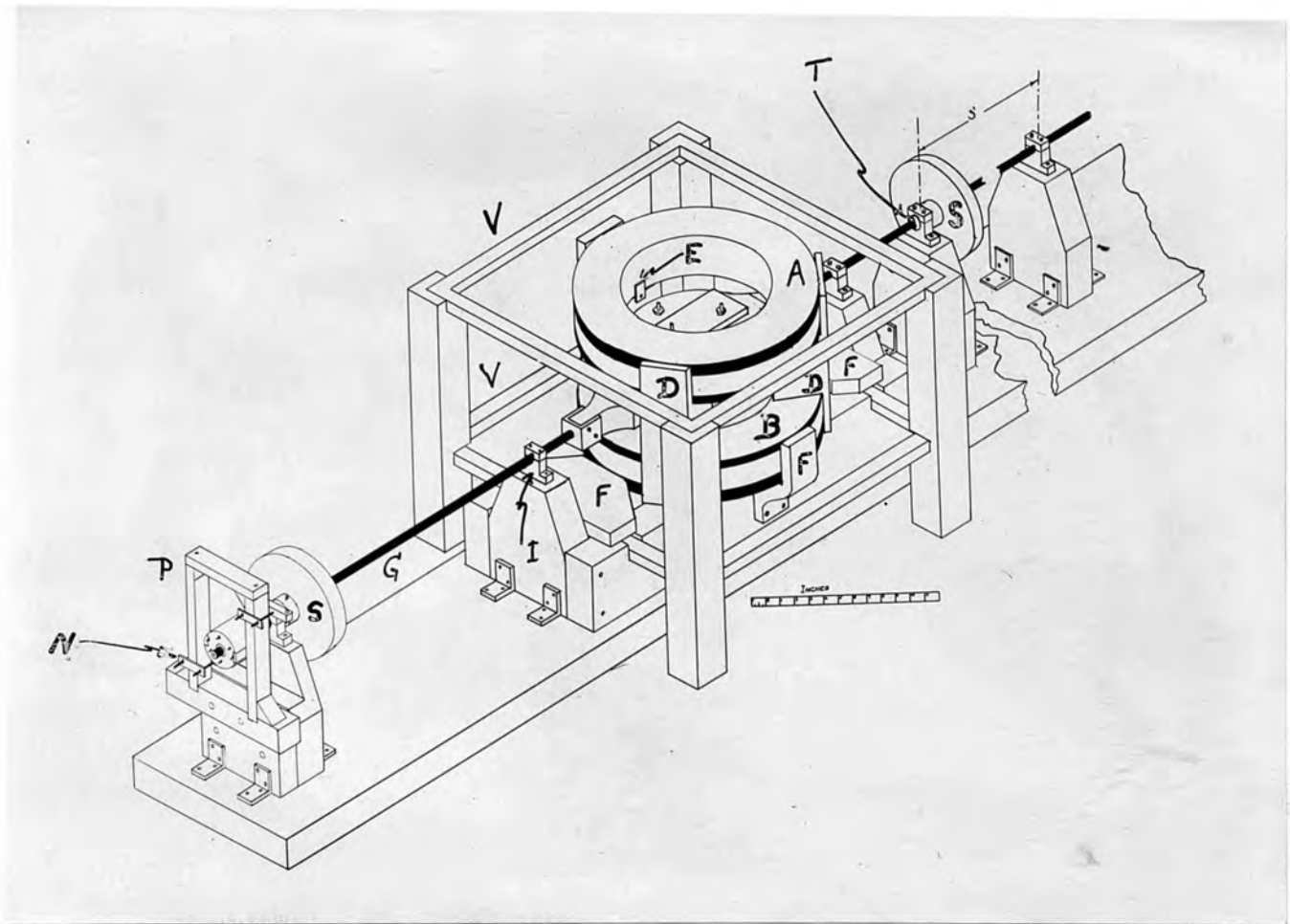


Figure V. The Layout of the Apparatus.

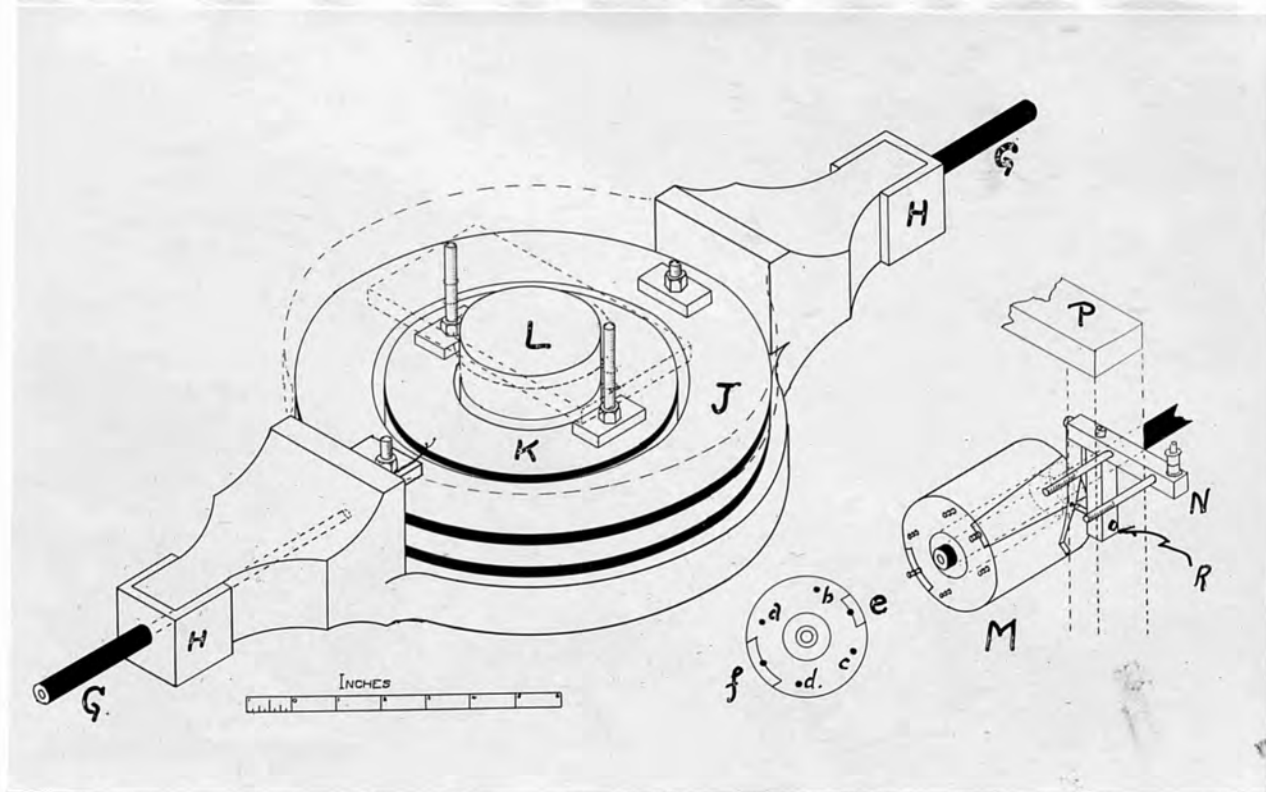


Figure VI. The Rotor, Commutator, and Brush.

§ 5. The Apparatus.

The general layout of the apparatus is similar to that of the original paper and will be followed from Figs.V and VI.

The twin field coils, A and B, were constructed of dextronite. The channels of radial depth and axial depth 3.6 cms. were each filled with 504 turns of d.s.c. copper wire, of s.w.g. 16, the mean diameter of windings being 32.8 cms. The coils were separated by distance pieces D and were held firmly and coaxially by brass buttons E so that they accurately satisfy the inductance criterion of the Helmholtz position given in § 3. The twin coil unit was firmly supported and locked in position by locking pieces F, such that its axis of symmetry is the axis of rotation of the shaft G.

The differential rotor, Fig.VI, consists of an oak holder, held by the shaft G and by two stout brass channel pieces H. The whole was made to rotate truly upon its bearings I. The two rotating coils, J and K, which constitute the differential rotor, were adjusted and fixed by brass bolts and nuts on the bed so that they were both concentric and coplanar, and rotate symmetrically about the axis of the shaft. The method of setting is dealt with in § 6.

The outer coil J consisted of 189 turns of d.s.c. copper wire, of s.w.g. 24, wound in 12 layers in a channel 1 cm. wide and of internal diameter, 21 cms. The inner coil K

consisted of 198 turns in 12 layers of the same wire in an identical channel but of internal diameter 12.5 cms. The total resistance of the rotor was 15.9 ohms.

These coils were locked in position by brass bolts and nuts and boxwood plates. Over the whole is an oaken cover to make the rotating unit balanced for rotations.

Through a hole in this rotating unit of diameter 7.5 cms. was fixed the conical angle coil L, Figs. III and VI. which was held firmly by boxwood plates, secured by brass bolts and nuts. It was set symmetrically with regard to the axis of rotation, and coaxially and concentrically with the rotating coils by adjustment of the boxwood plates and nuts. During rotations, its leads were wound round the bolts and firmly secured.

The brass shaft G, of external diameter 1.27 cms., was solid towards the motor and had a small hole drilled through it along a diameter, 0.5 cms. from its end near the motor. Attached to the motor shaft by set screws was another piece of this shaft with a similar hole near its end. The two shafts could be connected or disconnected with ease, by a flexible coupling which consisted of a length of s.w.g.20 steel wire, bent into four lengths forming a double bow, which passed through these holes.

The shaft G towards the commutator M, was hollow, of bore 0.5 cms., and carried two bell flexes from the rotating coils to the commutator terminals a, b, c, d.

The commutator M, of ebonite of diameter 5.5 cms., was secured to the shaft by set screws through a stout brass tube through its centre. Brass sectors with terminals e and f, attached, were let into the ebonite. One of these consisted of a tapering sector with an arc of contact varying from some 36° to 45° . The other sector which was set diametrically opposite to the first, was parallel along its length and of arc of contact, 48° . By connecting suitable terminals of a, b.,c, and d to the sector terminals e and f, the rotating coils could be used separately, or in conjunction, or in opposition.

The brush holders N,(Figs,V and VI) were of brass and could be adjusted and bolted firmly on to the ebonite framework F, by a brass plate (not shown in Fig.VI.) The brush was of spring brass and was held in position at the required pressure by a brass screw R from which a dowel pin projected into a hole through the brush. Each brush was set so that the differential rotating coil passed through the same sweep of mutual inductance, approximately, with the field coils during the make and break on the tapering sector. Also, the mutual inductance between the differential coils rotating coils and the twin field coils at making contact through the brushes was approximately, numerically, the same as at the instant of breaking contact, and the arc of contact was about 39° . The exact position of make and break

Figure VII overleaf.

Figure VII. The circuit.

The circuit is shown in Figure VII. It consists of a series of components connected in a loop. The components include a power source, a resistor, an inductor, and a capacitor. The current flows clockwise through the circuit. The voltage across each component is indicated by arrows. The total voltage is equal to the sum of the voltages across the individual components. The circuit is used to study the behavior of electrical components in a series configuration.

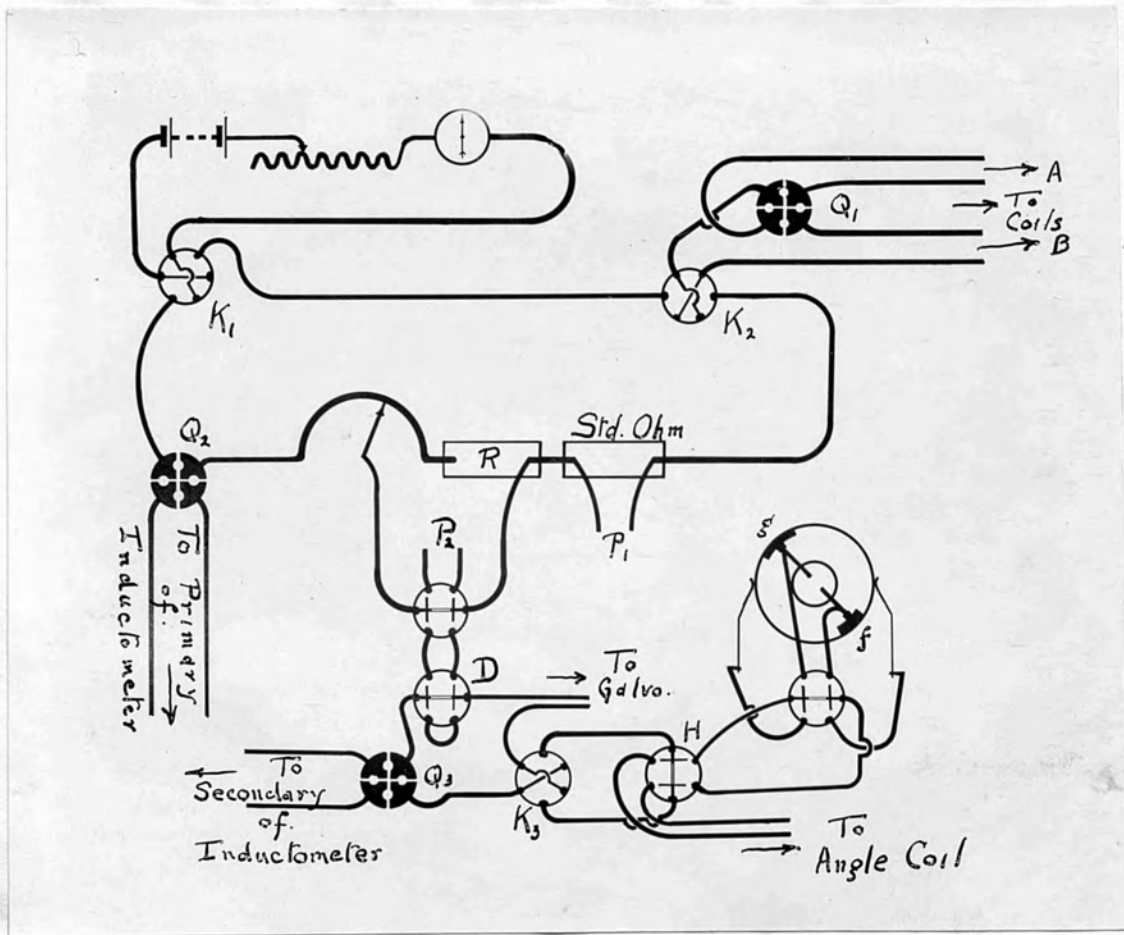


Figure VII. The Circuit.

was determined by connecting the brush holder terminals to a dry cell and a suitably shunted micro-ammeter. When the brush was in the critical position, a fluctuation occurred in the pointer of the instrument and this position was so precise that the rotating coil unit could be set to within 10 micro-henries by this device, that is — to within 0.05° of revolution.

The fly wheels S, and the locking device on the shaft T, to stop play along its length, are shown in the figure.

The large horizontal coils V, of sides 49×55 cms. are placed in the equivalent Helmholtz position and serve to neutralise the earth's vertical flux through the rotating coils when fed by a small current derived from accumulators.

The main electrical circuit will be readily understood from Fig.VII. The current of about 1 ampere, derived from accumulators, adjusted by rheostat and recorded on ammeter, can be sent in either direction through the twin field coils A and B, the quadrant key Q, serving to join them in conjunction or in opposition, but always in series. The same current passes through a standard ohm (N.P.L.Cert.58954) and through a variable resistance R, to be adjusted and measured. Each of these can be connected by potential leads with a good thermo-electric potentiometer which reads from 0 to 90 milli-volts so that the value of R can be checked within a few minutes of measuring the resistance absolutely.

The resistance R , consisted of a box of manganin resistances of nominal values varying from 0.005 ohms to 2.0 ohms in series with a short semi-circular copper wire of s.w.g. 18, with a movable potential contact.

The e.m.f. across R can be adjusted to neutralise that across the brushes e and f of the coils, spinning uniformly, and a balance is obtained on a galvanometer provided with a tapping key.

The circuit also permits of the measurement of the mutual inductance between the rotating coils, when stationary, and the twin field coils, as well as the measurement of the angle between them. For this purpose, the primary of a Campbell mutual inductometer is switched in at quadrant key Q_2 to be in series with the fixed twin coils. The rotating coils, through the sector terminals e and f , are thrown into series with the secondary of the mutual inductometer at the quadrant key Q_3 , and the galvanometer is included in this loop and detached from R by rocking over the mercury switch D . Alternatively, the angle coil may be thrown into the secondary circuit at H instead of the rotating coils, and hence the angle between it and the twin field coils can be determined by measuring the mutual inductance between this angle coil and the field coils. Balance is obtained to within $0.1 \mu\text{H}$ which represents in the case of the angle coil, $3''$ of arc.

The constant known speed of revolution was obtained by using a synchronised television motor. This is fully dealt with in Appendix I.

§ 6. Experimental Tests.

Attainment of a satisfactory rotor.

When the twin field coils, A and B, had been locked in the Helmholtz position, determined by the criteria given in Eq. 20. § 3., the number of turns on the inner and outer coils of the differential rotor were adjusted by three successive approximations after which the final form was attained. The complete data is as follows:-

The maximum mutual inductance between the inner, outer, and resultant coils with the twin coils were 7850, 18470, and 10620 nominal micro-henries respectively. This gives values of z approximating to those of Eqs. 25 and 28 in Table 2, and corresponds to an area ratio of $q_2/q_1 = 2.43$. Great care was taken to set the coils on the rotating table so that, approximately, they were, separately and jointly, at zero mutual inductance with the field coils in conjunction and at zero mutual inductance everywhere with the field coils in opposition. Finally, the angle coil was set to satisfy the same conditions of zero mutual inductance with the field coils ----the zero mutual inductance with the field coils

in conjunction, being at the same position of rotation as with the differential rotor. Whereas the angle coil could be set almost perfectly for symmetry, it was found impossible to avoid a maximum opposition residual inductance of two or three micro-henries for the differential rotor.

Readings of the mutual inductance M between the differential rotor and the field coils, and readings of the corresponding mutual inductance M_{θ} between the angle coil and the field coils were taken over all four quadrants at closely corresponding positions. The maximum angle coil in inductances were also observed and the angle θ of displacement from the angle coil zero were then evaluated from the Sine Law for all positions. The values of M and M_{θ} corrected for stud errors are given in nominal micro-henries in Table 4. together with θ in minutes of arc.

Table 4. Values of M for rotor and M_{θ} for angle coil at various angles θ in minutes of arc.

Quadrant 1			Quadrant 2		
M	M_{θ}	θ	M	M_{θ}	θ
10620.0	10812.0	4/1 max.	10619.7	10811.7	2/3 max
4244.8	4224.4	1379.963	4237.5	4209.0	1374.645
3942.5	3937.6	1281.48	3942.1	3929.5	1278.72
3847.5	3847.3	1250.71	3848.2	3839.7	1248.12
3804.6	3806.5	1236.84	3802.6	3796.6	1233.47
3735.6	3740.3	1214.37	3731.3	3728.3	1210.31
3659.5	3667.2	1189.63	3647.1	3647.2	1182.88
3549.1	3560.6	1153.67	3529.8	3534.6	1144.92
3428.4	3444.2	1114.55	3419.5	3427.7	1109.02
3225.0	3246.6	1048.48	3229.3	3243.5	1047.44
2825.8	2856.2	919.07	2827.2	2850.1	917.06
2411.1	2445.9	784.50	2409.1	2436.9	781.56
2012.4	2047.6	655.02	2017.2	2045.8	654.43
1014.2	1038.4	330.68	1013.3	1031.1	328.35
10.1	13.8	4.39	10.7	7.7	2.45
Quadrant 3			Quadrant 4		
M	M_{θ}	θ	M	M_{θ}	θ
10619.7	10811.7	2/3 max.	10619.7	10822.0	1/4 max.
4232.9	4211.9	1375.65	4237.8	4209.9	1374.96
3942.7	3937.1	1281.31	3932.7	3921.4	1275.95
3850.2	3848.6	1251.15	3846.5	3839.1	1247.92
3809.2	3809.5	1237.85	3804.4	3798.9	1234.25
3726.5	3730.1	1210.92	3723.4	3721.2	1207.90
3637.6	3645.0	1182.13	3639.9	3641.0	1180.78
3529.0	3540.5	1146.90	3518.4	3524.4	1141.49
3420.1	3435.3	1111.56	3412.6	3422.1	1107.14
3220.4	3241.3	1046.71	3218.9	3234.2	1044.34
2825.9	2855.3	918.78	2822.3	2846.1	915.74
2411.1	2444.5	784.04	2413.7	2442.0	783.22
2008.9	2043.4	653.66	2012.9	2042.3	653.30
1009.0	1031.9	328.61	1009.1	1027.7	327.26
14.4	17.1	5.44	13.5	11.8	3.75

By adding the corresponding values of M and θ in quadrants 1 and 2, the sweeps of inductance M' over the angles θ' can be obtained. Likewise, the sweeps M'' over the angle θ'' are found for quadrants 3 and 4.

The values of M'/θ' and M''/θ'' are tabulated in Table 5. together with the mean value of the angle of displacement θ and the value of $K = \frac{1}{2}(M'/\theta' + M''/\theta'')$.

Table 5. Values of M'/θ' for quadrants 1 and 2, and of M''/θ'' for quadrants 3 and 4 at mean angles of displacement θ , together with the mean value M/θ equivalent to the constant K in Eq.1.

M'/θ' μH per minute.	M''/θ'' μH per minute.	θ degrees	K μH per minute.
3.07649	3.07698	5.48	3.07674
3.07732	3.07722	10.90	3.07727
3.07792	3.07849	13.06	3.07820
3.07876	3.07884	15.29	3.07880
3.079466	3.07946	17.45	3.07946
3.07969	3.07960	18.51	3.07964
3.07967	3.07963	19.11	3.07965
3.07969	3.07989	19.73	3.07979
3.07954	3.07997	20.18	3.07976
3.07945	3.07981	20.59	3.07963
3.07972	3.07983	20.82	3.07977
3.07968	3.07962	21.32	3.07965
3.07931	3.07958	22.94	3.07945

These results, showing the rise of K towards a constant value between 18.5° and 21.3° of arc are considered most satisfactory in spite of small differences between the two

pairs of quadrants and irregularities, outside the accuracy of measurement, which is due to lack of perfect symmetry as revealed by the small residual inductances when the field coils are in opposition. For it must be pointed out that, in an actual resistance test, the value of K is measured immediately after the spin, by observations at the four boundaries of make and break between the sectors and the brushes, and these boundary settings are reproducible time after time, to within 0.05° of arc.

Attainment of a constant speed of revolution suitable for the measurement of a resistance very nearly equal to an international ohm.

With Koenig tuning forks of nominal frequencies 320, 384, and 512, and a bar of frequency, 288.6, and with synchronising wheels having 20, 24, 30, 36, and 44 teeth, a variety of speeds of revolution were obtained, enabling a number of resistances ranging from 0.71 ohms to 0.96 ohms to be measured absolutely.

A crucible steel bar of approximate dimensions $2.54 \text{ cms} \times 1.27 \text{ cms} \times 38.1 \text{ cms}$. was prepared and drilled for nodes at positions from its ends equal to 0.224 times its length. After slight adjustment by filing and rubbing, a frequency of 451.11 per second at 18.2°C was obtained and measured with the aid of the 44 toothed wheel. This bar was then used at the same temperature and setting with the 30 toothed wheel

and a speed of revolution of the apparatus of 15.037 per second was obtained. This enabled a balancing resistance of $1.0007_2 \times 10^9$ c.g.s. units equivalent to 1.0002 international ohms, to be set up and compared on a potentiometer very accurately with a standard international ohm (N.P.L. Cert. 58954)

Typical resistance test with data.

The brush contacts are cleaned with petrol and tested. A preliminary spin is then made with no current flowing through the field coils in order to adjust the current in the earth coils to neutralise exactly the earth's vertical flux through the rotor.

The speed of revolution of 15.037 per second in accordance with the preceding paragraph, was obtained and maintained, the field coil current of 1 ampere was started and a preliminary balance on a milli-voltmeter was obtained between the generated e.m.f. and that drawn off the adjustable resistance R. The final balance on a sensitive galvanometer is obtained by the fine adjustment of R until the reversal of the current by key K, Fig. VII produces no change in the position of the spot on the scale. This balance is checked with the reversal of both keys K₂ and K₃. Having set R to a value corresponding to a definite synchronised speed of revolution, comparison is made at once, on a potentiometer, of R with the standard international ohm.

The leads of the angle coil and rotor are now arranged for the determination of the constant K by observation of M and M_0 at the four sector edges which are defined to within 0.05° by the method of setting described in § 5.

Finally, the frequency of the bar is checked at 18.2°C . For this purpose, the motor is disconnected from the shaft of the rotor and the 44 toothed wheel is put on in place of the 30 toothed synchronising wheel.

The value of R in c.g.s. units is calculated from Eq.1 in the practical form:-

$$R = 216 \times 10^5 \times K \times N/t.$$

where N is the frequency of the controlling bar or fork, and t is the number of teeth on the synchronising wheel. K is in true micro-henries per minute. The correcting factor from nominal to true micro-henries is obtained with a 10 milli-henry standard (N.P.L.Cert.No.).

The data for a typical experiment are given in Table 6, and the tabulated results of all experiments to determine resistance absolutely with this apparatus, are given in Table 7.

Table 6. Data of a typical resistance experiment.

Bar frequency-----451.11; 30 toothed wheel.

Potentiometer reading off R -----	90.022
Potentiometer reading off 0.99998	
international ohm. -----	90.000
Value of R in international ohms -----	1.00022
" " R " true ohms -----	1.00074

Determination of K:

Quadrant	M	M_{θ}	θ in minutes
1/4 max.	(10619.4)	10811.7	
2/3 max.	(10619.5)	10811.7	
1	3628.1	3635.4	1178.90
2	3666.5	3667.7	1189.81
3	3621.5	3628.4	1176.54
4	3748.9	3747.2	1216.72

Whence:- $M'/\theta' = 3.07957$ nominal μH per minute.

$$M''/\theta'' = 3.07965 \quad " \quad \mu\text{H} \quad " \quad " \quad .$$

$$K = 3.07961 \quad " \quad \mu\text{H} \quad " \quad " \quad .$$

$$= 3.08146 \quad \text{true} \quad \mu\text{H} \quad " \quad " \quad .$$

Determination of N: With the bar at 18.2°C and the 44 toothed wheel in position, two identical counts of 36,909 were registered in two successive hours. $N = 451.11$

$$R = 1.00085 \times 10^9 \text{ c.g.s. units.}$$

Table 7. Showing tabulated data of experiments to determine resistance absolutely and the results.

N frequency of fork or bar	t teeth of wheel	N/t revs. per sec.	K true μH per minute	$R \times 10^9$ c.g.s. units	R by comp. true ohms.
288.57	20	14.429	} 3.08134 {	0.96032	0.96022
288.57	24	12.024		0.80026	0.80019
511.85	44	11.633	} 3.08114 {	0.77420	0.77415
511.85	36	14.318		0.94625	0.94618
384.06	30	12.802	} 3.08131 {	0.85205	0.85202
384.06	36	10.669		0.71004	0.71008
319.95	30	10.665	} 3.08137 {	0.70984	0.70987
319.95	24	13.331		0.88730	0.88727
451.11	30	15.037	} 3.08146 {	1.00085	1.00070
451.11	30	15.037		1.00085	1.00074
451.11	30	15.037	} 3.08145 {	1.00085	1.00072
451.11	36	12.531		0.83404	0.83396

The present
rather than
Computational

5

The
absolutely
varying
a
by
the

Figure 1. (overleaf).

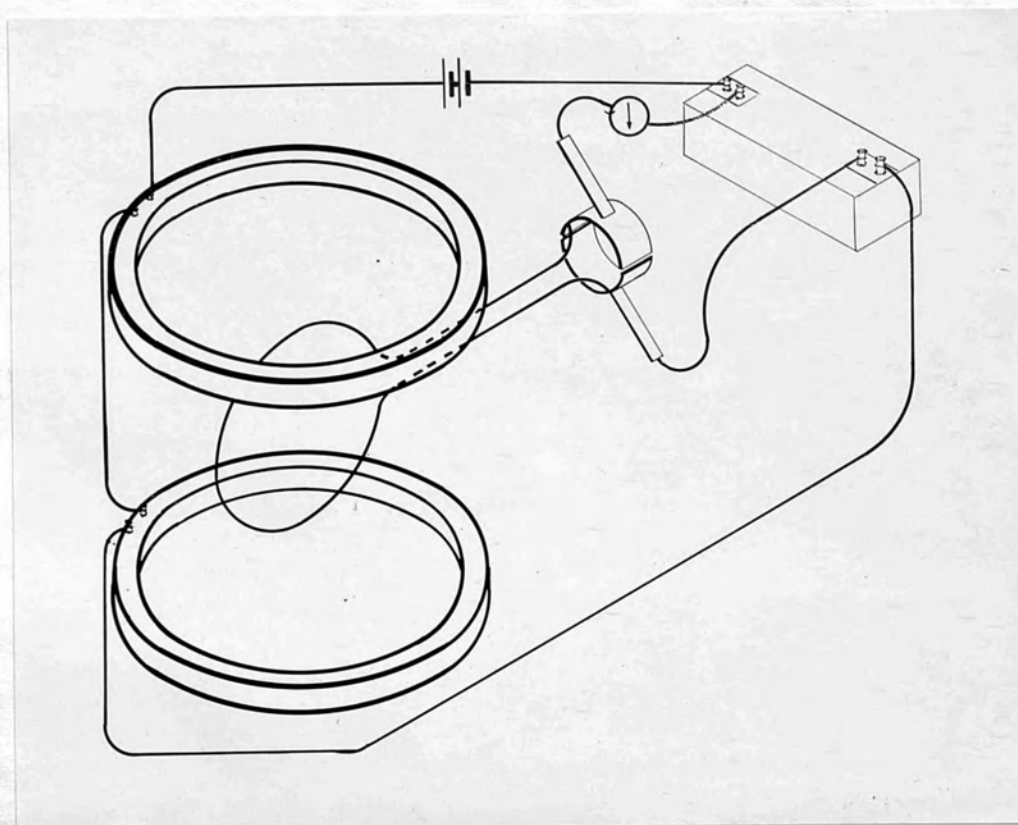


Figure 1 overleaf.

Figure 1. (overleaf)

PART II.

The Absolute Measurement of Electrical Resistance by a Method using the average Electro-Motive Force of a Commutating Generator.

§ 1. Introduction.

The method of measuring an electrical resistance absolutely, (Fig.1), by balancing the e.m.f. across it when conveying direct electric current against the average e.m.f. across a commutating generator, the field coils of which are traversed by the same current, was suggested by Rosa (Bull. Ber. Standards 5.499.1902.). Although Rosa proposed a form of apparatus and a method of procedure of which an account is given in the Dictionary of Applied Physics Vol.II.p.226., difficulties have been encountered and no experimental data derived from any form of this method, have ever been published.

The method is attractive on account of the high sensitivity due to the large e.m.f. involved and to the simplicity of the basic expression for R, the resistance to be measured viz:-

$$R = 4 n.M_0 \text{-----(1)}$$

where n is the frequency of revolution of the generator and M_0 is the maximum mutual inductance between the rotor and the field coils.

On the other hand, a number of difficulties arise, most of which are associated with the break of contact through the

detecting galvanometer, due to the gaps in the commutator. One of these difficulties viz. the break in the e.m.f. of the commutating generator, is readily overcome by arranging that the mutual inductance between the rotor and the field coils is very flat around the maximum value so that the e.m.f. is sensibly zero over considerable range and commutation may be effected without considerable loss. There still remains, however, the greater difficulty associated with the interruption of the steady e.m.f. drawn off the resistance R and accordingly Rosa suggested the use of a differential galvanometer as the detector, one coil of which was in uninterrupted connection with R while other similar coils were in connection with his composite rotor. This use of a differential galvanometer, permitting quantities of electricity, whose aggregate values over a period of revolution are not zero, to flow through the coils, adds considerable complication to the method and prevents the realisation of a simple null balance which is independent of the resistance of the detector circuit. A further difficulty, inherent in the method, is associated with the effect of the self-inductance of the rotor through which a current is reversed at each interruption when a simple detector is used; on the other hand, if the rotor is connected to the coil of a differential galvanometer and allowed to give uni-lateral current, other inductive complications arise.

In the present experiment the a.c. supply was zero impedance and a galvanometer was connected in series with the generator. By means of an extraordinary sensitive field coils, the resistance in the neighbourhood of the

Further details are given in Figures 2 and 3 overleaf.

actual commutator resistance across the supply line the rotor coil resistance and the

Figure 2. Showing the variation of mutual inductance between one rotor coil and the twin field coils at different displacements for three different distances of the rotor away from the axis of rotation. The dotted line is indicated by the dotted line. Values are given in Table 2, page 34, 35.

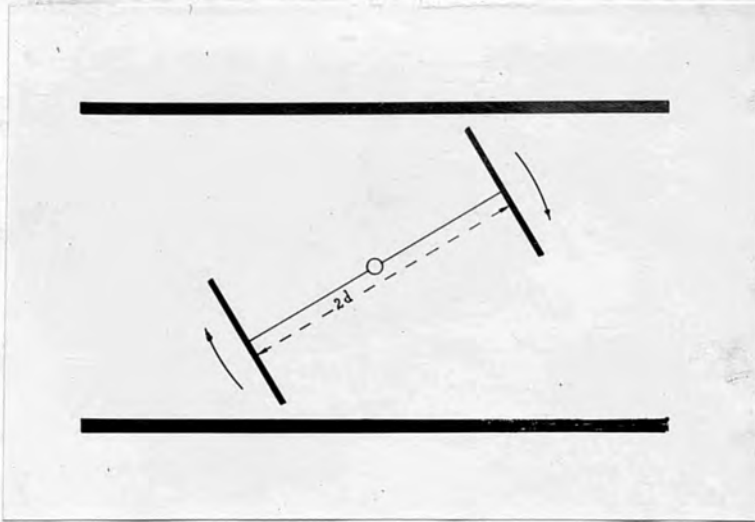


Figure 2.

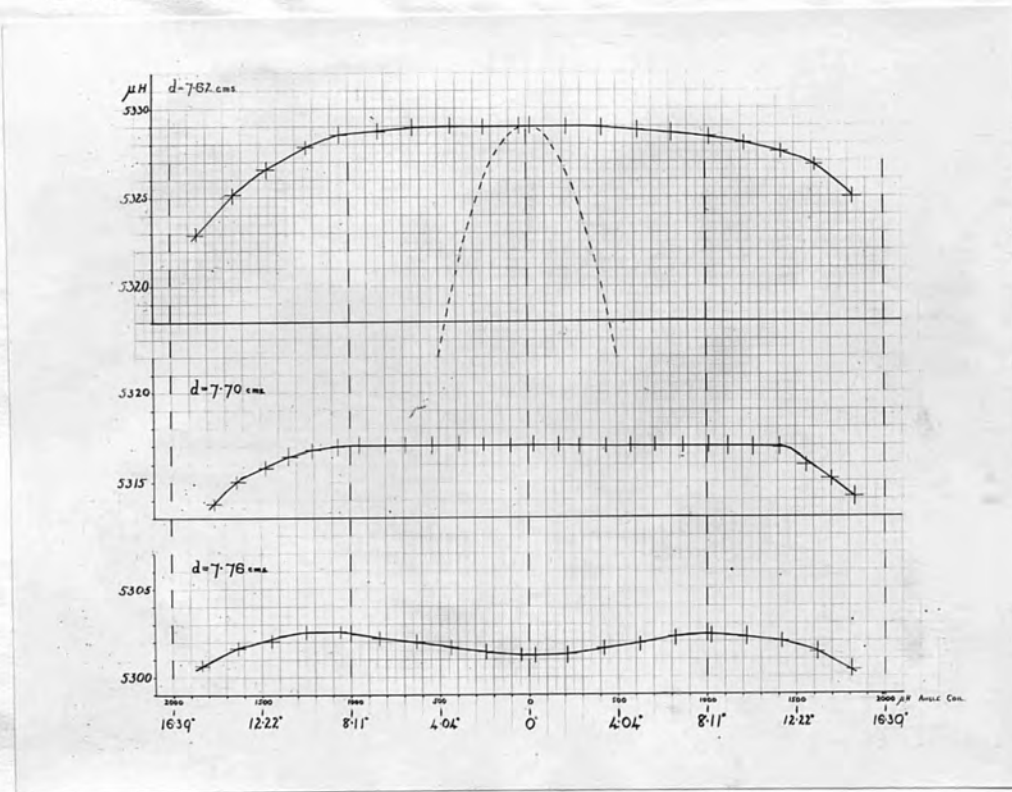


Figure 3. Showing the variation of mutual inductance between one rotor coil and the twin field coils at different displacements for three different distances of the rotor away from the axis of rotation. The contrast with the Sine Law is indicated by the dotted line. Values are given in Table 8, page 54, § 6.

In the present preliminary investigation here described, the e.m.f. across the resistance R is directly balanced for zero aggregate quantity of electricity through a ballistic galvanometer against the average e.m.f. of a commutating generator. By a new method, the rotor is made to have such an extraordinary flat maximum of mutual inductance with the field coils, that, over a range of some 15° of arc in the neighbourhood of commutation, no e.m.f. is generated.

Further, over a portion of this range covering the actual commutation, the rotor together with an adjustable resistance is short-circuited, thus enabling the e.m.f. across the resistance R to suffer no interruption but to supply instead the extra quantity of electricity, absorbed by the rotor on reversal of the current through it at commutation due to its self-inductance. In this way, the two main errors arising from the effects of commutation and self-inductance are set against each other and neutralisation is readily affected by experimental test.

§ 2. Attainment of a flat maximum of mutual inductance between a rotor and twin field coils.

Rosa (loc.cit.) proposed the use of twin field coils set somewhat further apart than the Helmholtz distance of separation with a rotor of large diameter, set symmetrically between them. A more convenient and efficient arrangement has been tested by the writer on the suggestion of Dr Nettleton and consists of two field coils (Fig.2.) A and B,

... by a small distance, and two relatively small
... rotor coils, C and D, with their planes parallel and
... by a distance d from the axis of rotation, where
... is approximately equal to half the radius of the
... By adjustment of d, the maximum value of
... is readily obtained.

The degree of
... the maximum
... from the
... of mutual inductance
... rotor coils and

Figure 4 overleaf.

... with the rotor
... is shown by
... the

... to give a range of 15° of
... Figure 4. Showing
... Curve I, M/300, the mutual inductance of the rotor

... with the twin field coils, against the
... angle of displacement of the rotor in
... degrees from the conjugate position.
... Curve II, M/300 of the rotor against the same angle
... The values from which these curves are derived are shown
... in Tables 9 and 10 on pages 26 and 27.
... variation of M with θ over a quadrant is shown by Curve I
... Fig. 4, and the corresponding values of M/300 proportional

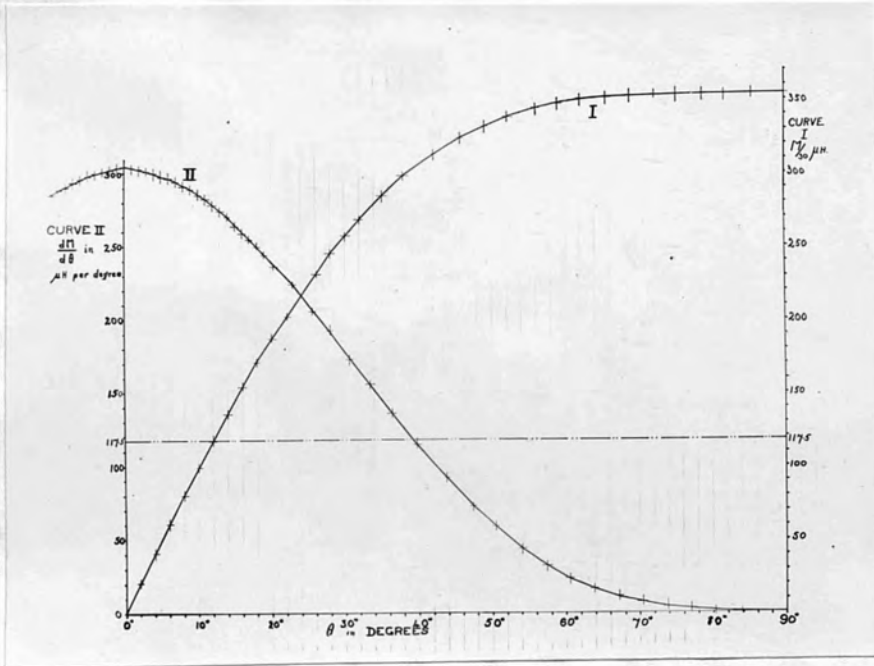


Figure 4. Showing in:-

Curve I. $M/300$, the mutual inductance of the rotor with the twin field coils, against the angle of displacement of the rotor in degrees from the conjugate position.

Curve II. $dM/d\theta$ of the rotor against the same angle
 The values from which these curves are derived are shown in tables 9 and 10 on pages 56 and 57.

separated by any small distance, and two relatively small twin rotor coils, C and D, with their planes parallel and displaced by a distance d from the axis of rotation, where d is approximately equal to half the radius of the field coils. By adjustment of d , the maximum flat of mutual inductance is readily attained.

The degree of flat of mutual inductance obtainable around the maximum with the rotor shown in Fig.11., will be gathered from the curves of Fig.3. which show the variation of mutual inductance with displacement between one of the rotor coils and the field coils at three slightly different distances d away from the axis of rotation. The contrast with the Sine Law which occurs approximately when d equals 0, is shown by the dotted curve. In the position of maximum flat, the extreme variation of mutual inductance is only $0.2 \mu\text{H}$ in 5317 over a range of 16° of arc.

§ 3. The wave form of the e.m.f. from the generator and of the currents through the detector and rotor at null balance.

With the aid of an angle coil obeying the Sine Law inserted in the hole in the oak bed (Fig.11) with the plane of its windings parallel to those of the rotor coils, the mutual inductance M between the two rotor coils in series and the twin field coils, was determined at various angles of displacement θ from the common conjugate positions. The variation of M with θ over a quadrant is shown by Curve I Fig.4. and the corresponding values of $dM/d\theta$ proportional

to the generated e.m.f.
shown in Curve II of Fig. 5
curve and the axis is
The experimental data

Figure 5. Showing the steady e.m.f. opposed by a variable
resistance in the commutating generator.
The actual experimental results are given in Table I and II.

the detector
shown in Fig. 5
resistance
e.m.f. having

Figures 5, 6, and 7 overleaf.

The results
detected

Figure 6. Showing the wave form of the current through
the detector at balance.

the detector
at 20° on each
to witness
the negative

Figure 7. Showing the wave form of the current through
the rotor at balance for zero aggregate quantity of
electricity through the detector.

the rotor at balance for zero aggregate quantity of
electricity through the detector.
The rotor at balance for zero aggregate quantity of
electricity through the detector.

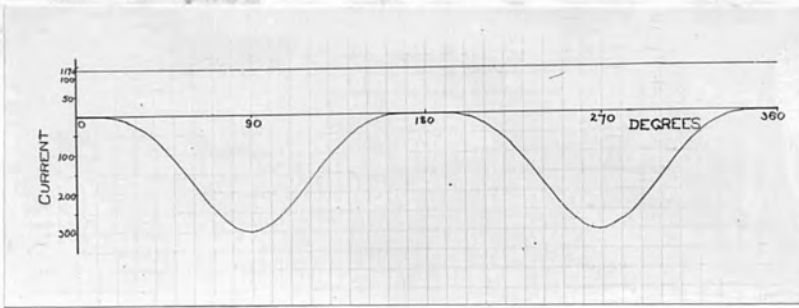


Figure 5. Showing the steady e.m.f. opposed by a variable e.m.f. derived from the commutating generator.

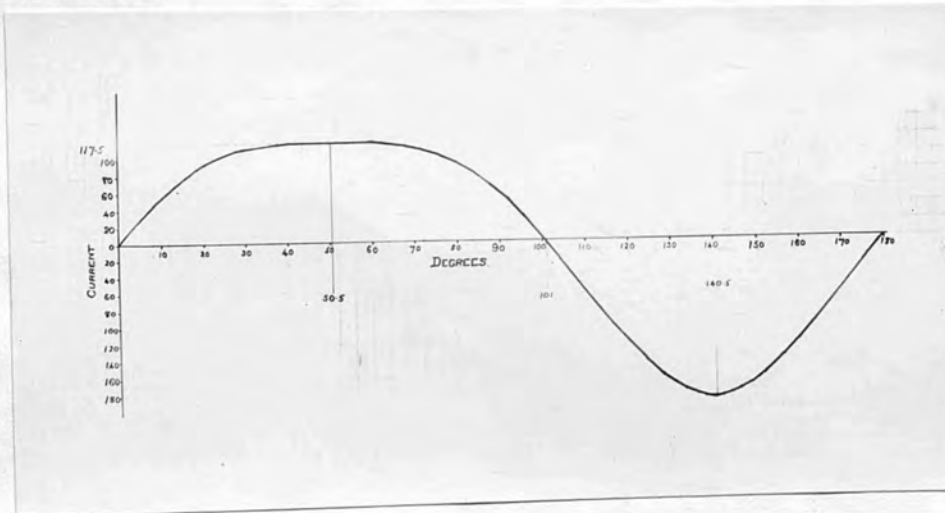


Figure 6. Showing the wave form of the current through the detector at balance.

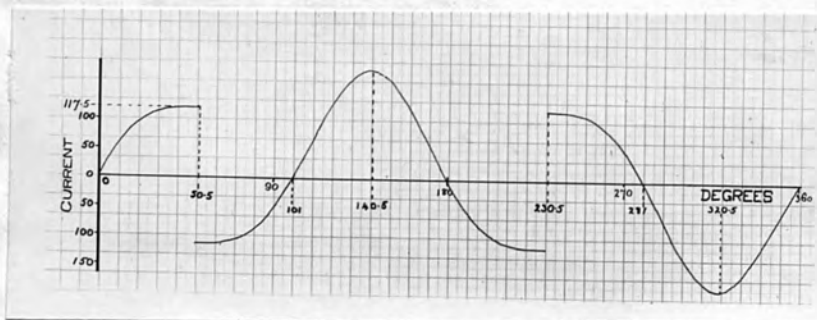


Figure 7. Showing the wave form of the current through the rotor at balance for zero aggregate quantity of electricity through the detector.

to the generated e.m.f. under uniform rate of rotation is shown in Curve II of the same figure. The area between this curve and the axis is equal to that of the rectangle shown. The experimental data from which these curves are derived, are given in Tables 9 and 10, in §6, under experimental tests.

In the actual experimental measurement of resistance, a balance for zero aggregate quantity of electricity through the detector must be obtained from the two opposing e.m.f.'s. shown in Fig.5.; the one is a steady e.m.f. drawn off the resistance R under test, and the other, the variable commutated e.m.f. having zero value in the neighbourhood of commutation.

The resultant wave form of the current through the detector at balance for zero aggregate quantity of electricity is shown in Fig.6. and has a period equal to half the time of revolution of the rotor. Over a range of 50.5° on each side of the maximum of mutual inductance, a current flows through the detector in the positive direction, and over the range of 39.5° on each side of the zero of mutual inductance an equal quantity of electricity flows through the detector in the negative direction. The current magnitude c_0 is equal to that through the detector when the rotor is at rest.

The wave form of the current through the rotor at balance for zero aggregate quantity of electricity through the detector, is shown in Fig.7. and has a period equal to the time of revolution of the rotor. The current c_0 is reversed through the rotor in the neighbourhood of each commutation.

§ 4. Theory of the method.

Simple theory. The time of commutation and self-inductance negligible.

In the arrangement the consists of a steady voltage resistance to be measured, includes the field coils, and the potential leads of the galvanometer of resistance

Figure 8. overleaf.

rotor of resistance r , and additional resistance Y in the total value of the resistance of this secondary circuit to X .

Let v be the instantaneous value of the opposing generated e.m.f. from the rotor and let C and c be the instantaneous values of the primary and secondary currents. Let C_0 and c_0 have values C_0 and c_0 respectively when v is zero for several degrees around the positions of commutation. Then, by applying Kirchhoff's Law to the primary and secondary circuits and neglecting the minute back e.m.f. due to the current c reacting in the primary circuit, we have:

$$CX + (C - c)R = E \quad \text{----- (1)}$$

$$cY = (C - c)R = E - CX \quad \text{----- (2)}$$

Whence $C = (E + cR)/(X + R) \quad \text{----- (3)}$

$$C_0 = E/(X + R) \quad \text{----- (4)}$$

$$c = ER/Y = c_0(X + R)/Y \quad \text{----- (5)}$$

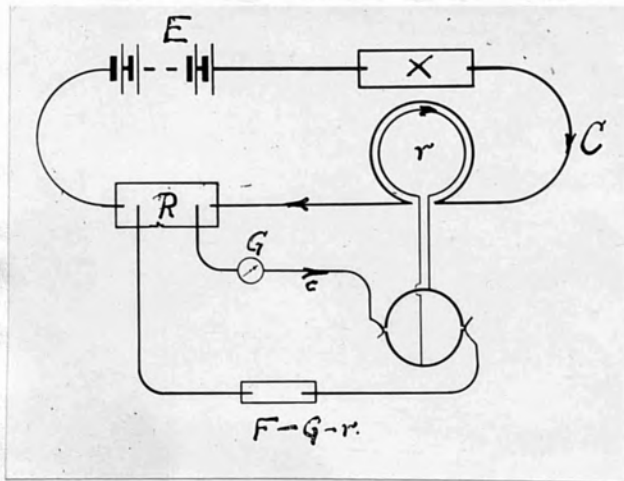


Figure 8. overleaf.

Figure 8.

§ 4. Theory of the method.

Simple theory: The time of commutation and self-inductance negligible.

In the arrangement shown in Fig.8. the primary circuit consists of a steady voltage E , in series with R , the resistance to be measured, and additional resistance X which includes the field coils. The secondary circuit, drawn off the potential leads of R , includes a high resistance ballistic galvanometer of resistance G , the low resistance commutating rotor of resistance r , and additional resistance bringing the total value of the resistance of this secondary circuit to F .

Let e be the instantaneous value of the opposing generated e.m.f. from the rotor and let C and c be the instantaneous values of the primary and secondary currents. Let C and c have values C_0 and c_0 respectively when e is zero for several degrees around the positions of commutation. Then, by applying Kirchhoff's Laws to the primary and secondary circuits and neglecting the minute back e.m.f. due to the current c reacting in the primary circuit, we have:-

$$CX + (C - c)R = E \quad \text{-----}(2)$$

$$cF - (C - c)R = -e \quad \text{-----}(3)$$

Whence $C = (E + cR)/(X + R) \quad \text{-----}(4)$

$$C_0 = E(F + R)/Y \quad \text{-----}(5)$$

$$c = ER/Y - e(X + R)/Y \quad \text{-----}(6)$$

$$c_o = ER/Y \text{ -----(7)}$$

where $Y = FX + FR + XR$.

Now the back e.m.f., $e = d(CM)/dt$ where M is the mutual inductance at a time t between the rotor and the field coils. The value of M changes from the negative maximum M_o in the position just after a commutation shown at the time $t=0$, in Fig.9. to the positive maximum M_o at a time $t = \frac{1}{2}T$ (the time of half a revolution) where the next commutation will take place. Whence integrating Eq.6. over half a period:-

$$\int_0^{\frac{T}{2}} c_o dt = \frac{ER}{Y} \cdot \frac{T}{2} - \frac{X+R}{Y} \int_{C_o(-M_o)}^{C_o(+M_o)} d(CM) \text{ -----(8)}$$

$$\text{or } \int_0^{\frac{T}{2}} c_o dt = \frac{ER}{Y} \cdot \frac{T}{2} - \frac{X+R}{Y} \times 2C_o M_o \text{ -----(9)}$$

whence for zero aggregate quantity of electricity through the detector over any number of half cycles:-

$$R = 4n.M_o.C_o(X+R)/E \text{ -----(10)}$$

where $n = 1/T$ is the frequency of revolution.

By the application of Eqs. 5 and 7, this is simplified to:-

$$R = 4n.M_o(1 + C_o.R/E) \text{ -----(11)}$$

$$\text{and } R = 4n.M_o \left[1 + R^2 / (FX + FR + XR) \right] \text{ -----(12)}$$

This shows that the value of the correcting term will only amount to a few parts in a million and that Eq.1. is highly accurate.

Consideration of the time of commutation and the self-inductance of the rotor.

It is impossible to estimate the time of break at commutation with accuracy or to diminish this time so that

the loss of flow in the positive direction due to the current c_0 is negligible. It is, however, easily possible to arrange that the rotor is short-circuited for such a time that M does not change from its value M_0 during it and yet it overlaps each commutation; this allows a current c_s , very slightly greater than c_0 , to flow through the detector. If r is the resistance short-circuited, we have:-

$$c_s = c_0 \left[1 + r / (F + R - r) \right] \text{-----(15)}$$

Let L be the self-inductance of the rotor, and let t_1 and t_2 be the times of short-circuit covering the two commutations in each revolution. Let $t_1 + t_2 = t$.

Consider the first commutation. Just prior to the short-circuit, a current c_0 is passing through the rotor in an anti-clockwise direction. On short-circuiting, this current decays through r only to a value $c_0 \cdot \epsilon^{-\frac{t_1 r}{L}}$, and on reopening the circuit, the clockwise current c_0 is established. The total change of flux built up at the expense of the energy through the detector is thus $c_0 (L + L_1)$ where $L_1 = L \cdot \epsilon^{-\frac{t_1 r}{L}}$, and in consequence there is a deficit in quantity of electricity through the detector in the positive direction of magnitude $c_0 (L + L_1) / (F + R)$.

Likewise, the second commutation covered by a short-circuit of time t_2 , will result in a deficit of the quantity of electricity through the detector of magnitude $c_0 (L + L_2) / (F + R)$ where $L_2 = L \cdot \epsilon^{-\frac{t_2 r}{L}}$. The total loss of quantity of electricity passing through the detector in one revolution due to

self-inductance is thus $c_o(2L + L_1 + L_2)/(F + R)$. If during each short-circuit, the rotor is momentarily on open circuit L_1 and L_2 will be zero.

Consider now the total quantity of electricity flowing through the detector in one complete revolution of period T . During the time $T - t$, we have for the time integral of the current:-

$$\int_0^{T-t} c \cdot dt = c_o(T-t) - \frac{(X+R)}{Y} \cdot 4C_o \cdot M_o \quad \text{-----(14)}$$

for owing to the flat of mutual inductance during the periods of commutation and short-circuiting, the value of M does not vary from M_o over the time t . During the time t of both commutations, the quantity of electricity flowing through the detector is:-

$$\int_0^t c \cdot dt = c_s \cdot t - c_o(2L + L_1 + L_2)/(F + R) \quad \text{-----(15)}$$

Hence, substituting for c_s from Eq.13.:-

$$\begin{aligned} \int_0^T c \cdot dt &= c_o \cdot T - \frac{X+R}{Y} \cdot 4C_o \cdot M_o + c_o \cdot t \cdot r / (F + R - r) \\ &\quad - c_o(2L + L_1 + L_2)/(F + R) \quad \text{-----(16)} \end{aligned}$$

The last two terms which show the effect of time of short-circuit and of self-inductance of the rotor respectively are very small if F is large and have opposite signs. By neutralisation, their combined effect is zero if:-

$$t = (2L + L_1 + L_2) \left[1/r - 1/(F + R) \right] \quad \text{-----(17)}$$

or very approximately, if:-

$$t = (2L + L_1 + L_2)/r \quad \text{-----(18)}$$

whatever the value of c_o .

Under these conditions, we have for zero aggregate

quantity of electricity through the detector over any number of periods, the Eqs. 14, 15, 16, and 17 as before.

The attachment of below circuit and self-inductance

A device, enabling a short-circuit, is shown revolving on the same larger brass sector e , brass sector, f , which The adjustable resistor

of the rotor, is circuited.

Figure 9 overleaf.

In order to obtain by Eqs. 17 and 18, the field is neutralised by

which is adjusted as in Figure 9. Showing the device and wiring to enable each

the field coils are shown out of the primary circuit in connection to be covered by a short-circuit of the rotor and the detector D is replaced by a string S , of the same resistance. With the rotor at rest in any position in which the short-circuit at N is inoperative, a steady current i_0 is passed as shown and the e.m.f. due to this current across the resistance S is balanced on a potentiometer. On spinning the rotor, there is an increase in the average value of i_0 due to the short-circuit, and a decrease in the average value of i_0 due to self-inductance. The latter effect becomes larger in magnitude as the frequency increases. By adjusting

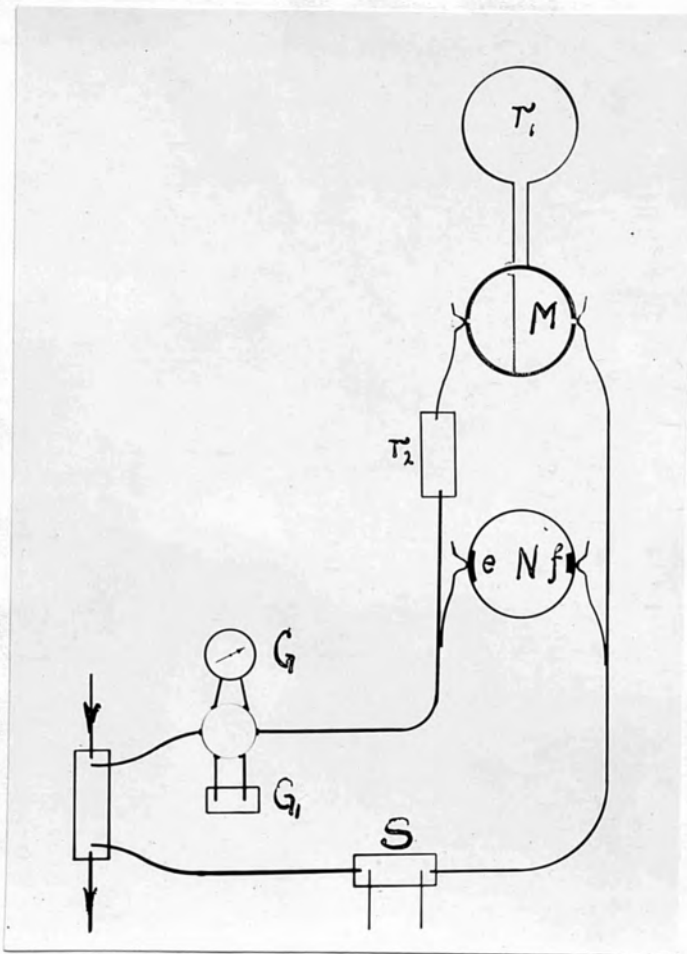


Figure 9. Showing the device and wiring to enable each commutation to be covered by a short-circuit of the rotor.

quantity of electricity through the detector over any number of periods, the Eqs. 10, 11, 12, and 1 as before. The attainment of balance between the effects of short-circuit and self-inductance.

A device, enabling each commutation to be covered by a short-circuit, is shown in Fig.9. The ebonite cylinder N, revolving on the same shaft as the commutator M, has a larger brass sector, e, connected to a smaller and tapered brass sector, f, which are both touched by additional brushes. The adjustable resistance r_2 , together with the resistance r_1 of the rotor, constitute the resistance r which is short-circuited.

In order to attain the balance conditions represented by Eqs. 17 and 18, the vertical component of the earth's field is neutralised by a current in the coils V and V which is adjusted as in § 6, Part II.

The field coils are thrown out of the primary circuit and the detector G is replaced by a dummy G, of the same resistance. With the rotor at rest in any position in which the short-circuit at N is inoperative, a steady current c_0 is passed as shown and the e.m.f. due to this current across the resistance S is balanced on a potentiometer. On spinning the rotor, there is an increase in the average value of c_0 due to the short-circuit, and a decrease in the average value of c_0 due to self-inductance. The latter effect becomes larger in magnitude as the frequency increases. By adjusting

the size of arc of the tapered disc, r , at the brush contact and the value of ω , ω is easily possible to arrange that, at the specified speed of revolution n , prepared for a resistance R , the dynamical potential balance across S is identical with the static balance. Under these conditions, we have below:

$$\int_0^T e \cdot dt = e_0 \cdot T \quad \text{----- (19)}$$

Now, since $M=0$; equation 16 becomes :-

$$\int_0^T e \cdot dt = e_0 \cdot T + e_1 \cdot \frac{R}{R+E} \cdot T = e_0 \cdot \frac{R_1 + R_2 + R_3}{R+E} \cdot T \quad \text{----- (20)}$$

and we obtain the required neutralising conditions of Eqs. 17 and 18. It should be noted that the circuit in degrees as determined

Figure 10 overleaf.

and 18 may be written:-

$$\theta = 180 \cdot n (EL + L_1 + L_2) \cdot f$$

or, very approximately when $f =$

$$\theta = 180 \cdot n (EL + L_1 + L_2) \cdot f \quad \text{----- (21)}$$

Use of a potentiometer to measure the average value of a variable current.

In Fig. 10, a current which is variable and periodic traverses a resistance S connected in series to a resistance R on a potentiometer. If at a time t , the instantaneous values of the current are as shown, we have from Kirchhoff's second law for the detector and potentiometer loops:-

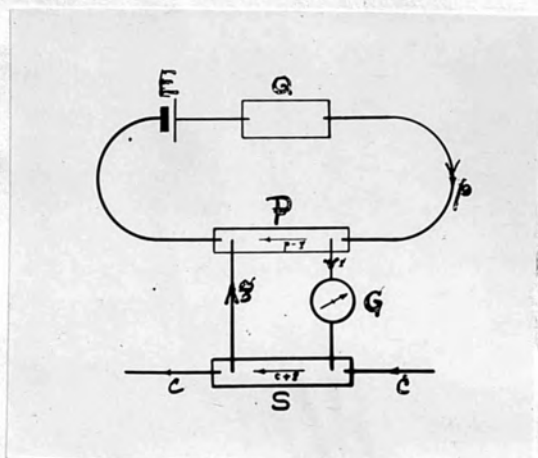


Figure 10 overlaid.

Figure 10.

the size of arc of the tapered sector, f , at the brush contact and the value of r_2 , it is easily possible to arrange that, at the synchronised speed of revolution n , prepared for a resistance test, the dynamical potentiometer balance across S is identical with the statical balance.

Under these conditions, as shown below:-

$$\int_0^T c \cdot dt = c_0 \cdot T \quad \text{-----(19)}$$

Now, since $M=0$; equation 16 becomes :-

$$\int_0^T c \cdot dt = c_0 \cdot T + c_0 \cdot \frac{tr}{F+R-r} - c_0 \cdot \frac{2L+L_1+L_2}{F+R} \quad \text{-----(20)}$$

and we obtain the required neutralising conditions of Eqs. 17 and 18. It should be noted that if θ is the arc of short-circuit in degrees as determined by the width of the sector f , since $\theta/180 = t/T = nt$, the balancing conditions of Eqs. 17 and 18 may be written:-

$$\theta = 180 \cdot n(2L+L_1+L_2) \left[\frac{1}{r} - \frac{1}{(F+R)} \right] \quad \text{-----(21)}$$

or, very approximately when F is large

$$\theta = 180 \cdot n(2L+L_1+L_2)/r \quad \text{-----(22)}$$

Use of a potentiometer to measure the average value of a variable current.

In Fig.10., a current which is variable and periodic traverses a resistance S connected as shown to a resistance P on a potentiometer. If at a time t , the instantaneous values of the current are as shown, we have from Kirchhoff's second law for the detector and potentiometer loops:-

$$g(G+S+P) + cS - pP = 0 \quad \text{-----(23)}$$

$$\text{and } p(Q+P) - Pg = E \quad \text{-----(24)}$$

Integrating these equations over the periodic time T and putting $\int_0^T g \cdot dt = \text{zero}$ for balance through the galvanometer for zero aggregate quantity of electricity, we have:-

$$\int_0^T c \cdot dt = P/S \int_0^T p \cdot dt \quad \text{-----(25)}$$

$$1/T \int_0^T p \cdot dt = E/(P+Q) \quad \text{-----(26)}$$

Now, if c_0 is the steady current through S which is permanently balanced on the potentiometer so that $g=0$ in Eqs. 23 and 24, p is steady at a value p_0 and

$$c_0 = p_0 \cdot P/S \quad \text{-----(27)}$$

$$p_0 = E/(P+Q) \quad \text{-----(28)}$$

Whence

$$\int_0^T c \cdot dt = c_0 \cdot T \quad \text{-----(29)}$$

§ 5. The Apparatus.

The general layout of the apparatus is the same as that in Part I of this thesis (Fig. V). The two field coils, A and B, are as described in Part I §5. They were used much closer together, however, in a position where the mean distance between them was

The complete

Fig. 11. Each of

copper wire of

having a diameter

7.55 cms. Each

self-inductance

the Figure 11 overleaf.

the self-inductance

in series, was

The oak bar

bolts, is accurately square in section and is

rotate about its longitudinal axis in the space

already described in Part I §5. Pairs of pieces of oak, each

pair being cut from one plank but of varying thickness, were

used to pack the coils, A and B, to the requisite distance

from the axis of rotation. These coils were adjusted and

tested for symmetry by measuring the mutual inductance of

each separately, with the twin field coils which were always

in series but were used first in opposition and then in

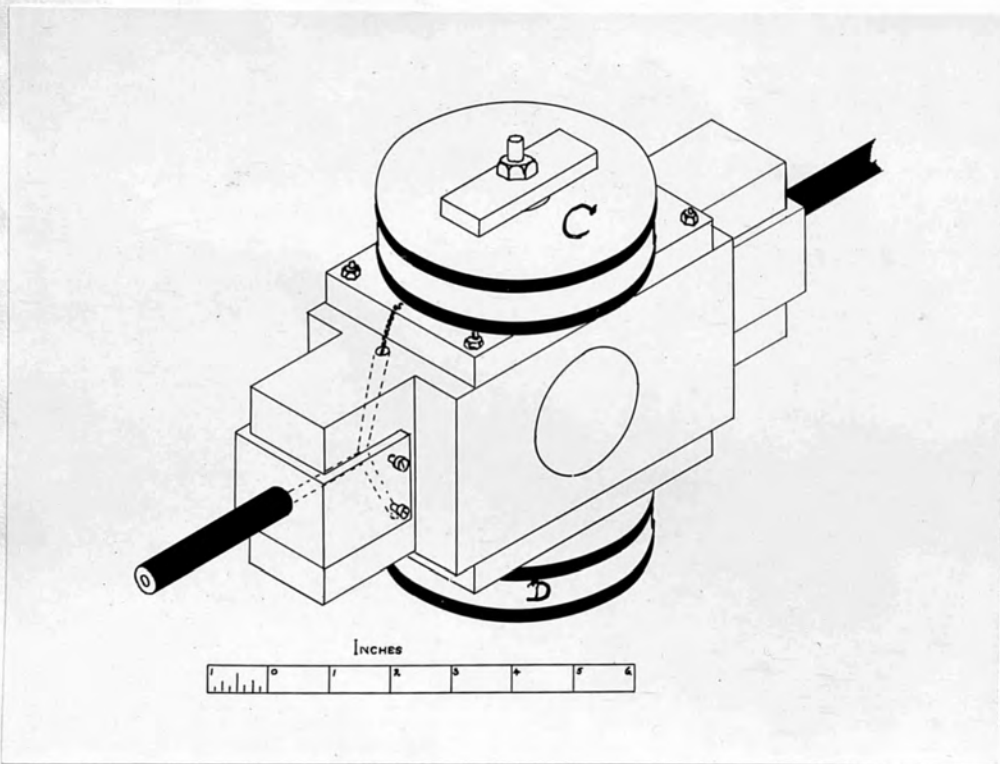


Figure 11. The Rotor.

§ 5. The Apparatus.

The general layout of the apparatus is the same as that in Part I of this thesis (Fig.V). The twin field coils, A and B, are as described in Part I §5. They were used much closer together, however, in a position where $x/a = 0.247$ and the mean distance of separation was 8.1 cms.

The complete rotor, as used in experiments, is seen in Fig.11. Each of the coils, C and D, has 334 turns of d.s.c. copper wire of s.w.g.26, and was wound on a mahogany former having a channel 1.2 cms. wide, and an internal diameter of 7.55 cms. Each coil was of resistance, 8.9 ohms, and had a self-inductance of 13660 μH . The mutual inductance between these coils in the final position of setting was 155 μH , and the self-inductance of the entire rotor, embracing both coils in series, was 27630 μH .

The oak bed upon which these coils are secured by brass bolts, is accurately square in section and is adjusted to rotate about its longitudinal axis in the brass channel holders already described in Part I §5. Pairs of pieces of oak, each pair being cut from one plank but of varying thicknesses, were used to pack the coils, C and D, to the requisite distance from the axis of rotation. These coils were adjusted and tested for symmetry by measuring the mutual inductance of each separately, with the twin field coils which were always in series but were used first in opposition and then in

conjunction.

The hole, 5 cms. in diameter, in the cast bed, was for the introduction of the main coil which was placed symmetrically with respect to the main field coils when investigating either the nature of the flat or mutual inductance, as described in § 51, or the wave form of the e.m.f. curve of the generator, as it is dealt with in § 52.

The coils of the main field which passed through the spindle to the commutator

Figure 12 overleaf.

in diameter, was to an absolute tube through which water was passed to the hollow shaft. The diameter of the tube was about 0.5 cms.

Further along the same shaft was fixed, in the same way, the shorting commutator, S, which consisted of an absolute cylinder of 3.5 cms. diameter, into which were placed two sectors of brass. These sectors were set diametrically opposite, the one tapering along its length from 6" to 12" and the other, parallel along its length, of 15" to 18". This was to supply the short-circuit of definite duration which was always longer than the break or short-circuit due to

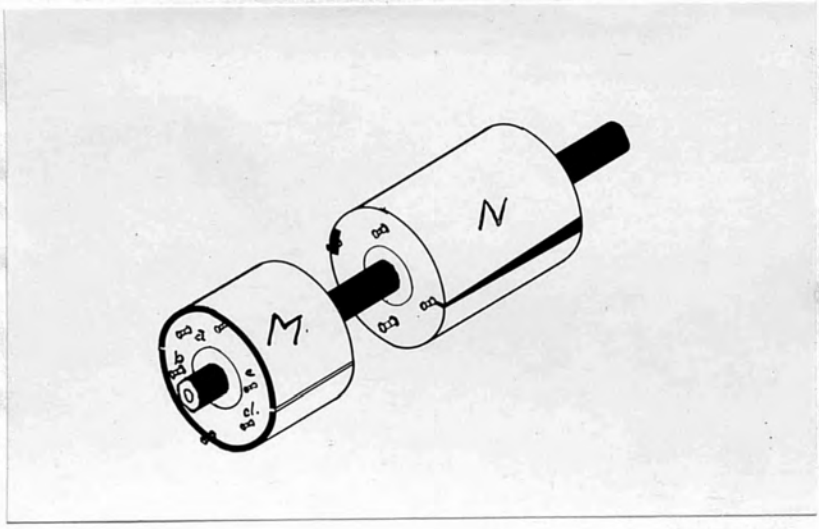


Figure 12. The Commutator.

conjunction.

The hole, 5 cms. in diameter, in the oak bed, served for the introduction of the angle coil which was fixed symmetrically with respect to the twin field coils when investigating either the nature of the flat of mutual inductance, as described in §2., or the wave form of the e.m.f. curve of the generator, as it is dealt with in §3.

The coils of the rotor were connected by twin bell flex which passed through the wooden bed and the hollow brass spindle to the insulated terminals, a, b, c, d, (Fig.12) on the commutator.

The commutator, M, (Fig.12) was a brass tube, 5.5 cms. in diameter, which was halved along its length and secured to an ebonite base. In the centre of this was a stout brass tube through which were set screws which secured the whole to the hollow shaft. The gap between the sectors was narrow, about 0.5 mms.

Further along the same shaft was fixed, in the same way, the shorting commutator, N, which consisted of an ebonite cylinder of 5.5 cms. diameter, into which were inlaid the two sectors of brass. These sectors were set diametrically opposite, the one tapering along its length from 0° to 8° . and the other, parallel along its length, of 15° of arc. This was to supply the short-circuit of definite duration which was always longer than the break or short-circuit due to

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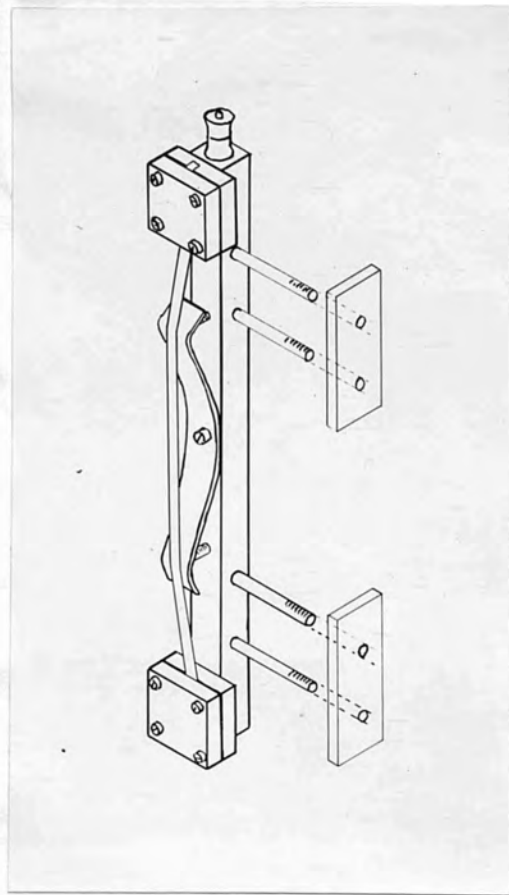


Figure 13. The Fiddlebow Brush.

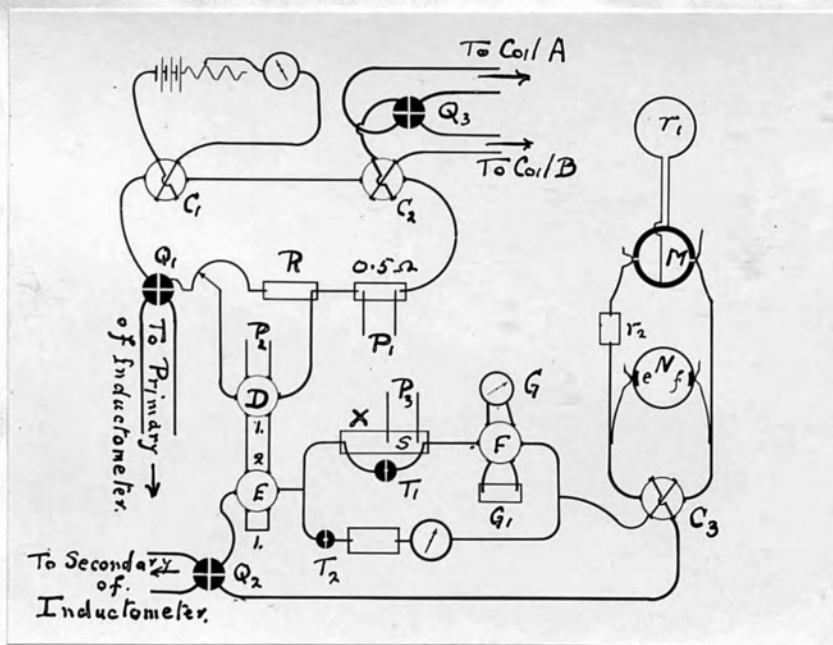


Figure 14. The Circuit.

the main commutator, M. The electrical connections of these commutators is shown in Fig.9. and their purpose is dealt with in §4.

The four brushes employed with these commutators, were of the fiddle bow type (Fig.13), the contact being made by brass ribbon, 0.5 cms. wide, rolled out of s.w.g. 22 wire. They were held by brass plates and nuts on the ebonite framework in Fig.V of Part I.

In the same diagram are seen the large coils, V and V, for neutralising the vertical component of the earth's magnetic field.

The motor coupling is dealt with in §5 of Part I and the motor with its synchronising and timing units is dealt with in Appendix I.

§ 6. Experimental Tests.

The experimental method of recording the flats of mutual inductance between the rotor coil and the twin field coils and the angles of rotation from the conjugate position.

The full circuit is shown in Fig.14. The angle coil is placed in the hole in the oak bed of the rotor (Fig.11) and is arranged so that its axis is perpendicular to that of the rotor coils. Its leads were joined to one side of a throw-over switch (not shown in Fig.14), and the leads from the terminals on the sectors of the commutator, M, (Fig.12) which were joined to the leads from one of the rotor coils

Table 8. Values of the mutual inductance between

through two of the insulated terminals, a, b, c, d, were also connected to the other side of the throw-over switch; the main leads of this switch were joined to C₃. The shorting commutator was disconnected. The quadrant keys, Q₁ and Q₂, were adjusted to put the primary and secondary coils of the inductometer in series with the main and rotor circuits respectively. Key E was placed on the shunting side, and of the plug keys, T₂ and T₁, the former was open and the latter was used to shunt out the box X. The galvanometer, G, was in the circuit. The inductometer was now adjusted until no throw occurred on the galvanometer when the current in the main circuit was reversed by key C₁. Thus the mutual inductance between the rotor coil or the angle coil and the field coils could be obtained at any setting of the position of the rotor. The maximum mutual inductance between the angle coil and the field coils was also obtained.

The values of the mutual inductance between one rotating coil and the field coils in nominal micro-henries at different displacements, d, of the rotor away from the axis of revolution, are given in Table 8, together with the angular rotation of the rotor from the position of maximum mutual inductance, in degrees. These results are shown graphically in Fig. 3.

2	430	3.70	5501.7	425	3.45	5501.6
1	240	1.25	5501.4	227	1.84	5501.3
0	44	0.57	5501.3	42	0.54	5501.2

Table 8. Values of the mutual inductance between :-

- i. One rotor coil and the twin field coils (M).
- ii. The angle coil and the twin field coils (M_e) at the same angle (θ) of rotation from the maximum for three different distances, d cms., of the rotor from the axis of rotation. Shown graphically in Fig.3.

	Left			Right		
	M_e	θ	M	M_e	θ	M
d = 7.62 cms.	1848	15.12	5322.9	1839	15.09	5325.1
	1641	13.39	5325.1	1621	13.23	5326.9
	1450	11.80	5326.5	1433	11.67	5327.6
	1230	10.00	5327.8	1228	9.98	5328.1
	1047	8.49	5328.5	1022	8.29	5328.7
	828	6.68	5328.7	813	6.56	5328.7
	636	5.15	5328.9	627	5.08	5328.8
	421	3.40	5329.0	427	3.45	5328.9
	243	1.97	5329.0	229	1.85	5329.0
	36	0.29	5329.0	28	0.23	5329.0
	d = 7.70 cms.	1761	14.39	5313.8	1707	13.94
1636		13.35	5314.9	1567	12.77	5315.8
1472		11.99	5315.7	1414	11.51	5316.8
1343		10.93	5316.3	1267	10.30	5316.8
1207		9.81	5316.6	1123	9.12	5316.8
1078		8.75	5316.8	1010	8.19	5316.8
945		7.66	5316.9	865	7.01	5316.9
796		6.45	5317.0	715	5.79	5316.9
685		5.55	5317.0	579	4.69	5316.9
532		4.30	5317.0	442	3.56	5316.9
382		3.09	5317.0	290	2.34	5316.9
243	1.97	5317.0	171	1.38	5317.0	
96	0.76	5317.0	35	0.28	5317.0	
d = 7.76 cms.	1835	15.01	5300.7	1817	14.86	5300.5
	1633	13.33	5301.6	1623	13.24	5301.4
	1446	11.77	5302.0	1422	11.57	5302.0
	1250	10.13	5302.5	1227	9.97	5302.3
	1054	8.55	5302.5	1027	8.33	5302.4
	837	6.76	5302.2	833	6.72	5302.2
	625	5.06	5302.0	629	5.09	5301.9
	430	3.70	5301.7	423	3.42	5301.6
	242	1.96	5301.4	227	1.84	5301.3
	46	0.37	5301.3	42	0.34	5301.3

Maximum mutual inductance between angle coil and field coils = 7083 μ H.

Experimental determination of the M/θ curve and hence the wave form of the generated e.m.f.

To obtain the M/θ curve, the above arrangement was retained with the sole exception that a small angle coil was constructed so that it could be inserted in the hole in the oak bed of the rotor (Fig.11.) with its windings parallel to those of the rotor coils which were now used in series and in conjunction. A series of values of the mutual inductance between the rotor coils in series and the field coils were taken through a quadrant, and at each setting, the mutual inductance between the angle coil and the field coils was measured. The maximum mutual inductance between the angle coil and the field coils was also measured.

These readings of mutual inductance between (i) the rotor and the field coils, and (ii) the angle coil and the field coils are given in Table 9 in nominal micro-henries, together with the corresponding angular rotation of the rotor from the conjugate position in degrees. The results revealed a lead of the angle coil on the rotor of 1.137° and the angle θ° is obtained from the expression :-

$$\theta^\circ = \sin^{-1} M_\phi / M_{\max} - 1.137.$$

These results are shown graphically in Curve I, Fig.4.

the e.m.f. generated by the rotor, with the angle of displacement of the rotor in degrees from the conjugate position. This is shown graphically in Curve II, Fig.4.

Table 9. Showing the values of the mutual inductance in nominal micro-henries between:-

- i. the rotor and the field coils, M .
- ii. the angle coil and the field coils, M_{ϕ} .
- iii. the angle of displacement of the rotor in degrees from the conjugate position, θ .

M	M_{ϕ}	θ	M	M_{ϕ}	θ
-869.4	-110.1	-2.880	5674.3	1320.9	20.275
-732.7	-81.5	-2.427	5955.6	1391.4	21.479
-362.3	-3.4	-1.191	6569.8	1550.1	24.230
-19.5	67.1	-0.074	7118.7	1700.6	26.898
326.1	139.5	1.073	7573.3	1831.6	29.275
660.2	209.2	2.178	8050.3	1980.2	32.044
857.2	250.6	2.835	8493.6	2128.2	34.891
1130.4	307.6	3.740	8931.1	2289.7	38.121
1507.0	387.2	5.006	9358.2	2467.5	41.860
1602.1	406.5	5.314	9714.1	2639.9	45.716
1896.8	468.8	6.308	9954.1	2778.4	49.027
2206.3	533.9	7.349	10139.9	2903.1	52.217
2552.9	608.6	8.547	10300.3	3036.9	55.931
2870.7	677.2	9.650	10401.0	3142.2	59.139
3190.7	746.7	10.773	10470.1	3231.8	62.139
3552.6	826.6	12.069	10524.8	3324.6	65.619
3911.9	905.8	13.361	10558.5	3398.8	68.804
4200.0	971.9	14.445	10580.2	3468.5	72.319
4216.4	974.4	14.486	10590.7	3517.5	75.307
4528.3	1046.8	15.680	10596.7	3562.5	78.788
4729.8	1093.9	16.461	10598.7	3591.7	81.912
4958.8	1146.8	17.342	10599.7	3612.3	85.563
5059.7	1171.8	17.760	10600.2	3618.3	88.863
5364.0	1244.4	18.979	10600.2	3618.3	
			(max.)	(max.)	

An examination of these results leads to the Table 10. which gives the values of $dM/d\theta$, (which is proportional to the e.m.f. generated by the rotor), with the angle of displacement of the rotor in degrees from the conjugate position. This is shown graphically in Curve II, Fig.4.

Table 10. Showing the values of $dM/d\theta$ in micro-henries per degree which is proportional to the e.m.f. generated by the rotor, with the angle, θ , of displacement of the rotor from the conjugate position in degrees.

θ degrees	$dM/d\theta$ in μH per degree	θ degrees	$dM/d\theta$ in μH per degree
0	304.0	48	69.7
4	299.8	52	52.1
8	291.1	56	36.3
12	278.8	60	24.9
16	258.2	64	15.1
20	238.4	68	9.3
24	216.7	72	5.1
28	190.7	76	2.1
32	164.5	80	0.6
36	138.5	84	0.2
39.5	117.5	90 _D	0.0
44	91.1		

Procedure for a typical resistance test with data.

- i. The current in the large coils, V , was adjusted to neutralise the vertical component of the earth's field. The main circuit was broken and the rotor circuit was completed through the galvanometer, G , the plug key, T_1 , the shunt at E_1 , and a single plug at Q_2 . On spinning at any speed, the galvanometer, G , will record no deflection when the earth's vertical flux is neutralised by the earth coil flux through the rotor coils.
- ii. At the synchronised speed of revolution required for the experimental determination of absolute resistance, the effect of the shorting of the commutator and of the self-inductance of the rotor coils was balanced in accordance with Eq.17 of §4. This was done by detaching the field coils from the main circuit, and by arranging a short length of wire across key C_2 . Key E was placed on side 2, and both plug keys, T_1 and T_2 , were open. The dummy, G_1 , was put in circuit in place of the galvanometer. The potential leads, P_3 , were used to obtain identical statical and dynamical balances on the potentiometer, as described on p.46, by adjustment of the arc of brush contact of the tapered sector, f , and of the value of r_2 (Figs.9 and 14). The approximate magnitude and constancy of the short-circuit could be tested by observing the micro-ammeter readings when the current was passed through the shorting device only, under both statical and dynamical conditions, and

indicated a shorting contact of 6° in 180° .

- iii. The galvanometer, of resistance 810 ohms and of sensitivity 27 cms per micro-ampere, was adjusted for symmetry of its moving coil so that no deflection occurred when pure alternating current is passed through it from a valve oscillator with a condenser in the circuit. In the main experiment, the spot of the galvanometer is drawn out into an ellipse whose major axis is about twice its minor axis but under these conditions, it remains quite steady and any deflection due to faulty adjustment of the zero quantity of electricity passing through it, is immediately detected.
- iv. The main field coils were replaced in the circuit and a current of 1 ampere is now passed through it. The synchronised speed of revolution was set up and maintained. With F open, E on side 2, and T₂ closed, a preliminary balance was obtained on the micro-ammeter by adjustment of resistance R. The balance was tested and should be maintained when C₁ was commutated or when C₂ and C₃ were commutated together. The final accurate balance was obtained on the galvanometer by a fine adjustment of R.
- v. The rotor was stopped from spinning and the throw-over switch, D, was placed on the potential lead side, P₂, and the resistance R was compared, on a good potentiometer with the standard resistance, 0.5 ohms, through the

potential leads, F_3 .

- vi. Key F was opened, T_2 closed, and E put on side 2. The rotor was rotated by hand until fluctuations of the micro-ammeter needle indicated that the shorting brushes were just about to make or break. The circuit was then arranged to measure the mutual inductance between the rotor and the field coils as it is described earlier in this article; the shorting brushes of the shorting commutator were arranged to be inoperative by breaking the connection between the shorting sectors. This was repeated at each of the four junctions of the brushes with the sectors of the shorting commutator. At each setting, the mutual inductance between the rotor coils and the field coils was measured and the average of these four readings was taken as the maximum mutual inductance between the rotor coils and the field coils as it was well within the flat of mutual inductance.
- vii. Finally, the speed of revolution was determined by disconnecting the motor from the apparatus at the coupling, and by obtaining it by the method described in Appendix I.

The results of a typical experiment are as follows:-

Konig fork of nominal frequency, 320.

Sectors on synchronising wheel, 30

Mean maximum mutual inductance -----10,612.2 true μH .

Frequency of fork at 18.4°C ----- 319.96₃

Revolutions per second ----- 10.665₄

Resistance = $10.665_4 \times 4 \times 10,612.2$ -----from Eq.1.

$$= 0.45272 \times 10^9 \text{ c.g.s. units.}$$

Resistance by comparison = 0.45273 ohms.

The results of the series of experiments performed are given in Table 11.

Table 11.

Frequency of bar or fork N	Teeth of wheel t	Revs. per second N/t	$R \times 10^9$ c.g.s. units	R in ohms by comparison
255.913	30	8.5304 ₃	0.36193	0.36196
288.65(bar)	30	9.6216 ₇	0.40814	0.40819
319.96 ₃	30	10.665 ₄	0.45272	0.45273
511.81	44	11.632 ₀	0.49362	0.49365
384.04	30	12.802	0.54324	0.54321
320.00 ₅	24	13.333 ₅	0.56580	0.56577
511.82 ₅	36	14.217 ₄	0.60335	0.60325

The valve circuitry is shown in the
system, 2, to light the lamp I, and to control the
Figures 15 and 16 overleaf.

Figures 15 and 16 overleaf.

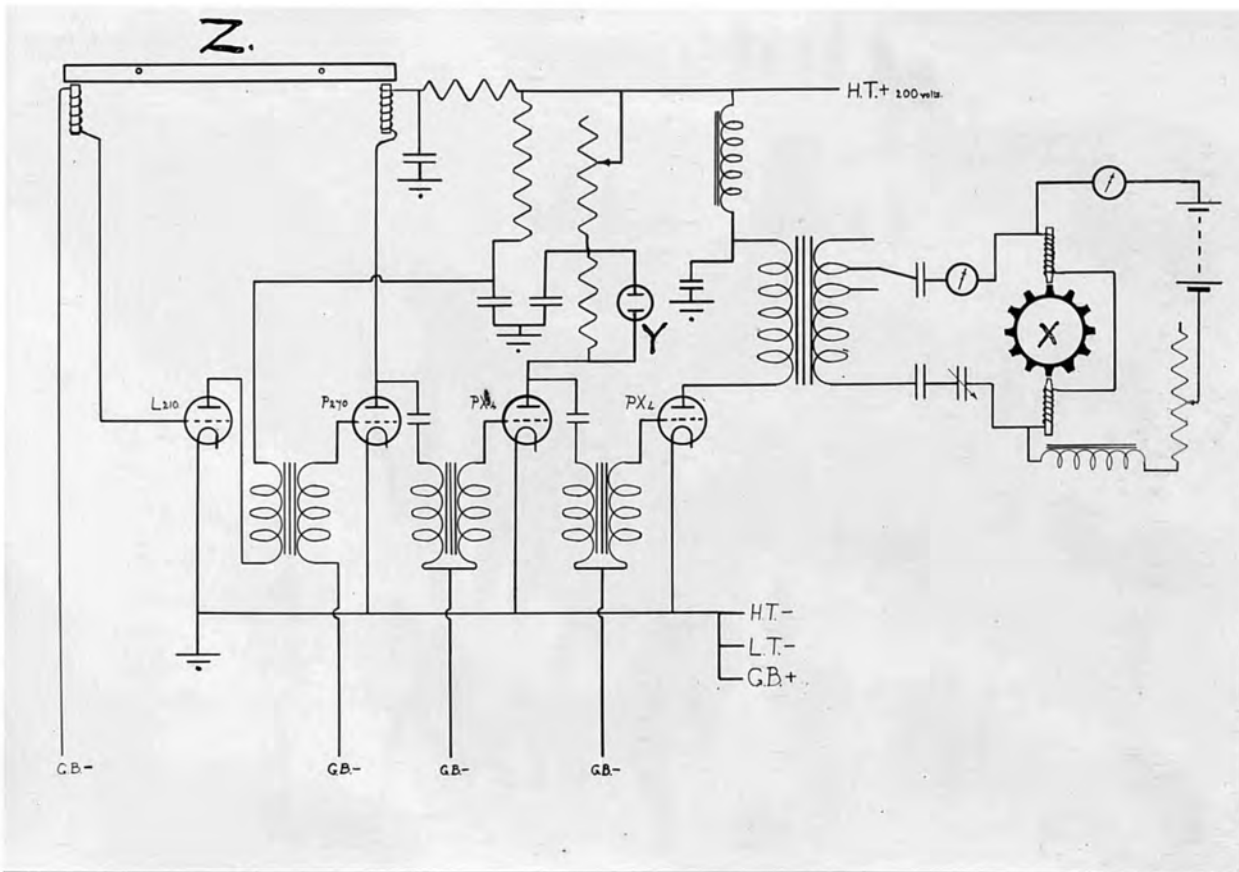


Figure 15. The Synchronising Circuit.

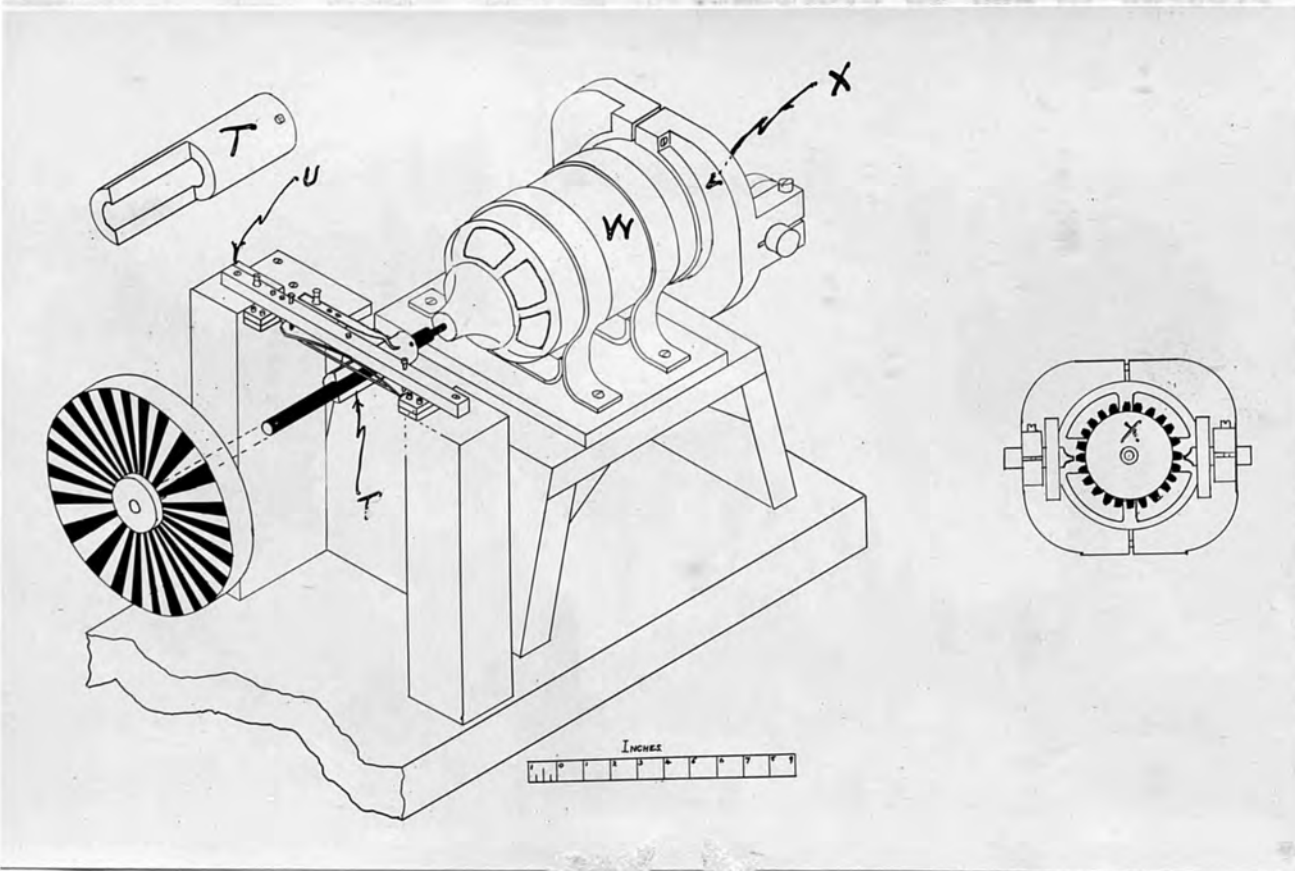


Figure 16. The Motor Unit.

Appendix I.

The determination of the frequency of a valve maintained tuning fork or bar and hence the determination and maintenance of the speed of revolution of the rotor, together with other incidental applications of this method.

A valve maintained tuning fork or bar was arranged to supply, through transformers and valves, uni-directional pulses of electricity which lighted a neon lamp by which a stroboscopic disc was viewed, and which controlled a phonic wheel on the spindle of the motor which drove the rotor.

The valve circuit used to maintain the iron vibrating system, Z, to light the neon lamp, Y, and to control the phonic wheel, X, of the television motor, W, is shown in Fig.15. The motor, W, with spindle attached, the fiddle-bow brush, U, and the device, T, for making one contact per revolution with a direct current circuit is shown in Fig.16. This circuit had in series with it, the following units:- the source of e.m.f., a resistance box, a milli-ammeter, the fiddle-bow brush making one contact per revolution, a brush making continuous contact with the spindle, a make and break key supplied with a screw for fixing it, and a post office telephone counter. There was also in this circuit, the usual capacitance device for quenching the sparking at the make and break of the intermittent contact. The counter is

designed to count 25 contacts per second for short intermittent periods of time but it was found to be inadvisable to exceed counts of about 14 per second over long and continuous periods. The timing was done by a standard clock which had been checked against Greenwich time signals over a period of some weeks. The pendulum of the clock was observed by telescope.

Experimental detail.

The part of the circuit which maintained the vibrating system and lit the neon lamp, was switched on. The vibrating system should be self starting to obtain the best results. The remaining part of the circuit which was connected with the pole pieces of the phonic wheel was now switched on and the capacitance in series with this part of the circuit was adjusted until a maximum deflection was obtained on the alternating current milli-ammeter. The direct current to these pole pieces was switched on and adjusted on the milli-ammeter in order to make the above alternating current uni-directional. Finally, the motor was started and its current, adjusted until the stroboscopic disc and neon lamp indicated that the speed of revolution was uniform.

The counter circuit was now started by depressing the make and break key at a definite second, observed by a telescope on the pendulum of the standard clock. A count could be made over almost any period of time by screwing up the make and break key. The count could be ended by

reversing the procedure of starting viz. by holding the key depressed whilst unscrewing the fixing screw, and by releasing this key at a precise second, observed by the telescope on the pendulum, on the standard clock.

In order to ensure that the revolutions per second were within the limits of the counter, ordinary cogged wheels of mild steel were softened, drilled and supplied with a central sleeve and set screws, and were used as synchronising wheels. These were obtained with diameters of from $1\frac{1}{2}$ inches to $2\frac{1}{2}$ inches with the following numbers of cogs:-4, 6, 8, 12, 20, 30, 44, 60, and 100. The synchronising pole pieces could be adjusted in position to suit the wheel employed for any count. These wheels could be used to measure frequencies from 32 to 1200 at speeds of revolution from about 8 to 14 per second. To obtain the cogged wheels of 4, 6, 8, and 12, teeth, wheels of more cogs had some removed. Thus a wheel with 36 teeth could be made into a 12 toothed wheel by removing two teeth and leaving one all the way round.

The 100 toothed wheel was not satisfactory as the teeth were too small owing to the fact that the available space between the pole pieces was limited by the clamping device on the motor.

Vibrating systems employed:-

- i. Tuning forks. The counts were of any suitable duration for two or three successive periods. Some typical results are given in Table 12.

Table 12. Showing the experimental details and frequencies obtained in some typical counts of tuning forks.

Temp. in degrees C	Duration in minutes	Teeth on cog wheel	Counts	Frequency
18.4	66	44	28,795 28,796	319.95
18.4	60	44	23,043 23,044	384.06
18.4	44	44	30,711 30,711	511.85
15.3	60	30	30,711 30,710 30,711	255.92 ₅
18.5	15	4	11,236 11,235	49.94
18.6	10	6	4,993 4,991	49.92

It was found that, with the electro magnets maintaining the fork in opposition, the first overtone of the fork could be produced, maintained, and its frequency measured. Thus the fork of nominal frequency 50, gave two successive five minute counts with a 30 toothed synchronising wheel of 3120 and 3119. The frequency of the first overtone of this fork was 311.9₅. The second overtone was also obtained by the adjustment of the position of the electro magnets which had to be the correct way on in this case. The frequency of this overtone could have been obtained in the same way.

ii. Bars. Bars of crucible steel, mild steel, and soft iron were drilled at their nodal points, 0.224 of their total length from each end, and were supported on rubber pads by horizontal rods through the holes. These were maintained in vibration and their frequencies measured. By adjustment of the position of the maintaining electro magnets, or by putting them in opposition, the overtones of the bars could be maintained and measured. The frequency of one bar was determined both in a steam jacket and in a cold water jacket, and hence the coefficient of frequency change with the temperature was obtained. Some typical results for bars are given in Table 13.

Table 13. Giving the dimensions of bars and their frequencies.

Crucible steel.							
Temp m °C	Length cms.	Breadth cms.	Depth cms.	Time minutes	Teeth	Count	Frequency
19.8	55.18	1.27	0.635	1	12	555	111
				2	12	1111	111.1
				20	12	11106	111.06
18.7	47.6	2.54	1.27	40	30	23092 23092	288.65
20.4	60.6	1.90	1.27	10	60	3152	315.2
						8598	859.8
20.6	47.75	2.54	1.27	10	60	5725	572.5
20.7	46.5	1.90	1.27	10	60	6530	653.0

Table 13. (continued)

Mild steel							
Temp in °C	Length cms.	Breadth cms.	Depth cms.	Time in. minutes	Teeth	Count	Frequency
20.8	21.8	0.318	0.794	60	30	38720	322.67 ₅
"	"	"	"	"	"	38721	
98.8	"	"	"	20	"	12791	317.77 ₅
99.0	"	"	"	"	"	12789	317.72 ₅
99.1	"	"	"	"	"	12790	317.75
Soft Iron							
21.3	21.8	0.318	0.953	60	30	40532	337.76
"						40532	
21.8				60	30	40530	337.75

For the mild steel bar, taking frequencies as:-

322.67₅ at 20.8°C and

317.75 at 99.0°C .

Then the coefficient of change of frequency with temperature is ----- 0.000109

iii. Plates. A circular mild steel plate, held at its centre, was maintained in vibration and its frequency measured. The position of the electro magnets determined the type of vibration which resulted and dust figures could be produced on the surface

in the usual manner.

iv. A steel wire on a sonometer was maintained in vibration in 1, 2, 3, or 4 loops according to the position of the maintaining electro magnets and the frequency was measured.

In the theory of the angle coil in Part I 33, the values of $E_n(\cos \psi / \cos \psi)$ are required for the angle $\psi = 49^\circ 6' 24''$ approximately having $\cos \psi = 3/7$. The necessary values are given in Table I.

Table I.

n	$E_n(\cos \psi / \cos \psi)$
3	3
4	0
6	$-33/7$
8	$655/7^3$
10	$-10533/7^5$
12	$-7055/7^7$

In the theory of the angle coil in Part I 33, and in the theory of the differential rotor Part I 34, values of $E_n(\cos \psi)$ are required for the Helmholtz coil, $\psi = 35^\circ$ approximately having $\cos^2 \psi = 1/5$. These values have been calculated by recurrence formulae and check has been made by direct evaluation of $E_{10}(\cos \psi)$. They are given in Table II.

Appendix II.

Tables of functions for calculating the mutual inductance for special cases of non-coplanar coils.

In the theory of the angle coil in Part I § 3, the values of $P'_n(\text{Cos } \phi)/\text{Cos } \phi$ are required for the angle $\phi = 49^\circ 6' 24''$ approximately having $\text{Cos}^2 \phi = 3/7$. The necessary values are given in Table I.

Table I.

n	$P'_n(\text{Cos } \phi)/\text{Cos } \phi$
2	3
4	0
6	$-33/7$
8	$468/7^3$
10	$13365/7^4$
12	$-7956/7^4$

In the theory of the angle coil in Part I § 3, and in the theory of the differential rotor Part I § 4, values of $P'_n(\text{Cos } \psi)$ are required for the Helmholtz angle, $\psi = 63^\circ 26' 6''$ approximately having $\text{Cos}^2 \psi = 1/5$. These values have been calculated by recurrence formulae and check has been made by direct evaluation of $P_{40}(\text{Cos } \psi)$. They are given in Table II.

Table II.

n	$P'_n(\cos \psi)$	Logarithm
1	1	
3	0	
5	-1.8	0.255,272,5
7	2.464	0.391,640,7
9	-0.936	$\bar{1}$.971,275,8
11	-1.816,32	0.259,192,4
13	3.465,28	0.539,738,3
15	-2.334,72	0.368,234,8
17	-0.973,324,8	$\bar{1}$.988,257,7
19	3.848,396,8	0.585,279,8
21	-3.763,533,3	0.575,595,7
23	0.492,189,7	$\bar{1}$.692,132,5
25	3.483,426,9	0.542,006,7
27	-4.869,710,7	0.687,503,2
29	2.301,798,2	0.362,067,2
31	2.358,941,7	0.372,717,2
33	-5.370,382,4	0.730,005,2
35	4.126,801,7	0.615,613,6
37	0.594,094,4	$\bar{1}$.773,855,4
39	-5.084,331,1	0.706,233,9
41	5.626,688,3	0.750,252,8

In the theory of the differential rotor Part I § 4, the values of the following functions are required in which n is an odd positive integer:-

$$B = \frac{2}{n(n+1)} \cdot \frac{1^2 \cdot 3^2 \cdot 5^2 \cdot \dots \cdot (n)^2}{2^2 \cdot 4^2 \cdot 6^2 \cdot \dots \cdot (n-1)^2}$$

$$D = 1 + (n+2)(n-1)$$

$$E = 1 + 10(n+2)(n-1) + (n+4)(n+2)(n-3)(n-1)$$

$$F = 1 + 91(n+2)(n-1) + 35(n+4)(n+2)(n-3)(n-1) + \\ (n+6)(n+4)(n+2)(n-1)(n-3)(n-5).$$

Tables of logarithms of these functions up to $n=31$ have been given by Nettleton and Llewellyn (Proc. Phy. Soc. 44, 216, 1932). It has been necessary to extend these tables up to $n=41$ and Table III below gives the logarithms of these factors for odd values of n between 33 and 41 inclusive.

Table III.

n	$\log B$	$\log D$	$\log E$	$\log F$
33	2.580,010,4	3.049,605,6	6.098,436,3	9.146,496,5
35	2.554,810,9	3.100,025,7	6.199,361,5	9.298,010,8
37	2.530,994,6	3.147,676,3	6.294,734,4	9.441,177,0
39	2.508,417,4	3.192,846,1	6.385,135,1	9.576,869,0
41	2.486,956,6	3.235,780,9	6.471,057,0	9.705,830,3

621.317.332

N^o 1.

TWO SIMPLE METHODS OF ABSOLUTE MEASUREMENT OF ELECTRICAL RESISTANCE IN TERMS OF INDUCTANCE AND FREQUENCY

BY

H. R. NETTLETON, D.Sc., Lecturer in Physics, Birkbeck College

AND

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TWO SIMPLE METHODS OF ABSOLUTE MEASUREMENT OF ELECTRICAL RESISTANCE IN TERMS OF INDUCTANCE AND FREQUENCY

BY H. R. NETTLETON, D.Sc., Lecturer in Physics, Birkbeck College
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ABSTRACT. In the first method a sinusoidal alternating current of some 15 mA. derived from a valve oscillator and of frequency equal to that of a König tuning-fork is allowed to induce an equal current in a secondary circuit. The equality of amplitude of the primary and secondary currents is judged with the aid of a Westinghouse instrument rectifier. The resistance of the secondary circuit is given by the expression

$$S = 2\pi n \sqrt{(M^2 - N^2)},$$

where n is the frequency, N the self-inductance of the secondary and M the mutual inductance between the primary and secondary. With standard forks of frequencies 256, 320, 384 and 512, resistances have been measured ranging from $16\ \Omega$ to $67\ \Omega$.

In the second method equal primary and secondary currents of known frequency are also produced and are further adjusted to be in quadrature. A simple arrangement is thereby derived which enables Campbell's two-phase alternating-current method of measuring resistance to be carried out in the laboratory.

In both methods a visibly beating circuit is employed which enables the frequency of the current used to be tuned easily, with precision, to that of a valve-maintained fork. This beating circuit is also of value in checking the relative accuracy of forks whose frequencies are very approximately in simple ratio to one another.

§ 1. PRINCIPLE OF THE FIRST METHOD

A PRIMARY circuit consists of a valve oscillator, resistance R , relatively large inductance L and neutralizing capacitance K . A secondary circuit of small self-inductance N and having a mutual inductance M , greater than N , with the primary circuit, has a total resistance S . The amplitudes of the primary and secondary currents are adjusted to equality by varying S or M , while the frequency n of the currents is adjusted to that of a valve-maintained König tuning-fork so that the beats, rendered visible, are of the order of 1 a minute.

Under such conditions the resistance of the secondary circuit is given by the relationship

$$S = 2\pi n \sqrt{(M^2 - N^2)} \quad \dots\dots(1).$$

R
 L, K
 N, M
 S
 n

This expression readily follows* from the well-known equations of a primary and secondary circuit, namely

$$\left(L - \frac{I}{\kappa\omega^2}\right) \frac{di_1}{dt} + M \frac{di_2}{dt} + Ri_1 = Ee^{j\omega t} \quad \dots\dots(2a),$$

$$M \frac{di_1}{dt} + N \frac{di_2}{dt} + Si_2 = 0 \quad \dots\dots(2b),$$

i_1, i_2
 ω

where i_1, i_2 are the respective instantaneous values of the primary and secondary currents when a sinusoidal alternating e.m.f. of pulsance ω equal to $2\pi n$ is applied to the primary circuit. For these equations give, for the ratio of the currents,

$$i_2/i_1 = -Mj\omega/(S + Nj\omega) \quad \dots\dots(3).$$

Thus the ratio of the secondary and primary current-amplitudes is in general $M\omega/\sqrt{S^2 + N^2\omega^2}$, and equation (1) results when these amplitudes are equal.

§ 2. THE EXPERIMENTAL ARRANGEMENT FOR METHOD 1

The primary circuit. A simple valve oscillator V , figure 1, was allowed to give a current of some 15 mA. through capacitance K , a variable self-inductance L_1 ,

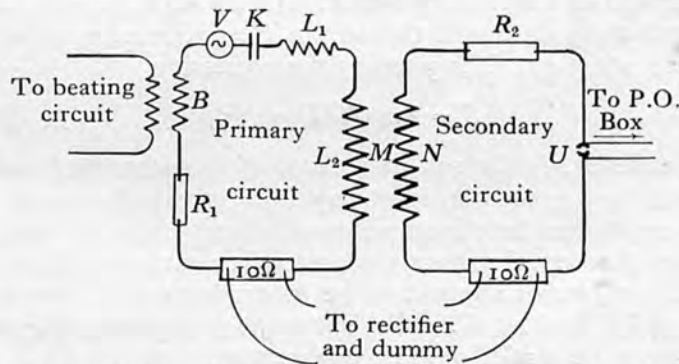


Figure 1. Simple circuit for absolute measurement of resistance.

a coil or coils of self-inductance L_2 linked with the secondary circuit, a coil B supplying energy to the beating circuit described below, a resistance box R_1 and a standard 10-ohm non-inductive resistance. The primary of a Campbell mutual inductometer could at any time be switched into the circuit.

The anode circuit of the oscillating valve (Mazda P 220) had adjustable capacitance including a variable $0.001\text{-}\mu\text{F}$. condenser. The inductance L_2 consisted of one or two twin coils of self-inductance 0.15 H. , resistance $15\ \Omega$, turns 500 and mean diameter 35 cm. The total effective inductance of the circuit was approximately neutralized, for the frequency used, by the capacitance K .

The secondary circuit. The secondary circuit included a coil N of 67 turns of s.w.g. 18 insulated copper wire about 31 cm. in diameter and of self-inductance $3242\ \mu\text{H}$. The coil had a maximum mutual inductance of $10500\ \mu\text{H}$. with either of the

* Cf. S. G. Starling, *Magnetism and Electricity*, pp. 359-61 (4th edition); A. Gray, *Absolute Measurements in Magnetism and Electricity*, pp. 251, 252; J. H. Jeans, *Electricity and Magnetism*, pp. 465, 466 (4th edition).

twin coils L_2 of the primary circuit. The rest of the circuit consisted of a non-inductive resistance box R_2 adjustable to 0.01Ω , a standard non-inductive $10\text{-}\Omega$ resistance similar to that in the primary circuit, and a plug key U which could be either closed to complete the secondary circuit or opened to permit connection with an accurate Post Office box, so that the total resistance S of the circuit could be measured. At any time the secondary of the Campbell mutual inductometer in series with a telephone could be included in the circuit.

The equal-amplitude tester. The $10\text{-}\Omega$ coils in the primary and secondary circuits were those of an accurate non-inductive ratio box and were connected, via potential leads, to a mercury rock-over key enabling either to be shunted at will by a circuit consisting of 500Ω , a full-wave instrument-type Westinghouse metal rectifier (specified as 4-1-1, 50 mA. Inst. unit) and a unipivot 0-120 d.-c. millivoltmeter. Equality of amplitude was judged by identity of deflection of the millivoltmeter as viewed through a microscope with an eye-piece scale on very quickly rocking over the switch, such identity of deflection having been carefully verified when the $10\text{-}\Omega$ coils were in series in the same circuit.

As the Westinghouse shunt circuit is not of infinite resistance, and as the primary and secondary currents are interdependent and have circuits of unequal impedance, special precautions have to be taken in accurately judging identity of current-amplitude. For the actual operation of the rock-over will very slightly modify the currents, and this effect, though small, is not symmetrical. Accordingly a dummy circuit of $1650\text{-}\Omega$ resistance was prepared and was always switched on to the primary 10Ω when the secondary 10Ω was on the Westinghouse circuit, and likewise always rocked on to the secondary 10Ω when the primary 10Ω was on the Westinghouse circuit. This dummy resistance was that of the Westinghouse circuit for the order of the deflection used and was such that the resistance of the secondary circuit, even when small, was the same whether the 10Ω in this circuit was shunted by the dummy or by the Westinghouse arrangement. This equality of resistance was judged by means of a P.-O.-box test with such a direct current, in either direction, as would produce the standard millivoltmeter deflection when the battery key was depressed.

Though this compensating device is most satisfactory it is no longer necessary if the voltages tapped off the $10\text{-}\Omega$ coils are switched in turn to the filament and grid of an amplifying valve, the anode current of which is passed through a transformer and thence to the Westinghouse circuit with added resistance. Rectifier M.B.S. 10 is now sufficiently sensitive.

The visible-beater. The output transformer from a valve-maintained tuning-fork was connected to a coil P_1 , figure 2, forming an adjustable mutual inductance with a secondary S_1 . Likewise the output from the coil B , figure 1, in the valve oscillator primary circuit was led to a coil P_2 , figure 2, forming an adjustable mutual inductance with the secondary S_2 . The secondaries S_1 , S_2 were connected in series through a resistance box R , having a telephone T across it, to a Westinghouse metal rectifier W and a unipivot 0-120 d.-c. millivoltmeter MV . If desired a push-pull wireless transformer having two equal primaries and a common secondary may be substi-

tuted for the coils P_1, P_2, S_1, S_2 , provided the inputs into the primaries are under control.

If the frequencies of the fork and valve-oscillator current are sufficiently near one another, beats may be heard in the telephone T and seen by the oscillations of the millivoltmeter pointer and so may be counted by both ear and eye. As the variable condenser on the oscillator is turned to bring the frequencies more closely into unison the beats become too slow to be recognized by ear, but the pointer oscillations increase in amplitude and are easily timed by eye. If the steady millivoltmeter deflections, due to each source separately, are made approximately equal at a quarter-scale reading, elegant full-scale swings of the pointer of many seconds' period may be produced when both sources are in operation, and the final tuning to unison is remarkably exact.

If both input circuits are derived from valve-maintained forks whose frequencies are very approximately in simple ratio to one another, beats between the common

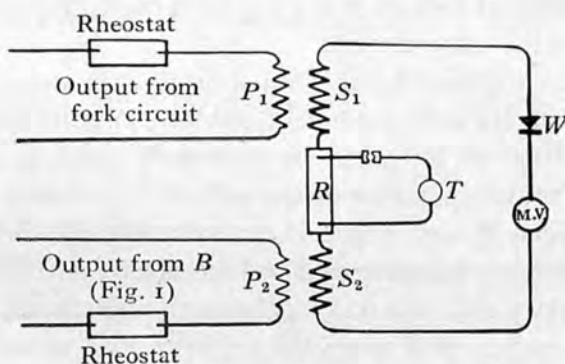


Figure 2. The visibly beating circuit.

harmonic components of the currents in the driving circuits, though of small amplitude, are readily seen and counted. Thus a König C 256 fork gave rise to beats of period 3.7 sec. when used in conjunction with a König E fork of frequency $320 + x$. These beats are between the current harmonics of frequencies 5×256 and $4 \times (320 + x)$ and were suppressed by a very minute load upon the E fork. A rapid and accurate check on the relative frequencies of the forks used can thus be easily effected, and data are given in table 5 below.

§ 3. EXPERIMENTAL TESTS (METHOD 1)

In performing an experiment the capacitance K , figure 1, was adjusted so as to neutralize approximately the total inductance of the primary circuit for the frequency used, while the anode condenser of the oscillator was chosen to produce visible beating with the standard valve-maintained fork, these adjustments being successive.

The resistance of the secondary circuit was set to produce an induced current approximately equal to the primary current, the final exact adjustment being made after tuning the oscillator to beat with the fork not more than once a minute. Any slight drift of frequency was easily corrected by a slight turn of the anode $0.001\text{-}\mu\text{F}$. condenser.

As soon as currents of equal amplitude, as judged by several rock-overs, had been obtained, the oscillator was switched off and the resistance of the secondary circuit was measured on a reliable Post Office box after removal of the plug key U , the resistance of the leads (as taken when the plug key was inserted) having been adjusted previously to an exact number of hundredths of an ohm. Whether the secondary 10Ω was shunted by the dummy or by the Westinghouse circuit the value of S thus obtained was in all cases the same to the nearest 0.01Ω .

The mutual inductance M between the primary and secondary was then measured with a Campbell mutual inductometer, the primary of which was switched into the primary circuit while the secondary and telephone were inserted in the secondary circuit. The usual impurity device was included and the frequency was maintained, by adjusting capacitance, at approximately that of the fork.

The self-inductance N of the secondary circuit is of smaller importance than M and was practically constant throughout the experiments. It was measured twice with the Heaviside-Campbell equal ratio bridge and found to be $3242\mu\text{H}$.

Variation of M for further tests was obtained by using one or both of the twin coils L_2 and by separating L_2 and N . As the range of the inductometer was limited to $11,100\mu\text{H}$, it was necessary when working above this range to add to it, at sufficient distance of separation, a $10,000\mu\text{H}$ standard of mutual inductance which could readily be cross-checked against the inductometer.

The results in tables 1 to 5 form a continuous series obtained when the mains were used with an eliminator as a source of high tension for the oscillator. Even in spite of slight fluctuations due to the d.-c. mains the equal-amplitude tester was very sensitive to changes of secondary resistance, which affect the primary and secondary currents in opposite directions. A change of $\pm 0.02\Omega$ clearly upset the equilibrium when the secondary resistance was some 25Ω , and a change of $\pm 0.05\Omega$ did so when the value of the secondary was 67Ω . We found later that if a high-tension battery was substituted for the mains and eliminator perfect quiescence was obtained, allowing a higher-power microscope to be used and aiding the ease and certainty of the equal-amplitude setting. Under such circumstances a secondary resistance of 32Ω could be set by the tester to within $\pm 0.01\Omega$.

Tables 1, 2, 3 and 4 give data for the four frequencies used. S is calculated from the relationship $S = 2\pi n \sqrt{(M^2 - N^2)}$ after M and N have been expressed in centimetres by multiplying the readings in microhenries by 10^3 . Table 5 gives the

Table 1. Results when $n = 512$, $N = 3242\mu\text{H}$.

M (μH .)	$S \times 10^{-9}$ (c.g.s.u.)	Resistance by bridge (Ω)
21090	67.04	67.09
18629	59.02	59.07
16626	52.46	52.50
10557	32.32	32.31
10510	32.16	32.14
8067	23.76	23.73
6025	16.34	16.30 ₅

Table 2. Results when $n = 384$, $N = 3242 \mu\text{H}$.

M (μH .)	$S \times 10^{-9}$ (c.g.s.u.)	Resistance by bridge (Ω)
21087	50.27	50.21
18790	44.66	44.66 ₅
16695	39.51	39.49 ₅
13608	31.89	31.87
10578	24.29	24.29
8085	17.87	17.88

Table 3. Results when $n = 320$, $N = 3242 \mu\text{H}$.

M (μH .)	$S \times 10^{-9}$ (c.g.s.u.)	Resistance by bridge (Ω)
10553	20.19	20.25
21097	41.91	41.87
18757	37.15	37.17
16718	32.97 ₅	32.96 ₅
13703	26.77	26.79

Table 4. Results when $n = 256$, $N = 3242 \mu\text{H}$.

M (μH .)	$S \times 10^{-9}$ (c.g.s.u.)	Resistance by bridge (Ω)
21067	33.48	33.51
18651	29.54	29.60
16634	26.24	26.26
13615	21.27	21.29
10746	16.48	16.50
10746*	16.48	16.47

* A second König fork, 256 B, was used in this experiment.

Table 5. Beats produced by valve-maintained tuning-forks

Beating* forks	Interval	Frequency of common octave	Time occu- pied by 10 beats in seconds	Corrected frequency of second fork
256 & 320	major third	1280	37	320.07
256 & 384	fifth	768	50.5	384.10
256 & 512	octave	512	88	512.11
256 & 256 B	unison	256	123	256.08

* The second fork in all cases required load for perfect harmony.

beats between the 256 fork and the others: the corrected value of the higher fork is relative only, the 256 fork being assumed to be correct. Such corrections have been neglected in the calculation of S .

§ 4. METHOD 2: A SIMPLE FORM OF CAMPBELL'S TWO-PHASE ALTERNATING CURRENT METHOD OF MEASURING RESISTANCE

If the arrangement shown in figure 1 be modified so as to take the form shown in figure 3 we have a circuit which is of value in teaching Campbell's method* of determining the ohm, and experiments may be carried out with a current of 15 mA. derived from a simple valve oscillator.

The primary circuit now includes the primary P' of a variable mutual inductance, preferably that of a Campbell mutual inductometer. The secondary circuit has two additions, viz. the resistance R whose magnitude is required, with potential leads, and the phase-adjuster consisting of a capacitance K' in parallel with a variable non-inductive resistance R_3 which permits of fine adjustment. The object of this multiple-arc arrangement is to neutralize the small self-inductance N of the secondary circuit so that the induced current shall be in quadrature with the primary current as well as ultimately equal to it in amplitude. Such compensation is attained when

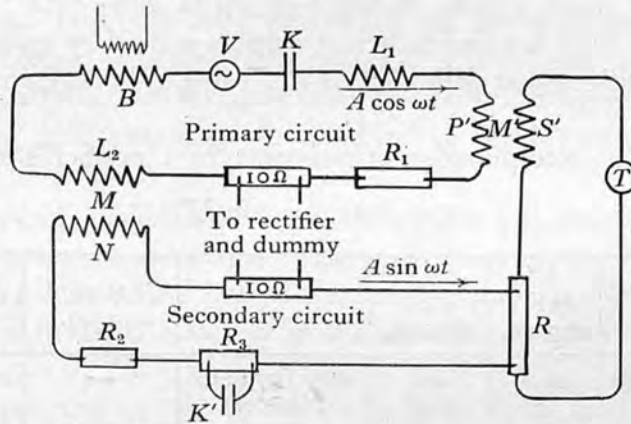


Figure 3. Simple form of Campbell two-phase alternating current method of measuring resistance.

$N = R_3^2 K'$ approximately and is almost independent of frequency, which only slightly influences the equivalent resistance. As the inductance-neutralizer introduces effective resistance into the circuit, and as the secondary current is limited when equal to the primary by the relationship

$$S = 2\pi nM$$

(N being now zero), it is well to use as a standard of frequency a valve-maintained fork of frequency not less than 512, and to set M at its largest value (in our case 21,000 μ H.). The resistance R is connected to the telephone T and the secondary S' of the mutual inductometer.

If by adjusting the mutual inductance M' silence can be obtained in the telephone when the primary and secondary currents are in quadrature as well as equal in amplitude, we have Campbell's relationship

$$R = 2\pi nM' \dots\dots(4).$$

* A. Campbell, *Proc. R. S.* 81, 450 (1908); 87, 398 (1912). For a brief account see the *Dictionary of Applied Physics*, 2, 224, 426.

In performing a first experiment the capacitance K in the primary circuit and the oscillator condenser were adjusted successively, as in § 3 above, to neutralize approximately the primary inductance and to produce visible beats with a valve-maintained König fork of frequency 512. The resistance R was one of the 10- Ω ratio arms of a Post Office box. K' was about $2\ \mu\text{F.}$ and the resistance R_3 was adjusted until, irrespective of equality of amplitude of the two currents, a reasonably sharp inductometer balance, as judged by approach to silence in the telephone T , was obtained. This occurred when R_3 was about $40\ \Omega$ and it showed the close approach to perfect quadrature, the secondary circuit being approximately non-inductive. The primary and secondary currents were now adjusted to equality with the box R_2 , as judged by the equal-amplitude tester, the frequency was controlled to obtain visible beats not exceeding one a minute, and R_3 was slightly adjusted for sharpness of inductometer-telephone balance. The adjustments were successive. A sharp reading for M' was thus obtained without the slightest difficulty, and a zero reading, equally sharp, was taken similarly with the 10- Ω plug of R inserted.

By making K' about $2.3\ \mu\text{F.}$, R_3 was reduced to some $38\ \Omega$, permitting R to be increased to $20\ \Omega$ while leaving sufficient surplus secondary resistance to allow of equal-amplitude adjustment with the box R_2 . Table 6 shows the readings obtained.

Table 6. Determination of resistance by Campbell's method

$$n = 512, R = 2\pi nM'$$

Nominal value of R	K' ($\mu\text{F.}$) approx.	R_3 (Ω) approx.	M' ($\mu\text{H.}$)	$R \times 10^{-9}$ (c.g.s.u.)	Resistance as measured with bridge (Ω)
zero (plugs inserted)	2	40	zero (by zero adjuster)	—	zero (lead reading taken)
10 Ω (left arm)	2	40	3101	9.976	9.98
10 Ω (right arm)	2	40	3102	9.979	9.98
20 Ω (both arms)	2.3	38	6208	19.97	19.96

§ 5. CONCLUDING REMARKS

The experiments described in this paper may be simplified and readily performed by advanced students. Inexpensive condensers may be used for capacitance in both methods. If no inductometer is available, the fundamental relationships involved in the methods, viz. those of equations (1) and (4), may be tested by resorting to the Owen and Carey Foster bridges for the measurements of self and mutual inductance respectively, though the absolute character of the methods is thereby sacrificed.

By virtue of the high inductance of the primary circuits, which ensures that the currents are minute until neutralizing capacitance for the fundamental frequency is introduced, error due to the presence of harmonics appears to be very small and inductometer balances are sharp.

§ 6. ACKNOWLEDGMENTS

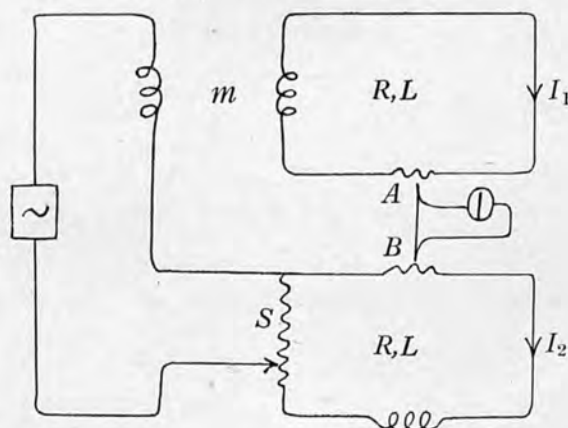
We express our gratitude to Prof. A. Griffiths who has generously provided us with apparatus and to Dr D. Owen, of the Sir John Cass Technical Institute, for the loan of a variable self-inductance. We thank Mr W. Wilson, Mr S. Baker and Mr R. Edgerton of the Physics Department, Birkbeck College, for suggestions and for assistance in our measurements.

DISCUSSION

Dr D. OWEN. The authors' first method utilizes ingeniously the fact that in the a.-c. transformer there is a simple relation between the resistance of the secondary, the mutual inductance, the self-inductance of the secondary, and the frequency, if the primary and secondary currents are exactly equal. From this relation the resistance of the whole of the secondary circuit may be calculated in absolute measure. It must be borne in mind that the effective a.-c. resistance of a coil is not equal to its d.-c. value, but always in excess of it, to an extent increasing with the frequency. It would have been of interest if results for the same circuit at various frequencies could have been recorded. The second method furnishes a simple means of carrying out A. Campbell's a.-c. determination of absolute resistance, which may with advantage be included in the laboratory course of the advanced physics student.

Mr A. CAMPBELL. The authors' second method is quite suitable for an ordinary laboratory and the results show that it can give good accuracy. Some time ago I introduced a somewhat similar system as a phase-splitter for a.-c. potentiometers*. It is rather simpler in detail than that of the authors, but not so self-contained for students' use.

In the accompanying figure the two loops have equal total resistance R and self-inductance L , being set by preliminary tests. The currents I_1 and I_2 will thus



be always in quadrature. By means of differential thermo-junctions and heaters A and B , by altering either S or m , we can get

$$I_2 = I_1.$$

* British Patent Specification No. 342,441.

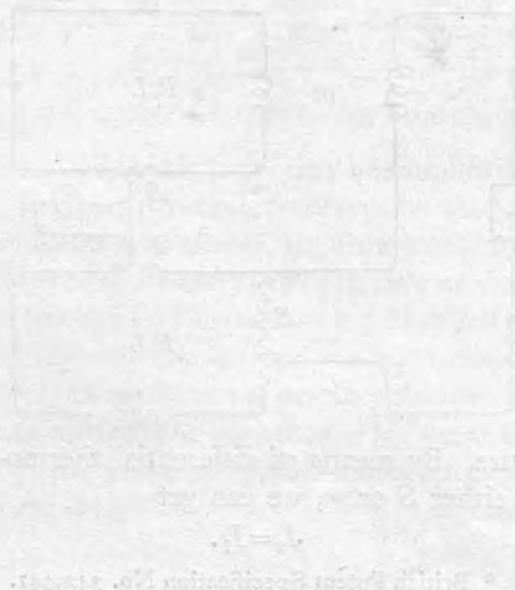
Then
$$S = \omega m = 2\pi n m,$$

which determines the resistance S in terms of m and n . With ordinary apparatus I have obtained accuracy to about 1 part in 2000.

I have suggested the system to Dr Hartshorn as an alternative to my M - R method, by which the National Physical Laboratory has recently made a determination of the ohm. It is sufficiently sensitive and requires very little apparatus, but it has one weak point—the accuracy is lowered by the presence of harmonics, for balance is obtained by making the effective (r.m.s.) values of I_1 and I_2 equal, and not, as in the M - R method, by use of a selective vibration galvanometer.

AUTHORS' reply. In reply to Dr D. Owen: We hope to carry out shortly further tests on the absolute measurement of resistance by a yet more simple a.-c. method. We shall certainly keep his suggestion in mind and measure the same resistance under different frequencies.

We are very interested to learn that Mr Albert Campbell has used a simple device in which two currents, always in quadrature, are adjusted to equality to enable a resistance to be measured absolutely in terms of a mutual inductance and a frequency. The circuit of figure 3 is designed for teaching purposes to resemble the original arrangement as closely as possible and it is perhaps an advantage that the student has to perform the adjustment for quadrature carefully to obtain sharpness of balance.



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No. 2.

THE ABSOLUTE MEASUREMENT OF ELECTRICAL
RESISTANCE BY A NEW ROTATING-COIL METHOD

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ABSTRACT. A rotating coil of mean radius α lies symmetrically between two fixed twin coils of radius a . If the ratio α/a lies between 0·58 and 0·53 it is easy to arrange, by merely adjusting the distance between the twin coils, that the mutual inductance M between the rotating coil and the two fixed coils shall be very accurately proportional to the angle θ of displacement from the conjugate positions over a range of some 10° of arc on either side of the zeros. The constant K of the relation $M=K\theta$ can then be accurately measured by a method here described in which θ is deduced from a mutual-inductance ratio. At the same time errors in the calibration of the inductometer used are eliminated.

If such a coil spins with an angular velocity ω , while a current C traverses the fixed twin coils, a uniform e.m.f. ωCK can be drawn off a commutator on the rotating shaft and made to balance an e.m.f. CR drawn off an adjustable resistance R carrying the same current. Thus $R=\omega K$.

If now the twin coils are brought rather closer together, the law of variation of M with θ takes the form $M/\theta=K+A\theta^2-B\theta^4$ over a displacement of 35° of arc from the zeros, where A and B are very small positive constants and M/θ attains a maximum value K' , where $\theta^2=A/2B$ and scarcely changes over a range of 3° in this neighbourhood. This allows larger sectors to be used on the commutator and the constant K' of the relationship $R=K'\omega$ can be accurately determined for a suitable commutator *in situ*.

The experimental work is mainly devoted to a study of the laws of inductance on which the method depends and to the determination of the constants K and K' , but preliminary spin experiments are very hopeful and resistances of between 0·32 Ω . and 0·64 Ω . have been measured absolutely by means of commutators with sector contacts of 23° and 47° of arc. Owing to the relatively large e.m.f.'s involved, the method is very sensitive and a fluxmeter can be used as the balance-detector for hand-controlled stroboscopic spins. In other experiments a synchronized television motor was used.

§ 1. INTRODUCTION

THE well-known method of measuring electrical resistance absolutely with the aid of a spinning coil was originally suggested by Weber and put forward independently by Kelvin to the Electrical Standards Committee of the British Association in 1863. It is very fully described in the reports of the British Association covering the period 1862-67; the experiments were carried out principally by Maxwell, Stewart, and Jenkin. Later determinations by this method were undertaken by Rayleigh and Schuster*; Rayleigh† and H. Weber‡.

* *Proc. roy. Soc.* 32, 104 (1881).

† *Phil. Trans.* 173, 661 (1882).

‡ *Der Rotations-inductor* (Leipzig, Teubner, 1882).

The method involves fundamentally the measurement of a coil-area, a galvanometer constant, a speed of rotation, and the deflection of a needle at the centre of the spinning coil. Corrections are necessary for the moment of the magnetic needle, the torsion of the supporting fibre, and the self inductance of the coil. It is not surprising, therefore, that the method has largely given way to the methods of Lorentz and Campbell which involve quantities more easy to determine and are applicable to the measurement of external resistances with potential-leads.

In 1880 Carey Foster* suggested an interesting null method involving the same principle as the British Association method. In this arrangement a steady current is passed through a tangent galvanometer of known principal constant and through the resistance to be measured. The e.m.f. across the potential leads of this resistance is balanced against that derived from a coil spinning in the earth's field, the balancing circuit being completed through commutators only over some 20° of arc. The middle of the period of contact was made to coincide with the instant when maximum e.m.f. was induced in the spinning coil, and the extreme variations of the e.m.f. during contact was 1.83 per cent. Though this null method has the great advantage of dispensing with the corrections necessary in the original method and is applicable to the measurement of an external resistance, the same fundamental quantities are involved. The correction necessary for the angle of contact, which must be measured, depends for its validity on a symmetrical setting of the commutators about the position of maximum e.m.f. which is difficult to locate with precision, and thermo-electric effects are likely to be troublesome.

The object of the present communication is to describe a preliminary investigation of a sensitive null method by which a resistance may be measured absolutely in terms of a mutual inductance and a frequency.

§ 2. THE THEORY OF THE METHOD

Simple form of the method. A coil spinning uniformly about a horizontal diameter lies between two larger fixed twin field coils having their planes horizontal and so separated in the theoretically simplest type of experiment that the mutual inductance M between the rotating and fixed coils in series and conjunction is, over a range of some 10° of arc on either side of zero, very accurately proportional to the angle θ of displacement from the position of zero mutual inductance. The axis of rotation lies in the magnetic meridian and the earth's vertical flux through the rotating coil is neutralized by a small current passing through large compensating coils in the Helmholtz position.

A steady current C of about one ampere is passed through the twin fixed coils and through a variable manganin resistance which is adjusted until the e.m.f. across its potential-leads is balanced by that across the commutating sectors of the rotating coil. These make contact through fixed brushes with a galvanometer over some 20° of arc, during which the e.m.f. arising from the uniform spin is constant.

* *B.A. Reports*, p. 426 (1881).

M Since the mutual inductance M at any position θ over the range of contact is given by the linear relationship

$$M = K\theta \quad \dots\dots(1),$$

K where K is constant, we have for the flux F through the spinning coil at any position θ

$$F = CM = CK\theta \quad \dots\dots(2),$$

C and for the numerical value of the e.m.f.

$$E, t \quad E = dF/dt = CK\omega \quad \dots\dots(3),$$

ω where ω is the constant angular velocity of rotation. Whence, equating this to the
 R e.m.f. CR across the balancing resistance R , we have

$$R = K\omega = 2\pi nK \quad \dots\dots(4),$$

n where n is the number of revolutions per second.

In view of the remarkable accuracy of the linear law connecting M and θ over considerable range the constant K can be determined with precision, and a method of doing this is described below which not only avoids the direct measurement of angles but at the same time eliminates errors in the stud-calibration of the mutual inductometer used. Further, on account of the linear law which gives rise to a uniform e.m.f. the exact angle of contact is not required and, although symmetry of contact is readily obtainable by means of inductance measurements, slight departure from symmetry is unimportant. Thermoelectric effects are rendered insignificant by adjusting the earth-coil current for zero galvanometer deflection when the current C is broken and the resistance R is connected across the contacts of the rotating coil spinning at the frequency n . On application of the current C relatively large opposing e.m.f.'s. come into action, giving high sensitivity, and on reversal of C throughout the balance is preserved.

Extension of the method to large angles of contact. So far we have supposed that the twin field coils are so separated that the linear law $M = K\theta$ is rigorous and that an unvarying e.m.f. $CK\omega$ is drawn off the rotating coil over an angle limited to some 20° of arc. Let us now consider a more general case applicable when the commutator sectors are enlarged to some 50° of arc.

M' Let M' be the change in the mutual inductance between the rotating coil and
 θ' the twin fixed coils over the angle of contact θ' of the sectors with the brushes in
 M'' the region of the first conjugate position, and let M'' be the change in mutual in-
 θ'' ductance over the corresponding angle of contact θ'' around the second conjugate
position. Then the flux changes CM' , CM'' occur in times θ'/ω , θ''/ω respectively. Thus the average e.m.f. during the contacts of one revolution is $C\omega (M'/\theta' + M''/\theta'')/2$; and if this is balanced on a ballistic galvanometer or fluxmeter by the opposing e.m.f. CR , drawn off the adjustable resistance R , we have:

$$R = \omega \frac{1}{2} \left(\frac{M'}{\theta'} + \frac{M''}{\theta''} \right) = \omega K' \quad \dots\dots(5).$$

Now while in general it would not be possible to measure precisely the sweeps of inductance M' and M'' over precise sector angles θ' and θ'' , it is easily possible

to measure with great precision the ratios M'/θ' and M''/θ'' provided that M/θ is rendered constant in the neighbourhoods of the sector-extremities. This, it has been found, can be readily accomplished by bringing the twin field coils rather closer together than when they are set to yield the linear law, so that their mutual inductance for a range of some 35° of arc on both sides of the zero obeys a law of the form

$$M/\theta = K + A\theta^2 - B\theta^4 \quad \dots\dots(6).$$

It will be seen below that the coils were so separated that

$$M/\theta = 124.932 + 3.665 \times 10^{-4}\theta^2 - 3.05 \times 10^{-7}\theta^4 \quad \dots\dots(7),$$

where M is in nominal microhenries and θ is in degrees of arc. This gives a maximum value for M/θ of $125.042 \mu\text{H. per degree}$ at $24^\circ.5$. The value 125.040 corresponds to both $22^\circ.8$ and $26^\circ.1$, so that over this range of $3^\circ.3$ of arc M/θ may be regarded as constant and equal to $dM/d\theta$.

By making one sector larger than the other so that the range of contact is determined by the smaller sector, θ' and θ'' were made equal at a value of about $46^\circ.8$. M'/θ' and M''/θ'' , rendered almost equal by symmetrical setting, were measured by observations of M and θ in the neighbourhoods of the sector edges, readings being taken when the sectors were (a) just on and (b) just off the contact brushes, the differences in angle between the on and the off readings being narrowed down to about $0^\circ.1$. Under such circumstances, the constant K' can be measured with high precision.

It should be observed that although in this method of working with a large angle of contact the e.m.f. drawn off the rotating coil is not quite constant, the extreme variation of e.m.f. is only 0.16 per cent over a contact of 47° as against 1.83 per cent over 22° of arc in Carey Foster's method. Moreover in this method the e.m.f. at the point of leaving the sectors is equal to the average e.m.f. over the whole contact.

Design and arrangement of coils. The design and arrangement of coils which provide the simple inductance laws stated above are based on the following theory. If two concentric circles have the ratio of their radii α/a equal to 0.506078^* the mutual inductance between them is so accurately proportional to the angle of displacement from the conjugate positions that the rising deviation from a straight-line law amounts to only 4.2 parts in a million at 7° of displacement and 18 parts in a million at 10° of displacement. With a slightly larger ratio of α/a , the deviation may be distributed so that over a range of 7° it never exceeds 0.75 parts in a million. On the other hand, if a coil of radius α lies between two circles of radius a separated by a distance $2x$, the ratio α/a must be increased to bring about a similar approach to linearity. Thus if $x/a = 0.19438$, the ratio α/a must be raised to 0.5461 to preserve the limiting linear law, the deviation at 7° being now 5.1 parts in a million. This deviation may likewise be distributed and diminished by slightly raising the ratio α/a .

The practical significance of this theory, coupled with the fact that the primary effect of multiplicity of layers is to alter slightly the effective radii of the coils, lies

* Nettleton and Llewellyn, *Proc. phys. Soc.* 44, 195 (1932).

in the result that if α/a lies between the limits $0.58 > \alpha/a > 0.52$, linearity within the accuracy of experimental measurements may be secured over some 12° with multiple-layered coils by merely adjusting the distance of separation between the larger twin coils. Such twin coils may at any time be joined in opposition, thus enabling the smaller coil to be set symmetrically.

In general we may express the mutual inductance between twin coils and a smaller coil displaced from the conjugate positions by an angle θ by a series of the type

$$M = K\theta + A\theta^3 + B\theta^5 + \dots \dots \dots (8),$$

where the constants K, A, B etc. depend on α, a, x and the number of turns, and may be evaluated with the aid of Legendre functions, though the process is laborious.

If α/a lies between the limits given, the coefficient A may be rendered zero by adjusting the separation $2x$. The succeeding coefficients are then small and negative, and a limiting linear law with accuracy of the order already stated results. If now the separation $2x$ is reduced, A assumes a small positive value and we have in practice with high accuracy over 30° of arc

$$M = K\theta + A\theta^3 - B\theta^5 \dots \dots (9),$$

which gives a maximum value of $M/\theta = K + A^2/4B$ at positions given by $\theta^2 = A/2B$. These positions are those proposed for the sector edges and define the ideal angle of contact for the new and closer distance of separation.

The e.m.f. on uniform rotation is everywhere proportional to $dM/d\theta$. Thus the minimum e.m.f. over the sector contacts is represented by K when $\theta = 0$, the maximum e.m.f. by $K + 9A^2/20B$ when $\theta^2 = 3A/10B$, and the average e.m.f. by $K + A^2/4B$ (which is the actual e.m.f. when $\theta^2 = A/10B$ or at the sector edges) where $\theta^2 = A/2B$.

Measurement of angle by a mutual inductance method. This method of measuring an angle of displacement is particularly suitable for the present purpose and is based upon the fact that the mutual inductance between a small solenoid of designed dimensions and two large twin coils between which it rotates can be rendered with great accuracy proportional to the sine of the angle of displacement from the conjugate positions.

The mutual inductance between twin circles A and B , figure 1, and a single layered solenoid C of radius α and length $2L$, rotated from the conjugate position by an angle θ , is given by the expression

$$M_\theta = Gq \sum_{n=1}^{n=\infty} \frac{2}{n(n+1)(n+2)} \left(\frac{s}{r}\right)^{n-1} P_n'(\cos \psi) \frac{P_{n+1}(\cos \phi)}{\cos \phi} P_n(\sin \theta) \dots \dots (10),$$

P_n, P_n'
 G
 q, s, r, ϕ, ψ

where P_n is the Legendre function of the first kind, of order n ; P_n' its differential coefficient; n an odd positive integer, G the galvanometer constant of A and B together at the origin of symmetry; q the total area of C ; and s, r, ϕ , and ψ are sufficiently defined by the figure.

If the angle solenoid C is of small radius, this series converges rapidly and only

the first three terms are of importance. They lead to the expression

$$M_{\theta}/Gq = \sin \theta + K_1 P_3(\sin \theta) + K_2 P_5(\sin \theta) \dots\dots(11),$$

where

$$K_1 = \frac{3 \cdot 4}{2} \cdot \frac{1}{r^4} \left[x^2 - \frac{a^2}{4} \right] \left[\frac{L^2}{3} - \frac{\alpha^2}{4} \right]$$

and

$$K_2 = \frac{5 \cdot 6}{2} \cdot \frac{1}{r^8} \left[x^4 - \frac{3}{2} x^2 a^2 + \frac{a^4}{8} \right] \left[\frac{L^4}{5} - \frac{L^2 \alpha^2}{2} + \frac{\alpha^4}{8} \right],$$

or when the result is expressed in powers of $\sin \theta$

$$M_{\theta}/Gq = \sin \theta [1 - 3K_1/2 + 15K_2/8] + \sin^3 \theta [5K_1/2 - 35K_2/4] + \sin^5 \theta [63K_2/8] \dots\dots(12).$$

If α/a is less than $\frac{1}{7}$, and K_1 is rendered sensibly zero by making $L^2/3$ equal to $\alpha^2/4$ or x^2 equal to $a^2/4$, K_2 also is always small and a close approach to the sine law results. If the coils are multiple-layered to the extent used in this research, this effect may be treated by a method due to Maxwell* and shown to be negligibly small.

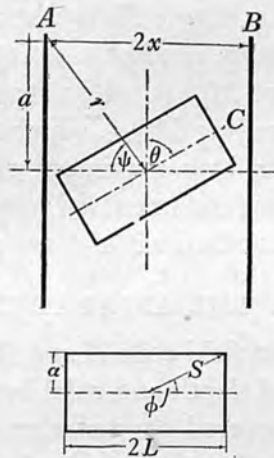


Figure 1.

In the angle solenoid used in our experiments $2L = 3.78$ cm. and $\alpha = 2.18$ cm. as found with the aid of a standard solenoid. The mean radius a of the twin coils was 16.41 cm. and in the least favorable position used, namely when the coils were closest and $\psi = 78^\circ$, these figures give $K_1 = -1.09 \times 10^{-5}$ and $K_2 = -3.20 \times 10^{-5}$. Whence if $M_{\max.}$ denotes the value of M when $\theta = 90^\circ$

$M_{\max.}$

$$\sin \theta = \frac{M_{\theta}}{M_{\max.}} [1 + 7 \times 10^{-7} - 2.5 \times 10^{-4} (\sin^2 \theta - \sin^4 \theta)] \dots\dots(13)$$

and the sine law is very accurate, but it should be observed that the value of K_1 is necessarily uncertain as errors in the measurements of L and α have large effects on its calculated value. Some experimental tests upon this angle coil have already been described by Llewellyn†, but in view of the special advantage of the method for the present purpose and its sensitivity to two or three seconds of arc over considerable

* Proc. phys. Soc. 42, 507 (1930).

† Thesis, Ph.D. degree, University of London.

range, further improvements in the method and in the tests of its accuracy are contemplated.

Method of eliminating local inductometer errors. Essentially the methods here described with either large or small angles of contact require the determination of a constant M/θ , the angle θ being measured from the relationship $\sin \theta = M_\theta/M_{\max}$. All mutual inductances are measured on a Campbell inductometer. This inductometer is first carefully calibrated so that all the stud readings are known in terms of 100 divisions of the scale, so that inductances can be read in nominal microhenries, subject to calibration errors. Further, the rotating coil is wound with such a number of turns that its mutual inductance with the twin field coils is so close to the value of the mutual inductance between the angle coil and the same field coils that up to at least 25° of arc the readings are within 100 microhenries; thus the same thousands and hundreds studs are in use for both readings. With the widening gap between the radian law and the sine law, the difference of reading at 40° of arc is still only some $300 \mu\text{H}$.

Under such circumstances it is easy to show that any errors of the order possible in the calibration are rendered quite negligible as regards the determination of the ratio M/M_θ , and the accuracy of M/θ is solely dependent on the accuracy of the inductometer at the angle-coil reading M_{\max} , which in our case was some $7000 \mu\text{H}$.

In future work we shall aim at a maximum angle-coil reading of just over $10,000 \mu\text{H}$, which will enable the fundamental dimensional length measurement to be checked by a 10 millihenry standard without any resort to the calibration curve.

§ 3. THE APPARATUS

The formers of the fixed twin field coils *A* and *B*, figure 2, were constructed of dexionite. The channels, of radial depth and axial breadth 3.6 cm., were each filled with 504 turns of double-silk-covered copper wire, s.w.g. 16, the mean diameter of winding being 32.8 cm. The coils, separated by distance pieces *D*, *D*, were locked together and firmly supported.

The rotating coil *C* consisted of 80 turns of double-silk-covered copper wire, s.w.g. 26, wound on a solid mahogany former, the channel having axial breadth 1 cm. and radial depth 0.3 cm. The mean diameter of winding was about 19.0 cm. A hole of diameter 5 cm. through the centre of the former served for the insertion of the angle solenoid when required. The hole was lengthened by attaching to each face of the former wooden rings, one of which is seen at *E*; to the ends of these rings were fastened small brass brackets with screw adjustments enabling the angle coil to be set centrally and secured in position.

The portions of the brass shaft to which the rotating coil *C* was attached terminated in pieces of channel brass *F*, *F* which held the coil by brass bolts. Some play within the channel pieces, which allowed room for packing, enabled the coil to be set symmetrically about the axis of rotation before the bolts were screwed up tightly. The brass shaft was of external diameter 1.27 cm. and had a central hole of bore 0.5 cm. which permitted bell flex leads from the rotating coil to be led through the portion *G* of the shaft to terminals *t*, *t* on the sectors of the commutator *M*.

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The commutator M consisted of a cylindrical ebonite piece some 6.5 cm. in diameter and 3.3 cm. long, through the centre of which passed stout brass tubing which enabled it to be slipped over the shaft and fixed thereon by heavy screws. The brass sectors let into the ebonite were diametrically opposite. In the particular commutator shown inset in figure 2 on a larger scale, one sector S consisted of a brass piece tapering along its length from 44° to 51° of arc while the other sector was parallel along its length and of width equivalent to 60° of arc. The framework N carried spring brass brushes adjustable in width and height and provided with terminals. The fly-wheel L , the stroboscopic disc R , the flexible unions U and the well-separated motor T are seen in the diagram.

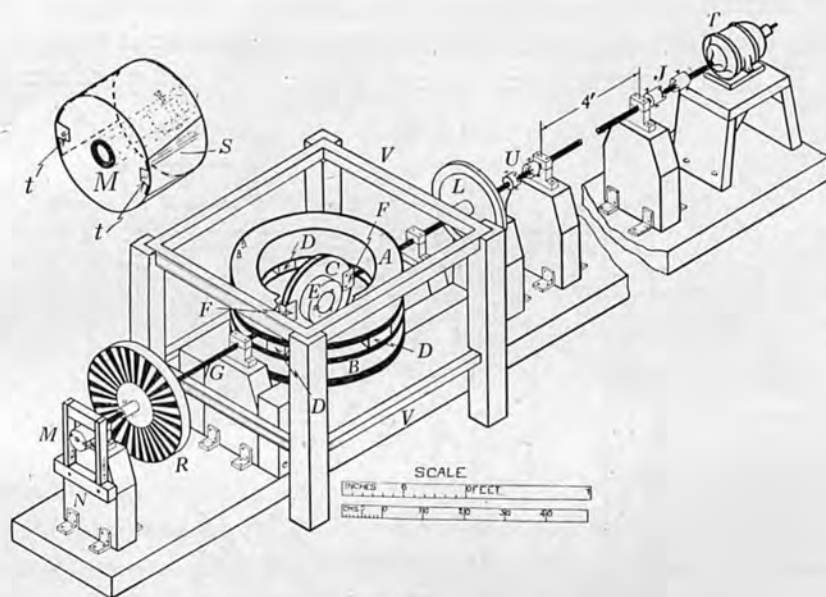


Figure 2.

The large horizontal coils V, V of sides 49×55 cm. are placed in the equivalent Helmholtz position and serve for the neutralization of the earth's vertical flux through the rotating coil when fed by a small current derived from accumulators.

The main electrical circuit will be readily understood from figure 3 with little description. The current of about 1 A., derived from accumulators, can be sent in either direction through the twin coils A and B , the quadrant key Q_3 serving to join them either in conjunction or in opposition but always in series. The same current passes through standardized resistances of $0.20029 \Omega.$ and $0.50006 \Omega.$ and through the variable resistance R to be adjusted and measured, all of which can be connected by potential leads with a good thermoelectric potentiometer which reads from 0 to 90 millivolts so that the value of R can be checked by comparison within a few minutes of measuring its resistance absolutely. The resistance R consisted of a box of manganin resistances of nominal values varying from $0.005 \Omega.$ to $2.0 \Omega.$ in series with a short semi-circular copper wire, of s.w.g. 18 provided with a movable potential contact. The e.m.f. across R can be adjusted to neutralize that across the

brushes b of the uniformly spinning coil, balance being observed on a galvanometer or fluxmeter provided with a tapping key.

The circuit also readily permits of the measurement of the mutual inductance between the rotating coil, when stationary, and the twin field coils as well as of the measurement of the angle between them. For this purpose the primary of a Campbell mutual inductometer is switched in at the quadrant key Q_1 to be in series with the fixed coils. The rotating coil through the sector terminals t, t is thrown into series with the secondary of the mutual inductometer at the quadrant key Q_2 and the galvanometer is included in this loop and detached from R by rocking over the mercury switch D . Alternatively, the angle coil may be thrown into the secondary circuit at H instead of the rotating coil and mutual inductance, and hence the angle between it and the twin coils can be measured. Balance is easily obtained to within $0.1 \mu\text{H.}$, which represents in the case of the angle coil $3''$ of arc.

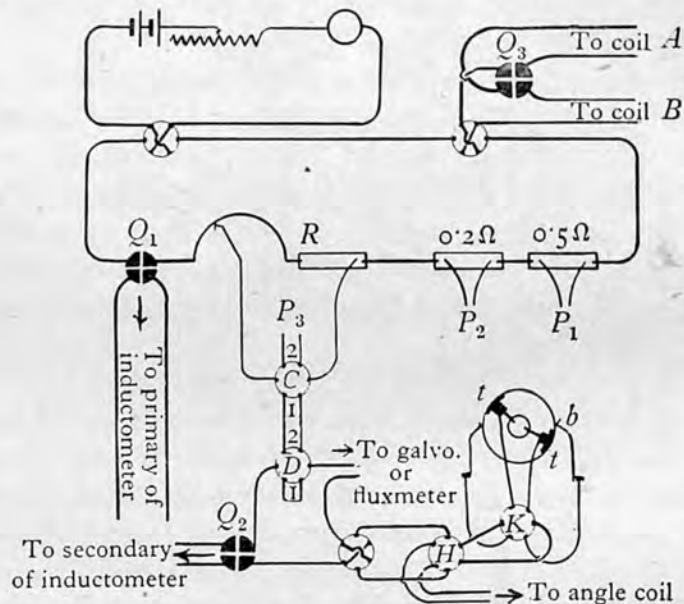


Figure 3.

The most constant speed of revolution was obtained by using a synchronized television motor, the cogged wheel of which had thirty narrow teeth separated by gaps four times the width of a tooth. The synchronizing impulse is fed to coils actuating an electromagnet pulling upon the teeth of the cogged wheel. The impulses used were derived from valve-maintained König tuning-forks which had been calibrated against one another by the method of visible beating*. In figure 4 the complete valve circuit is shown for maintaining the fork, synchronizing the motor and supplying a neon lamp with the tuning-fork frequency for viewing the stroboscopic disc R , figure 2, now provided with thirty lines. Because the television motor was rather weak in power, it was somewhat overrun and required careful rheostat adjustment to maintain synchronization for brief periods, but perfect

* Nettleton and Balls, *Proc. phys. Soc.* 45, 545 (1933).

balances were obtained on the galvanometer at three different speeds, viz. 8·533, 10·667 and 12·800 revolutions per second corresponding respectively to forks of frequencies 256, 320, and 384 vibrations per second.

For other speeds a stronger motor was used in conjunction with other stroboscopic disc rulings, viewed by means of the fork-controlled neon lamp, and the average speed was maintained as constant as possible with rheostats and hand-friction control. Though the imperfections of this control were manifested on a galvanometer by the oscillations of the spot of light, excellent balance was obtained by means of the following artifice. A Grassot fluxmeter of the silk-fibre-suspension

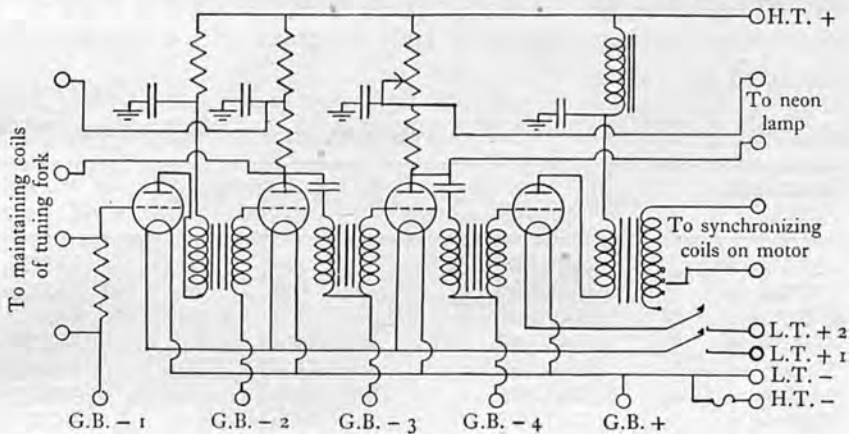


Figure 4.

type* shunted by some 80 Ω . was substituted by a rock-over switch for the galvanometer, and the tapping-key was depressed while the average known speed of revolution was maintained. Any advance of a stroboscopically viewed line was immediately rectified by increasing the friction and enforcing its return. Departure from balance was then manifested by the growing drift of the fluxmeter pointer in one or the other direction, and was rectified by adjustment of R until, after a considerable run, the pointer maintained its zero value. This method was proved to be satisfactory by deliberately allowing lines to escape and return in a time small compared with the period of the fluxmeter. Moreover the sensibility was good, the balance of R being sharp on the slide wire, while even small departures from the correct current in the earth coils used for neutralizing the earth's vertical component could readily be detected by this fluxmeter method.

§ 4. EXPERIMENTAL TESTS

Attainment of the linear law $M = K\theta$. The distance pieces between the twin field coils A and B were adjusted by trial, a few readings of the type given in table 1 sufficing until close approximation to the linear law was attained. Symmetry of setting was acquired by throwing A and B into opposition and moving them together until the mutual inductance between them and the rotating coil was as closely

* The recent control-less jewel-pivoted type is less satisfactory owing to solid friction.

as possible zero in all positions. The angle coil likewise was set symmetrically by opposition tests, and its positions of zero mutual inductance with the fixed coils in conjunction were made to agree very closely with the corresponding conjugate positions of the rotating coil. Readings of the mutual inductance M between the rotating coil and the field coils, and the readings of the corresponding mutual inductance M_θ between the angle coil and the field coils, were then taken over all four quadrants at closely corresponding positions. The maximum angle-coil inductance was observed also and the angle θ in seconds of arc was then evaluated from the sine law for all positions. The corresponding values of M and θ in all four quadrants were added together and the ratio M/θ was obtained from $\Sigma M/\Sigma \theta$. The values obtained for the final setting of the field coils for spins with a commutator of small angle are recorded in table 1.

Table 1. Values of M/θ for various angles θ . $x/a = 0.247$, $M_{\max} = 7085.17$

ΣM nominal ($\mu\text{H.}$)	$\Sigma \theta$ (seconds of arc)	θ , approximate (degrees)	M/θ nominal ($\mu\text{H./degree}$)
2560.6	74,726	5.2	123.359
3279.2	95,698	6.6	123.358
4033.9	117,726	8.2	123.355
4833.2	141,050	9.8	123.357
5649.5	164,875	11.45	123.355
6448.8	188,209	13.1	123.351
7320.8	213,675	14.8	123.341
8148.6	237,844	16.5	123.337
8947.9	261,176	18.1	123.336
9781.3	285,502	19.8	123.336
10659.4	311,146	21.6	123.331
12450.2	363,614	25.25	123.265
15043.9	439,795	30.5	123.144

A plot of M/θ against θ reveals peculiarities due to imperfect symmetry, but the approach to linearity is remarkable and the value of K for contacts of approximately known angle can be readily determined with precision.

With the coils in this position the semi-angle of the contacts used in the spin experiments was $11^\circ.6$, and accordingly the value of the constant K was taken as 123.355 nominal microhenries per degree. The correcting factor to convert to true microhenries was found with the aid of a standard inductance to be 1.00035, and hence for this arrangement of coils and contacts the experimental value of K is 123.398 $\mu\text{H.}$ per degree of arc.

Spin experiments with small angle of contact. The small commutator was readily adjusted systematically with the aid of the inductometer until the mutual inductance between the rotating coil and the fixed coils had approximately the same value, at the four boundaries of make and break between sectors and brushes, which are crossed during each revolution. The value in question was some 1425 $\mu\text{H.}$, corresponding to a semi-angle of $11^\circ.6$. The current in the earth coils was then adjusted, when the rotating coil was spinning with the main current broken, until no deflection was obtained on the galvanometer when closed through R and the commutator of the revolving coil. The speed of revolution was then maintained at a constant value,

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determined by the tuning-fork and the stroboscopic disc in use, while R was adjusted until accurate balance was obtained on the galvanometer or fluxmeter when a main current of the order of an ampere traversed the circuit in either direction. The balancing value of R was then immediately checked on the thermoelectric potentiometer by comparing it with one of the standard resistances—usually the $0.50006\text{-}\Omega$. resistance. The value of R in c.g.s. units is given by the expression

$$R = 36 \times 123.398 \times 10^4 \times n,$$

where n , the number of revolutions per second, is obtained by dividing the frequency of the fork by the number of lines on the stroboscopic disc. Table 2 gives the results obtained.

Table 2. Absolute measurement of resistance. $K = 123.398 \mu\text{H./degree}$

Frequency of fork (c./sec.)	Sectors on disc	n (rev./sec.)	$R \times 10^{-9}$ (c.g.s. units)	Resistance by comparison (Ω .)
256	30	8.533	0.37908	0.37889
320	30	10.667	0.47385	0.47365
384	30	12.800	0.56862	0.56842
384	36	10.667	0.47385	0.47386
512	36	14.222	0.63180	0.63193
384	40	9.600	0.42646	0.42647
512	40	12.800	0.56862	0.56879

The use of the synchronized television motor was limited of necessity to the first three experiments. Fluxmeter balances were taken in the other tests.

Adjustment of twin coils for use with large contacts. The twin coils A and B were now brought closer together and set symmetrically with respect to both the rotating coil and the angle coil by opposition inductometer tests as previously described. The variation of M and θ was then explored over all four quadrants by taking readings of M and M_θ in closely corresponding positions.

On adding the corresponding values of M and θ in the four quadrants we obtain from $\Sigma M/\Sigma \theta$ mean values of M/θ at various angles of displacement. These results are given in table 3 and are well represented by the relationship

$$M/\theta = 124.932 + 3.665 \times 10^{-4} \theta^2 - 3.05 \times 10^{-7} \theta^4$$

as will be seen from the fifth column, which gives the values calculated from this expression. The differences between the observed and calculated values of M/θ are given in the last column.

This setting of the coils with a maximum value for M/θ of 125.042 nominal $\mu\text{H.}$ per degree at $24^\circ.50$ appears ideal for use with a commutator having sector contacts of semi-angle between 23° and 26° . Accordingly the commutator M , figure 2, with smaller sector of angle tapering from 44° to 51° was adjusted on the shaft.

Determination of the constant K' for large angle of contact. The larger commutator was adjusted systematically and firmly fixed in position. The constant K' of equa-

Table 3. Values of M/θ at various angles θ . $x/a=0.212$, $M_{\max.}=7249.37$

M (nominal $\mu\text{H.}$)	$(\theta$ seconds of arc)	θ (degrees)	M/θ observed (nominal $\mu\text{H./degree}$)	M/θ calculated (nominal $\mu\text{H./degree}$)	Difference
3261.2	93,961	6.53	124.949	124.947	+0.002
4842.7	139,511	9.69	124.963	124.964	-0.001
7247.3	208,722	14.49	125.000	124.996	+0.004
9690.8	279,048	19.38	125.021	125.027	-0.006
10957.8	315,489	21.91	125.038	125.038	Zero
12255.2	352,824	24.50	125.045	125.042	+0.003
13625.1	392,298	27.24	125.033	125.036	-0.003
14927.4	429,860	29.85	125.014	125.016	-0.002
16834.9	485,007	33.68	124.958	124.955	+0.003

tion (5), equivalent to $(M'/\theta' + M''/\theta'')/2$, was found for it directly *in situ* by taking readings of M and θ at the sector edges over all four quadrants with the brushes (a) just on and (b) just off. The data are recorded in table 4.

Table 4. Direct determination of K' for contact of 47°

$$x/a=0.212, M_{\max.}=7249.37$$

Quadrant	Contact	M (nominal $\mu\text{H.}$)	M_θ (nominal $\mu\text{H.}$)	θ (seconds of arc)
1	On	2945.7	2890.0	84,579
1	Off	2968.1	2910.7	85,222
2	On	2924.6	2885.9	84,451
2	Off	2927.2	2888.1	84,520
3	On	2880.8	2827.8	82,653
3	Off	2888.3	2835.0	82,876
4	On	2937.2	2898.4	84,840
4	Off	2953.0	2912.5	85,278

The mean value of K' for the on and off positions, which owing to the closeness of the ratios involved is equivalent to $\Sigma M/\Sigma \theta$ for all readings, is 125.040_5 nominal $\mu\text{H. degree}$ or, when corrected by the factor 1.00035 to convert to true microhenries, $125.084 \mu\text{H. degree}$ for a semi-angle of contact of $23^\circ.4$.

Spin experiments with an angle of contact of 47° . A series of spin experiments was performed in the manner already described, and the balancing value of R in c.g.s. units was given by

$$R=36 \times 125.084 \times 10^4 \times n.$$

Table 5 shows the values of R so obtained and the corresponding potentiometer checks. The synchronized motor was used in the first three experiments.

Table 5. Absolute measurement of resistance. $K' = 125.084 \mu\text{H./degree}$

Frequency of fork (c./sec.)	Sectors on disc	n (rev./sec.)	$R \times 10^{-9}$ (c.g.s. units)	Resistance by comparison (Ω .)
256	30	8.533	0.38426	0.38398
320	30	10.667	0.48032	0.48018
384	30	12.800	0.57639	0.57614
256	24	10.667	0.48032	0.48008
320	24	13.333	0.60040	0.59982
256	36	7.111	0.32022	0.32047
256	36	14.222	0.64043	0.64036
512	36	14.222	0.64043	0.64026
320	36	8.889	0.40027	0.40002
384	36	10.667	0.48032	0.48033
512	40	12.800	0.57639	0.57627

§ 5. CONCLUDING REMARKS

As this method of measuring resistance absolutely is based on the constancy of the ratio M/θ in the neighbourhood of an angle θ which determines the approximate semi-angle of the contacts employed, the greater part of our time has been given to an investigation of the relationship between M and θ and to devising means of measuring the ratio accurately. Once the laws of inductance made use of have been established, a few readings suffice to determine the constant K for any commutator in position and to set its value between close limits. We favour the use of contacts of some 50° of arc owing to the great accuracy with which K can then be measured.

In view of the high sensitivity and the rapidity and ease with which balance can be obtained, we propose to undertake further work with coils of rather greater diameter (allowing more space between the fixed pair and the rotator) so wound that resistances of the order of an ohm may be conveniently measured. Further investigation is also being undertaken of the limits of accuracy of the sine law for the angle solenoid. It is proposed for the purpose of measuring θ to use two additional twin coils always in the Helmholtz position and possessing a maximum mutual inductance of some 10 mH. with the angle coil. The essential length and time measurements will then be dependent upon the value of a convenient standard inductance and upon a standard tuning-fork frequency.

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DISCUSSION

Dr D. OWEN. The method described is not really related to that of the B.A. revolving coil, as appears to be implied in the introduction to the paper. Its affinity is rather with the Lorentz method, as is clearly indicated by the formula $R = Mn$ which applies to both. The idea of using a momentary contact at the instant of maximum induced e.m.f. of the moving conductor was proposed by Lippmann, who used a coil rotating in the uniform field within a solenoid carrying the same current as that passing through the resistance to be measured. The advantage of the present method lies in the application of the investigation previously made by one of the authors of a type of variable mutual inductance in which, over a wide range of angular movement, the mutual inductance is very closely a linear function of the angle. This at once puts the determination of resistance on an altogether higher plane of accuracy. Compared with the Lorentz revolving-disc method, it is now possible to use a multilayered coil, and consequently the scale of size of the whole apparatus, or the speed of rotation of the coil, may be greatly reduced. These advantages may well engage the careful consideration of those concerned with future work on the determination of the ohm at the various national laboratories.

AUTHORS' reply. The method here described resembles the B.A., the Carey Foster and the Lippmann methods in that an alternating e.m.f. is generated in the revolving coil. It differs from them in that the e.m.f. is not sinusoidal and is very uniform over the contacts particularly at the contact edges. In Lorentz's method the magnetic lines of force are cut throughout a revolution at a constant rate and the e.m.f. is unvarying.

We thank Dr Owen for drawing our attention to Lippmann's method, the formula for which may be written $R = Mn$ if the contact is momentary. Our remarks in the paper on the correction for arc of contact in the Carey Foster method are equally applicable to the Lippmann method.